

# Multiple Equilibria, Adverse Selection and Loss Coverage in Insurance Markets

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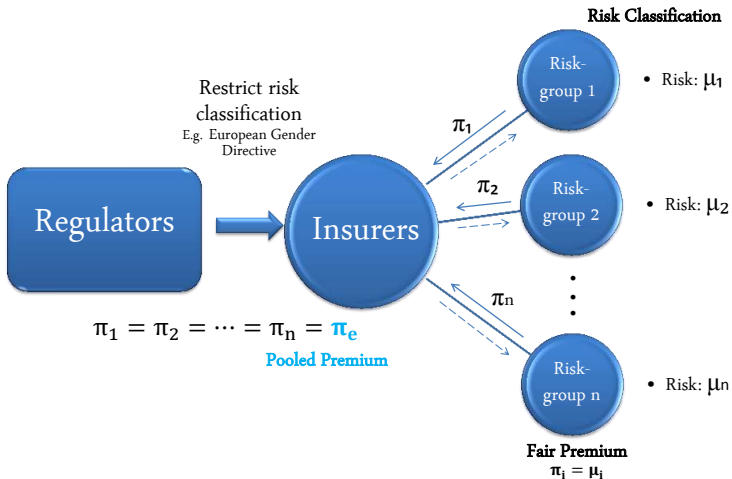
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# Background

## How insurance works and risk classification scheme



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## Definition

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- $\frac{\partial}{\partial \pi} d(\mu, \pi) < 0$ , which implies that demand falls as the premium rises.
- $\frac{\partial^2}{\partial \pi^2} d(\mu, \pi) > 0$ , which implies a decreasing rate of fall in demand as premium increases. I.e. individuals are risk averse.



# Demand Functions

## Iso-elastic demand

$$d(\mu, \pi) = \tau \left[ \frac{\pi}{\mu} \right]^{-\lambda}$$

$$\begin{aligned} \epsilon(\mu, \pi) &= -\frac{\pi}{d(\pi, \mu)} \frac{\partial}{\partial \pi} d(\mu, \pi) \\ &= \lambda \end{aligned}$$

## Negative-exponential demand

$$d(\mu, \pi) = \tau e^{(1 - \frac{\pi}{\mu})\lambda}$$

$$\epsilon(\mu, \pi) = \frac{\lambda}{\mu} \pi$$

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For two risk-groups,

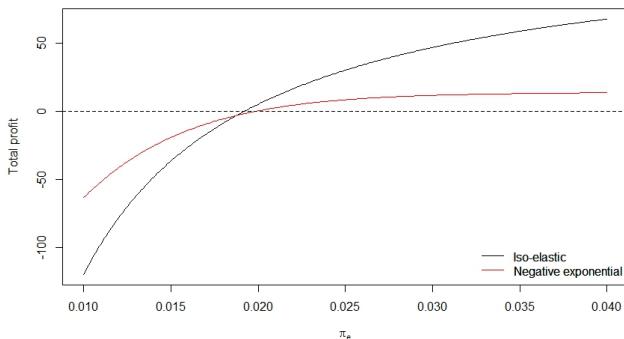
$$f(\pi_e) = d(\mu_1, \pi_e)p_1(\pi_e - \mu_1) + d(\mu_2, \pi_e)p_2(\pi_e - \mu_2). \quad (1)$$

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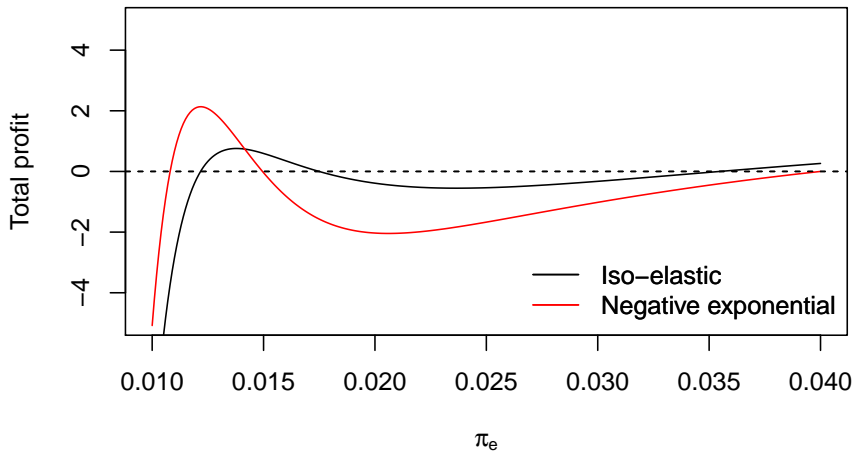
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## Theorem

Given  $(\mu_1, \mu_2), (\tau_1, \tau_2)$  and  $(\lambda_1, \lambda_2)$ , if  $c < c_1$  and  $\alpha(\pi_{01}) \leq \alpha \leq \alpha(\pi_{02})$ , there are **multiple equilibria**. Otherwise, there is a unique equilibrium premium.

