Social welfare under restricted risk classification

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Background

Adverse selection:

If insurers cannot charge **risk-differentiated** premiums, then:

- higher risks buy more insurance, lower risks buy less insurance,
- raising the **pooled** price of insurance,
- lowering the demand for insurance,

usually portrayed as a bad outcome, both for insurers and for society.

In practice:

Policymakers often see merit in restricting insurance risk classification

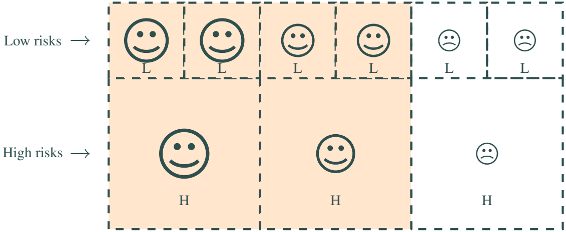
- EU ban on using gender in insurance underwriting.
- UK moratorium on the use of genetic test results in underwriting.

Question:

How can we reconcile theory with practice?

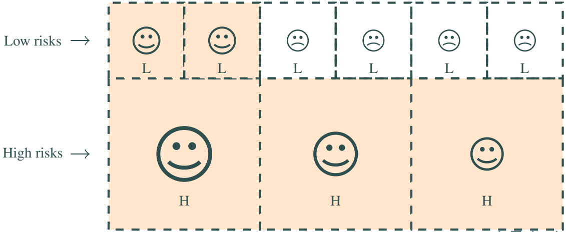
Motivation: Two risk-groups $\mu_L = 0.01$ and $\mu_H = 0.04$

Scenario 1: No adverse selection: Risk-differentiated premiums: $\pi_L = 0.01$ **and** $\pi_H = 0.04$



Utility increase:
66.2×10^{-4}

Scenario 2: Some adverse selection: Pooled premiums: $\pi_L = \pi_H = 0.028$



Utility increase:
71.2×10^{-4}

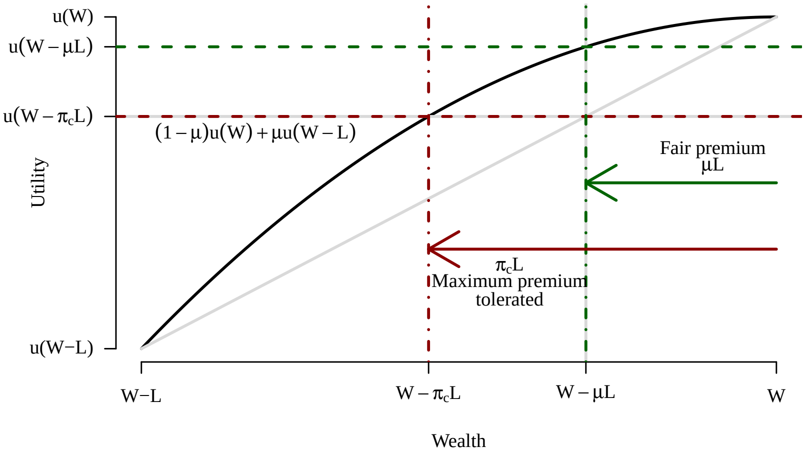
Why do people buy insurance?

Assumptions

Consider an individual with

- an initial wealth W,
- exposed to the risk of loss L,
- with probability μ,
- utility of wealth $u(w)$, with $u'(w) > 0$, and
- an opportunity to insure at premium rate π.

Utility of wealth and insurance purchasing decision



Demand for insurance

Normalization

As certainty equivalent is invariant to positive affine transformations, we assume $u(W) = 1$ and $u(W - L) = 0$ for all individuals.

Insurance purchasing decision:

Given a premium π, an individual will purchase insurance if:

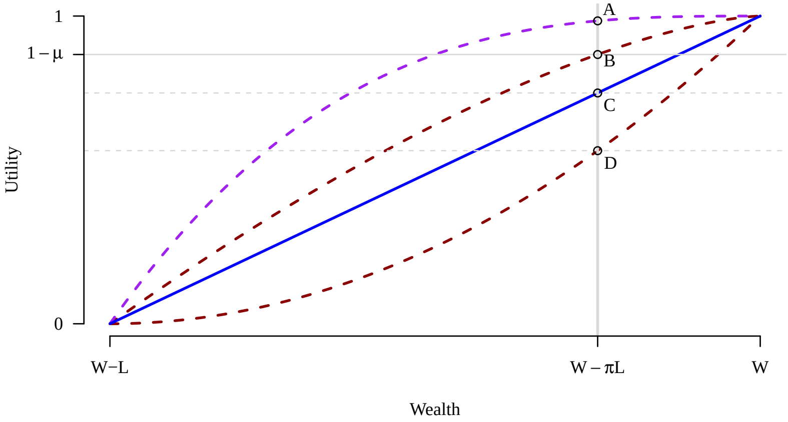
$$\underbrace{u(W - \pi\, L)}_{\text{Utility with insurance}} > \underbrace{(1 - \mu)\; u(W) + \mu\, u(W - L) = (1 - \mu)}_{\text{Utility without insurance}}.$$

