MATHEMATICS & STATISTICS/ A-LEVEL CORE FORMULA SHEET

University of Kent
School of Mathematics, Statistics and Actuarial Science

Algebra, Geometry and Functions

Quadratic Equation

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Logs and Exponentials

$y = b^x \Leftrightarrow x = \log_b(y), \quad$ for $b, y > 0$

$\log_b(p) + \log_b(q) = \log_b(pq)$

$\log_b(p) - \log_b(q) = \log_b(p/q)$

$\log_b(p^k) = k \log_b(p)$

$b^{\log_b(x)} = x, \quad \log_b(b^x) = x$

$\ln(x) = \log_e(x), \quad e^{\ln(x)} = \ln(e^x) = x$

Odd and Even Functions

$f(-x) = -f(x) \quad \Leftrightarrow \quad f(x)$ is odd

$f(-x) = f(x) \quad \Leftrightarrow \quad f(x)$ is even

Straight Lines

Line with gradient m through (x_1, y_1) has equation $(y - y_1) = m(x - x_1)$

Lines with gradients m_1 and m_2 are perpendicular if $m_1 m_2 = -1$

Circles

Circle with centre $C(a, b)$ and radius r has equation $(x - a)^2 + (y - b)^2 = r^2$

Sequences and Series

Arithmetic Sequences

For an arithmetic sequence with first term a, last term l, and common difference d:

nth term $= u_n = a + (n-1)d$

Sum to n terms $= S_n = \frac{1}{2}n(2a + (n-1)d) = \frac{1}{2}n(a + l)$

Geometric Sequences

For a geometric sequence with first term a and common ratio r:

nth term $= u_n = ar^{n-1}$

Sum to n terms $= S_n = \dfrac{a(1 - r^n)}{1 - r}$, for $r \neq 1$

Sum to infinity $= S_\infty = \dfrac{a}{1 - r}$, for $|r| < 1$

Binomial Series

Binomial coefficient is ${}^nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

For $n \in \mathbb{N}$,

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n$$

For $n \in \mathbb{R}$ and $|b| < |a|$,

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \ldots + \frac{n(n-1)\cdots(n-r+1)}{r!}a^{n-r}b^r + \ldots$$

Numerical Methods

Trapezium Rule

$$\int_a^b y\,dx \approx \frac{h}{2}(y_0 + y_n) + h(y_1 + y_2 + \ldots + y_{n-1}),$$

where $h = \dfrac{b-a}{n}$, $x_k = a + kh$, $x_n = b$, and $y_k = f(x_k)$

Newton-Raphson Iteration

To solve $f(x) = 0$, use $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Trigonometry

Radians

2π radians $= 360°$

For a sector of angle θ radians in a circle of radius r:

Arc length $= s = \theta r$

Sector area $= A = \frac{1}{2}\theta r^2$

Triangles

Sine rule: $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos(A)$

Area $= \frac{1}{2}ab\sin(C)$

Trig Identities

Pythagorean Identities:

$\sin^2(\theta) + \cos^2(\theta) \equiv 1$

$\tan^2(\theta) + 1 \equiv \sec^2(\theta)$

$1 + \cot^2(\theta) \equiv \operatorname{cosec}^2(\theta)$

Sum/Difference Identities

$\sin(a \pm b) = \sin(a)\cos(b) \pm \sin(b)\cos(a)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

$\tan(a \pm b) = \dfrac{\tan(a) \pm \tan(b)}{1 \mp \tan(a)\tan(b)}$

Double Angle Formulae

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

$\tan(2\theta) = \dfrac{2\tan(\theta)}{1 - 2\tan(\theta)}$

Small Angle Approximations

When θ (in radians) is small: $\sin(\theta) \approx \theta, \cos(\theta) \approx 1 - \frac{1}{2}\theta^2, \tan(\theta) \approx \theta$