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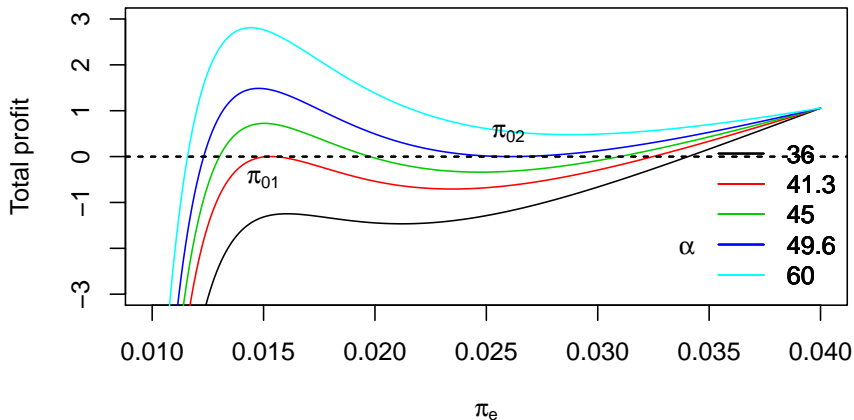
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For iso-elastic demand,  $c = \lambda_2 - \lambda_1, c_1 = -\frac{\sqrt{\mu_1} + \sqrt{\mu_2}}{\sqrt{\mu_2} - \sqrt{\mu_1}} < 0$ .

For negative-exponential demand,  $c = \frac{\lambda_2}{\mu_2} - \frac{\lambda_1}{\mu_1}, c_1 = -\frac{4}{\mu_2 - \mu_1} < 0$ .

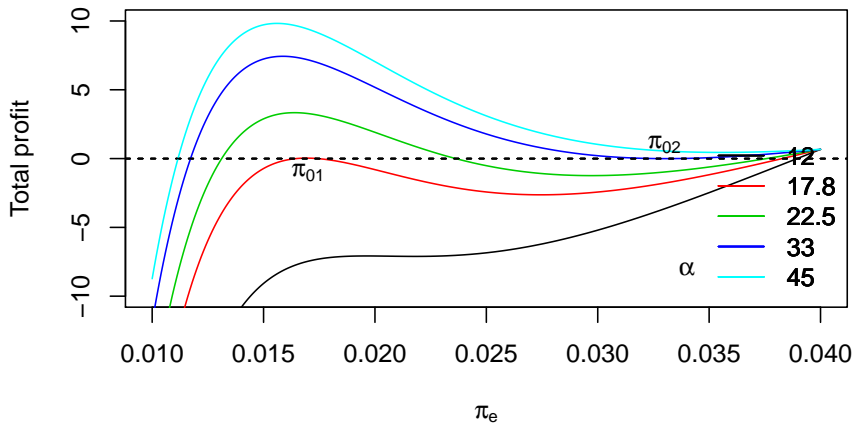
# Example

Iso-elastic demand



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# Adverse Selection

- $0, \pi_1, \pi_2, \pi_3, \pi_e, \dots, \pi_7, \pi_8, \dots, \pi_n, 1.$

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## Original definition

Purchasing decision is positively correlated with losses  
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- Empirical results are mixed and vary by market.



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Life Insurance	Cawley and Philipson (1999)	X
Auto Insurance	Chiappori and Salanie (2000) Cohen (2005)	X O
Annuity	Finkelstein and Poterba (2004)	X
Health Insurance	Cardon and hendel (2001)	X

# Adverse Selection

- Restricting risk classification -> Policy is over-subscribed by high risks **BAD?**

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$$S = \frac{E[QL]}{E[Q]E[L]}, \quad (2)$$

where

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$L$  : risk experience .

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- Restricting risk classification -> Policy is over-subscribed by high risks **BAD?**
- Model:

$$S = \frac{E[QL]}{E[Q]E[L]}, \quad (2)$$

where

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- A moderate degree of adverse selection can be PREFERABLE!**

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## Definition

$$\begin{aligned} \text{Loss Coverage} &= \frac{\text{insured expected losses}}{\text{population expected losses}} \\ \text{Loss Coverage Ratio} &= \frac{\text{loss coverage at a pooled premium } \pi_e}{\text{loss coverage at at fair premium } \pi_i} \\ &= \frac{\sum_{i=1}^n d(\mu_i, \pi_e) p_i \mu_i}{\sum_{i=1}^n d(\mu_i, \mu_i) p_i \mu_i} \end{aligned}$$

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 &= \frac{\sum_{i=1}^n d(\mu_i, \pi_e) p_i \mu_i}{\sum_{i=1}^n d(\mu_i, \mu_i) p_i \mu_i} > 1, \text{ **PREFERABLE!**}
 \end{aligned}$$

# Example

## Example

- A population of 1000
- Two risk groups
  - ▶ 200 high risks with risk 0.04
  - ▶ 800 low risks with risk 0.01
- No moral hazard