Source of randomness in demand:

Utility of insurance of an individual chosen at random, $u(W - \pi\, L)$, is a random variable, U_I. Given a premium π, insurance demand, $d(\pi)$, is:

$$d(\pi) = \mathrm{P}\left[U_I > 1 - \mu\right].$$

Demand for insurance



Insurance risk classification

Risk-groups

Suppose a population can be divided into 2 risk-groups where:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: p_1, p_2;
- iso-elastic demand for a given premium, π:

$$d_i(\pi) = \tau_i \left(\frac{\mu_i}{\pi}\right)^{\lambda_i}, \quad i = 1, 2;$$

- demand elasticity: $\epsilon_i(\pi) = -\frac{\partial \log(d_i(\pi))}{\partial \log \pi} = \lambda_i, \quad$ for $i = 1, 2$;
- fair-premium demand: $\tau_i = d_i(\mu_i)$ for $i = 1, 2$;
- premiums offered: π_1, π_2.

Note: The framework can be generalised for $n > 2$ risk-groups.

Market equilibrium

For a randomly chosen individual, define:

$$Q = I\,[\,\text{Individual is insured}\,]\,;$$
$$X = I\,[\,\text{Individual incurs a loss}\,]\,;$$
$$\Pi = \text{Premium offered to the individual.}$$

Simplifying assumption

The potential loss amount L is same for all individuals.

Expected premium, claim and market equilibrium

Market equilibrium: $E[Q\Pi] = E[QX]$, where,

Expected premium: $E[Q\Pi] = p_1\, d_1(\pi_1)\, \pi_1 + p_2\, d_2(\pi_2)\, \pi_2,$

Expected claim: $E[QX] = p_1\, d_1(\pi_1)\, \mu_1 + p_2\, d_2(\pi_2)\, \mu_2.$

Risk-classification regimes

Risk-differentiated premiums: $\underline{\pi} = (\mu_1, \mu_2)$

- Equilibrium is achieved when $\pi_1 = \mu_1$ and $\pi_2 = \mu_2$.
- No losses for insurers.
- No (actuarial/economic) adverse selection.

Pooled premium: $\underline{\pi} = (\pi_0, \pi_0)$

If risk-classification is banned, insurers charge same premium π_0 to both risk-groups.

- Market equilibrium $\Rightarrow$ No losses for insurers! $\Rightarrow$ No (actuarial) adverse selection.
- Pooled premium is greater than average premium charged under full risk classification $\Rightarrow$ (Economic) adverse selection.
- Aggregate demand (cover) is lower than under full risk classification $\Rightarrow$ (Economic) adverse selection.

Social welfare

Definition (Social welfare)

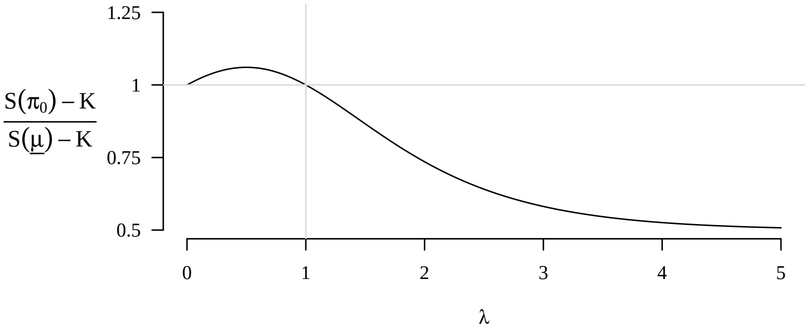
For any premium regime $\underline{\pi}$, social welfare is the expected utility for an individual selected at random from the population:

$$S(\underline{\pi}) = \mathrm{E}\Big[\underbrace{Q\,U_I}_{\text{Insured population}} + \underbrace{(1-Q)\,[(1-X)\,U_W + X\,U_{W-L}]}_{\text{Uninsured population}} \Big].$$

Without loss of generalisation , we can normalise utility function, so that: $U_W = 1$ and $U_{W-L} = 0$. Hence:

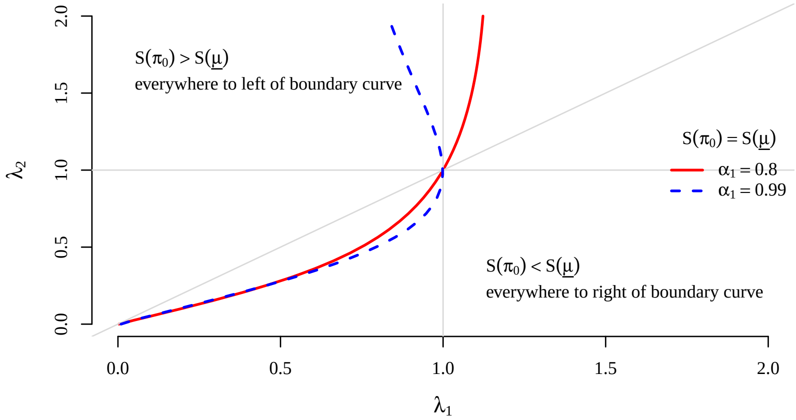
$$S(\underline{\pi}) = \mathrm{E}\,[\,Q\,U_I + (1-Q)\,(1-X)]$$

Iso-elastic demand with same demand elasticity

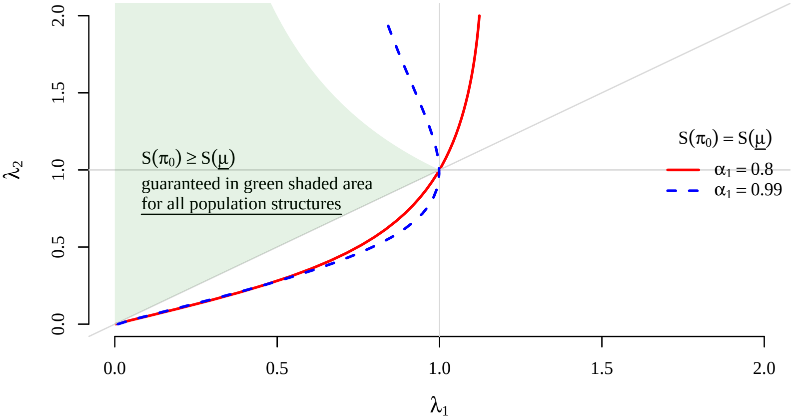


- $\lambda < 1 \Leftrightarrow S(\pi_0) > S(\underline{\mu}) \Rightarrow$ Risk pooling is *better* than full risk classification.
- $\lambda > 1 \Leftrightarrow S(\pi_0) < S(\underline{\mu}) \Rightarrow$ Risk pooling is *worse* than full risk classification.
- **Empirical evidence suggests $\lambda < 1$ in many insurance markets.**

Iso-elastic demand with different demand elasticities

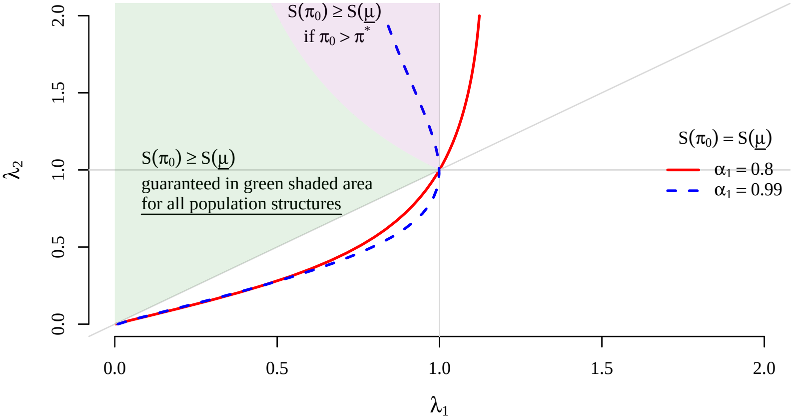


Iso-elastic demand with different demand elasticities



$$\lambda_1 \leq 1 \text{ and } \lambda_1 \leq \lambda_2 \leq \frac{1}{\lambda_1} \Rightarrow S(\pi_0) \geq S(\underline{\mu}).$$

Iso-elastic demand with different demand elasticities



$$\exists\, \pi^* \ni \lambda_1 \leq 1 \text{ and } \lambda_2 > \frac{1}{\lambda_1} \text{ and } \pi_0 \geq \pi^* \Rightarrow S(\pi_0) \geq S(\underline{\mu}).$$

Generalisations

The results can be generalised:

- For any number of risk-groups $n \geq 2$.
- For full take-up of insurance by the high risk-group.
- For general insurance demand function using arc elasticity of demand.

Conclusions

Adverse selection need not always be adverse.

Restricting risk classification increases social welfare if:

- $\lambda \leq 1$, when demand elasticity is the same for all risk-groups.
- $\lambda_1 \leq 1$ and $\lambda_1 \leq \lambda_2 \leq 1$, when demand elasticities are different.

Empirical evidence suggests $\lambda < 1$ **in many insurance markets.**

Reference

CHATTERJEE, I., MACDONALD, A.S., TAPADAR, P.& THOMAS, R.G. (2021). When is utilitarian welfare higher under insurance risk pooling?. *Insurance: Mathematics and Economics*, **101(B)**, 289–301.

https://blogs.kent.ac.uk/loss-coverage/