

Adverse Selection, Loss Coverage and Equilibrium Premium in Insurance Markets

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Background

How insurance works and risk classification scheme

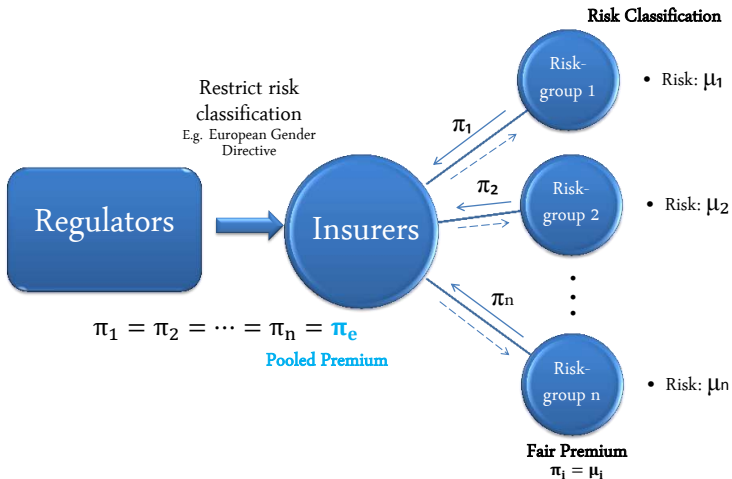


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Adverse Selection

- $0, \pi_1, \pi_2, \pi_3, \pi_e, \dots, \pi_7, \pi_8, \dots, \pi_n, 1.$

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Original definition

Purchasing decision is positively correlated with losses
-Chiappori and Salanie (2000) “Positive Correlation Test”

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- Empirical results are mixed and vary by market.

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- Empirical results are mixed and vary by market.

Life Insurance	Cawley and Philipson (1999)	X
Auto Insurance	Chiappori and Salanie (2000)	X
	Cohen (2005)	O
Annuity	Finkelstein and Poterba (2004)	O
Health Insurance	Cardon and Hendel (2001)	X

Adverse Selection

- Restricting risk classification \Rightarrow Policy is over-subscribed by high risks **BAD?**

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- **Good measurement?**
- Model:

$$S = \frac{E[QL]}{E[Q]E[L]} = \frac{\text{pooled premium } \pi_e}{\text{population-weighted fair premium}} \quad (1)$$

where

Q : quantity of insurance

L : risk experience .

Adverse Selection

- Restricting risk classification \Rightarrow Policy is over-subscribed by high risks **BAD?**
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Q : quantity of insurance

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- **$S > 1 \Rightarrow$ Adverse Selection.**

Example

Example

- A population of 1000
- Two risk groups
 - ▶ 200 high risks with risk 0.04
 - ▶ 800 low risks with risk 0.01
- No moral hazard

Example

No restriction on risk classification

Example

No restriction on risk classification

Table 1	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (fair premium)	0.01	0.04
Number insured	400	100
Adverse Selection	1	

Example

No restriction on risk classification

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No adverse selection.

Example

Restriction on risk classification-Case 1

Example

Restriction on risk classification-Case 1

Table 2	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (pooled premium π_e)	0.02	
Number insured	300(400)	150(100)
Adverse Selection	1.25 > 1	

Example

Restriction on risk classification-Case 1

Table 2	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (pooled premium π_e)	0.02	
Number insured	300(400)	150(100)
Adverse Selection	1.25 > 1	

Moderate adverse selection

Example

Restriction on risk classification-Case 2

Example

Restriction on risk classification-Case 2

Table 3	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (pooled premium π_e)	0.02154	
Number insured	200(400)	125(100)
Adverse Selection	1.3462 > 1	

Example

Restriction on risk classification-Case 2

Table 3	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (pooled premium π_e)	0.02154	
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Heavier adverse selection

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Heavier adverse selection

Adverse selection suggests pooling is always bad. But is it?