# Example

No restriction on risk classification

# Example

No restriction on risk classification

Table 1	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium ( <b>fair premium</b> )	0.01	0.04
Number insured	400	100
Insured expected losses	4	4
Loss Coverage	0.5	
Loss Coverage Ratio	1	

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**No adverse selection.**



# Example

## Restriction on risk classification-Case 1

# Example

## Restriction on risk classification-Case 1

Table 2	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium ( <b>pooled premium</b> $\pi_e$ )	0.02	
Number insured	300(400)	150(100)
Insured expected losses	3	6
Loss Coverage	0.5625	
Loss Coverage Ratio	<b>1.125 &gt; 1</b>	

# Example

## Restriction on risk classification-Case 1

Table 2	Low risk-group	High risk-group
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**Moderate adverse selection but preferable loss coverage.**

# Example

## Restriction on risk classification-Case 2

# Example

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Table 3	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium ( <b>pooled premium</b> $\pi_e$ )	0.02154	
Number insured	200(400)	125(100)
Insured expected losses	2	5
Loss Coverage	0.4375	
Loss Coverage Ratio	<b>0.875 &lt; 1</b>	

# Example

## Restriction on risk classification-Case 2

Table 3	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium ( <b>pooled premium</b> $\pi_e$ )	0.02154	
Number insured	200(400)	125(100)
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Loss Coverage	0.4375	
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**Severe adverse selection and worse loss coverage.**

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# Results

## Assumptions

- There are  $n$  risk-groups
- They have equal demand elasticities -> **Unique Equilibrium**

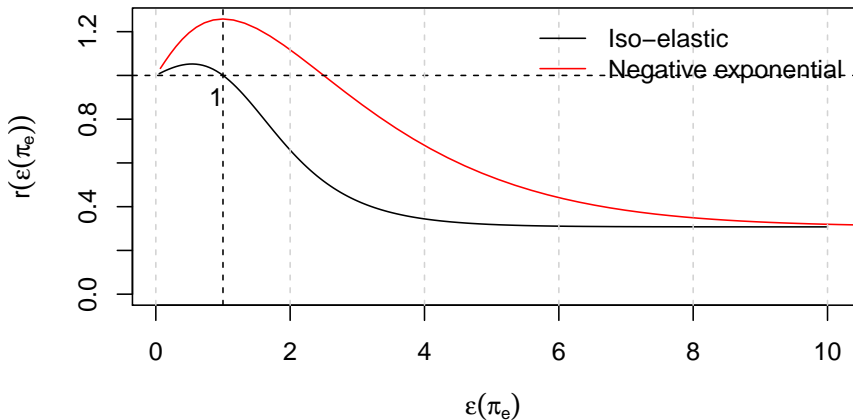
For  $i \neq j, i, j \in (1, n)$ ,

- ▶ Iso-elastic demand:  $\lambda_i = \lambda_j = \epsilon(\pi_e)$
- ▶ Negative-exponential demand:  $\frac{\lambda_i}{\mu_i} \pi_e = \frac{\lambda_j}{\mu_j} \pi_e = \epsilon(\pi_e)$



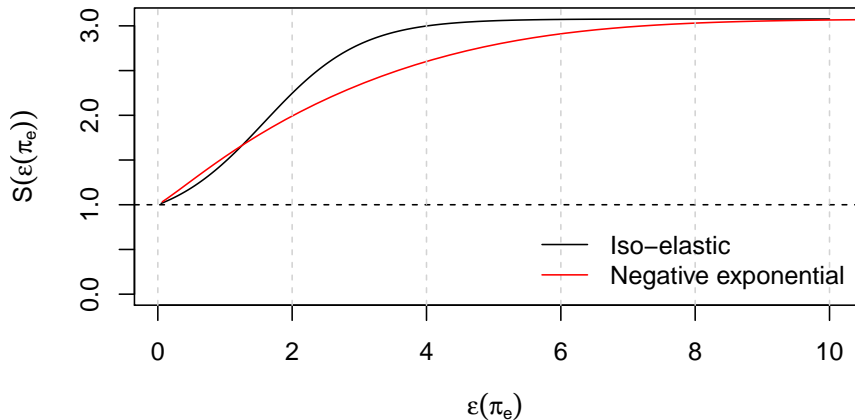
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- In a two-risk-group case, there are multiple **pooled premia**  $\pi_e$  if
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  - ▶ This result also holds when there are  $n$  risk-groups with risks  $\mu_1 < \mu_2 < \dots < \mu_n$ .
  - ▶ This result also holds when the risk  $\mu \sim \text{Beta}(\alpha, \beta)$ .

# Further Research

- Other/more general demand e.g.  $d(\mu, \pi) = \tau e^{1 - (\frac{\pi}{\mu})^\lambda}$ .
- Loose restriction on demand elasticities.
- Partial restriction on risk classification.

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# Questions?

Thank you!