Calculus

Table of Derivatives

Function	Derivative
x^n	nx^{n-1}
a^{kx}	$k\ln(a)a^{kx}$
e^{kx}	ke^{kx}
$\ln(kx)$	$\frac{1}{x}$
$\sin(kx)$	$k\cos(kx)$
$\cos(kx)$	$-k\sin(kx)$
$\tan(kx)$	$k\sec^2(kx)$
$\sec(kx)$	$k\sec(kx)\tan(kx)$
$\text{cosec}(kx)$	$-k\,\text{cosec}(kx)\cot(kx)$
$\cot(kx)$	$-k\,\text{cosec}^2(kx)$

Rules

Differentiation from first principles:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Chain rule:

$$y = f(g(x)) \quad\Rightarrow\quad \frac{dy}{dx} = f'(g(x))\times g'(x)$$

$$\text{or}\quad y = f(g(x)) = f(u) \quad\Rightarrow\quad \frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$$

Product rule:

$$y = uv \quad\Rightarrow\quad \frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

Quotient rule:

$$y = \frac{u}{v} \quad\Rightarrow\quad \frac{dy}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

Inverse rule:

$$1\Big/\frac{dy}{dx} = \frac{dx}{dy}$$

Parametric rule: For a curve given by $x = f_1(t)$ and $y = f_2(t)$,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Table of Integrals

Function	Integral (remember to add $+c$!)
x^n	$\frac{x^{n+1}}{n+1}\quad (n\neq -1)$
$x^{-1} = \frac{1}{x}$	$\ln(x)$
e^{kx}	$\frac{1}{k}e^{kx}$
$\sin(kx)$	$-\frac{1}{k}\cos(kx)$
$\cos(kx)$	$\frac{1}{k}\sin(kx)$
$\tan(kx)$	$\frac{1}{k}\ln\lvert\sec(kx)\rvert$
$\sec^2(kx)$	$\frac{1}{k}\tan(kx)$
$\text{cosec}^2(kx)$	$-\frac{1}{k}\cot(kx)$
$\cot(kx)$	$\frac{1}{k}\ln\lvert\sin(kx)\rvert$
$\frac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert$

Rules

Definite integration: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is the integral of $f(x)$

Integration by parts: $\int u\frac{dv}{dx}\,dx = uv - \int\frac{du}{dx}v\,dx$

Mechanics

Forces

Weight $= mg$

Friction: $F \leq \mu R$

Newton's 2nd law: $F = ma$

Kinematics

For 1D motion with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

For 1D motion with variable acceleration:

r (position), v, and a are all functions of t; the above *suvat* equations no longer apply, so use

$$v = \frac{dr}{dt},\quad a = \frac{dv}{dt},\qquad r = \int v\,dt,\quad v = \int a\,dt$$

Statistics

Probability

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A|B) = \frac{P(A\cap B)}{P(B)}$, for $P(B)\neq 0$

Summary Statistics

$$\text{Mean} = \bar{x} = \frac{\Sigma x_i}{n} = \frac{\Sigma f_i x_i}{\Sigma f_i}, \qquad \text{Variance} = \sigma^2$$

$$\text{Standard dev.} = \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - \bar{x}^2}$$

Outliers are any data outside of the interval $\bar{x}\pm 2\sigma$ or $Q_1 - 1.5\text{IQR}$ and $Q_3 + 1.5\text{IQR}$

Binomial Distribution

If $X\sim B(n,p)$, then $P(X=r) = {}^nC_r p^r(1-p)^{n-r}$

Mean of $X = np$, variance of $X = np(1-p)$

Normal Distribution

If $X\sim N(\mu,\sigma^2)$, then $Z = \frac{X-\mu}{\sigma}$ with $Z\sim N(0,1)$

Hypothesis test for the mean: if $X\sim N(\mu,\sigma^2)$, then

$$\bar{X}\sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad\text{and}\quad \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\sim N(0,1)$$

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