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Loss Coverage

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Definition

$$\text{Loss Coverage} = \frac{\text{insured expected losses}}{\text{population expected losses}}$$

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- Thomas (2008, 2009) “**Loss Coverage**”:

Definition

$$\begin{aligned} \text{Loss Coverage} &= \frac{\text{insured expected losses}}{\text{population expected losses}} \\ \text{Loss Coverage Ratio} &= \frac{\text{loss coverage at a pooled premium } \pi_e}{\text{loss coverage at at fair premium } \pi_i} \\ &> 1, \text{ **Favorable!**} \end{aligned}$$

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Table 1	Low risk-group	High risk-group
Population	800	200
Risk	0.01	0.04
Break-even premium (fair premium)	0.01	0.04
Number insured	400	100
Insured expected losses	4	4
Loss Coverage	0.5	
Loss Coverage Ratio	1	

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Number insured	300(400)	150(100)
Insured expected losses	3	6
Loss Coverage	0.5625	
Loss Coverage Ratio	1.125 > 1	

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Loss Coverage Ratio	1.125 > 1	

Moderate adverse selection but favorable loss coverage.

Example

Restriction on risk classification-Case 2

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Insured expected losses	2	5
Loss Coverage	0.4375	
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Heavier adverse selection and worse loss coverage.

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**Heavier adverse selection and worse loss coverage.
Loss Coverage might be a better measurement!**

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Demand Functions

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The demand function $d(\mu, \pi)$ is the demand of a single individual with risk μ , will buy insurance at premium π .

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- $\frac{\partial^2}{\partial \pi^2} d(\mu, \pi) > 0 \Rightarrow$ a decreasing rate of fall in demand as premium increases.

Definition

The demand elasticity $\epsilon(\mu, \pi) = -\frac{\partial d(\mu, \pi)}{d(\mu, \pi)} / \frac{\partial \pi}{\pi}$ i.e. sensitivity of demand to premium changes.

Demand Functions

Iso-elastic demand

$$d(\mu, \pi) = \tau \left[\frac{\pi}{\mu} \right]^{-\lambda}$$
$$\epsilon(\mu, \pi) = \lambda$$

Negative-exponential demand

$$d(\mu, \pi) = \tau e^{(1 - \frac{\pi}{\mu})\lambda}$$
$$\epsilon(\mu, \pi) = \frac{\lambda}{\mu} \pi$$

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Equilibrium Premium

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$$f(\pi_e) = E[\text{Total Profit}] = 0$$

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For two risk-groups,

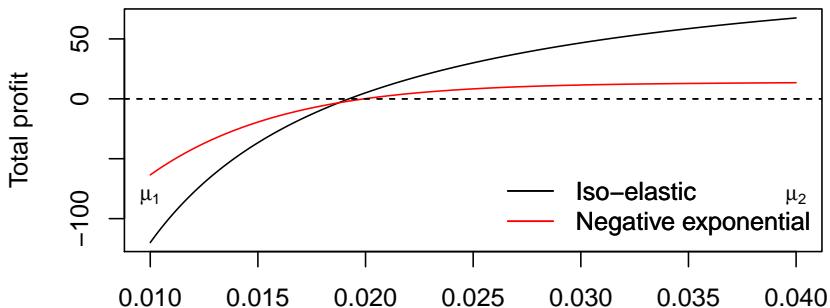
$$f(\pi_e) = d(\mu_1, \pi_e)p_1(\pi_e - \mu_1) + d(\mu_2, \pi_e)p_2(\pi_e - \mu_2) = 0. \quad (2)$$

Equilibrium Premium

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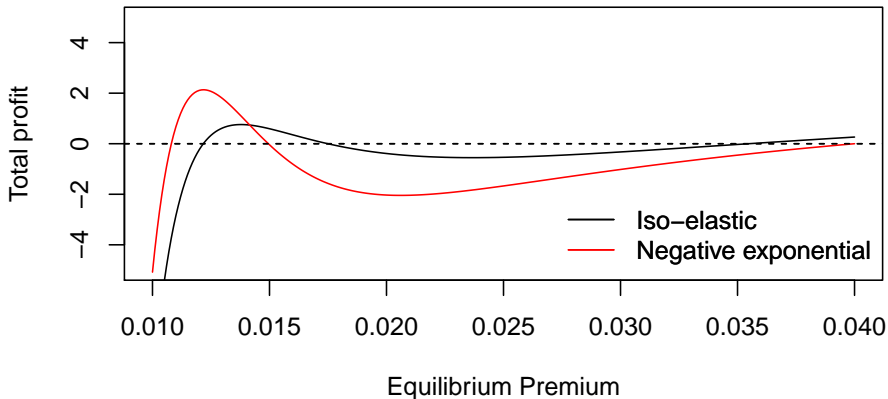


Equilibrium Premium

Multiple Equilibria

Only for extreme parameter values. E.g.

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01, \lambda_1 = 5; p_2 = 80, \tau_2 = 1, \mu_2 = 0.04, \lambda_2 = 1$



Multiple Equilibria

Theorem

Given (μ_1, μ_2) , (τ_1, τ_2) and (λ_1, λ_2) , there are **multiple equilibria** if and only if $c < c_1$ and $\alpha(\pi_{01}) \leq \alpha \leq \alpha(\pi_{02})$.

Where

- $\alpha = \frac{p_1}{p_2}$.
- π_{01}, π_{02} are solutions to $f(\pi_e) = 0, f'(\pi_e) \leq 0$.

Multiple Equilibria

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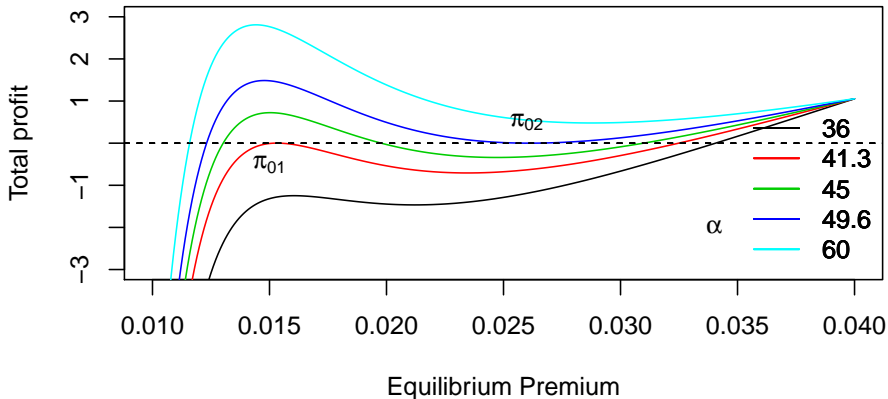
For iso-elastic demand, $c = \lambda_2 - \lambda_1, c_1 = -\frac{\sqrt{\mu_1} + \sqrt{\mu_2}}{\sqrt{\mu_2} - \sqrt{\mu_1}} < 0$.

For negative-exponential demand, $c = \frac{\lambda_2}{\mu_2} - \frac{\lambda_1}{\mu_1}, c_1 = -\frac{4}{\mu_2 - \mu_1} < 0$.

Example: Iso-elastic demand

$$\mu_1 = 0.01, \mu_2 = 0.04 \Rightarrow c_1 = -3;$$

$$\lambda_1 = 4, \lambda_2 = 0.5 \Rightarrow c = -3.5 < c_1$$



Example: Negative-exponential demand

$$\mu_1 = 0.01, \mu_2 = 0.04 \Rightarrow c_1 = -133.33 :$$

$$\lambda_1 = 2, \lambda_2 = 0.5 \Rightarrow c = -187.5 < c_1$$

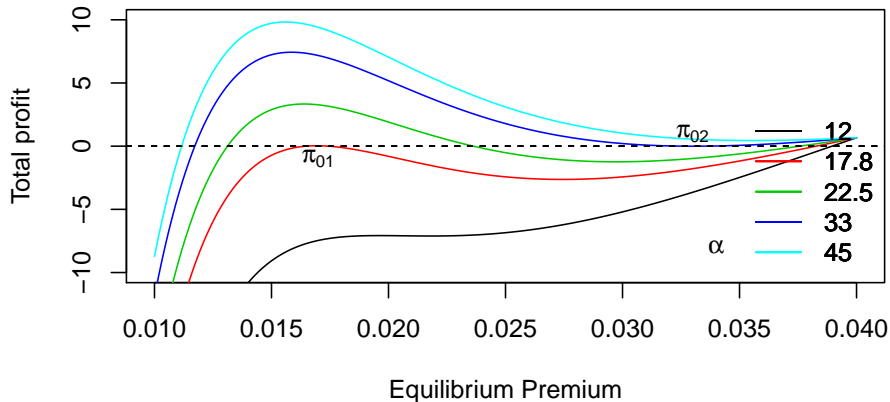


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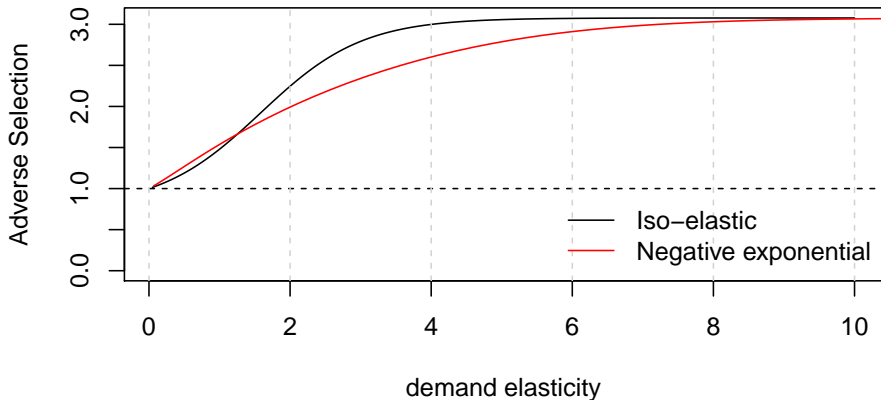
Results

Assumptions

- There are 2 risk-groups
- They have equal demand elasticities \Rightarrow **Unique Equilibrium**
 - ▶ Iso-elastic demand: $\lambda_1 = \lambda_2 = \epsilon(\pi_e)$
 - ▶ Negative-exponential demand: $\frac{\lambda_1}{\mu_2} \pi_e = \frac{\lambda_2}{\mu_2} \pi_e = \epsilon(\pi_e)$

Results: Adverse Selection

$$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04$$



Results: Loss Coverage

$$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04$$

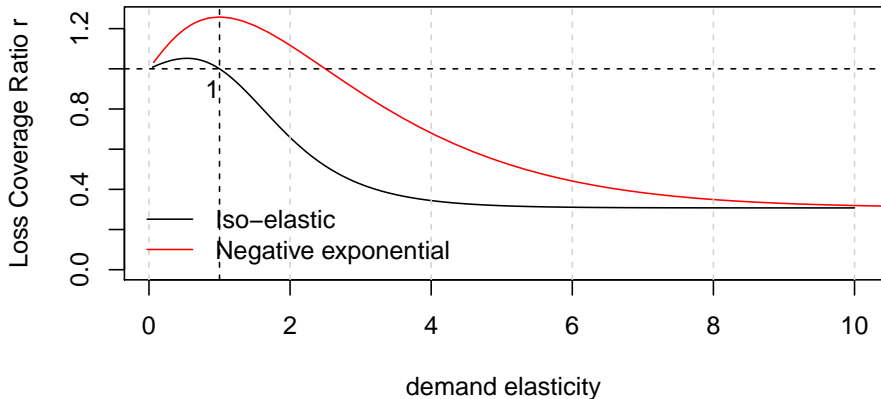


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Summary

- When there is restriction on risk classification, a **pooled premium** π_e is charged across all risk-groups.
- There will always be adverse selection \Rightarrow Adverse Selection may not be a good measurement.
- Loss Coverage is an alternative metric.
Using iso-elastic and negative-exponential demand,
- **Adverse Selection is not always a bad thing!**
A moderate level of adverse selection can increase loss coverage.

Further Research

- Other/more general demand e.g. $d(\mu, \pi) = \tau e^{1 - (\frac{\pi}{\mu})^\lambda}$.
- Loose restriction on demand elasticities.
- Partial restriction on risk classification.

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Questions?

Thank you!