

# LOSS COVERAGE IN INSURANCE MARKETS: WHY ADVERSE SELECTION IS NOT ALWAYS A BAD THING

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# Table of contents

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
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- Loss Coverage

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium



# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage
- Summary and Further research

# Table of contents

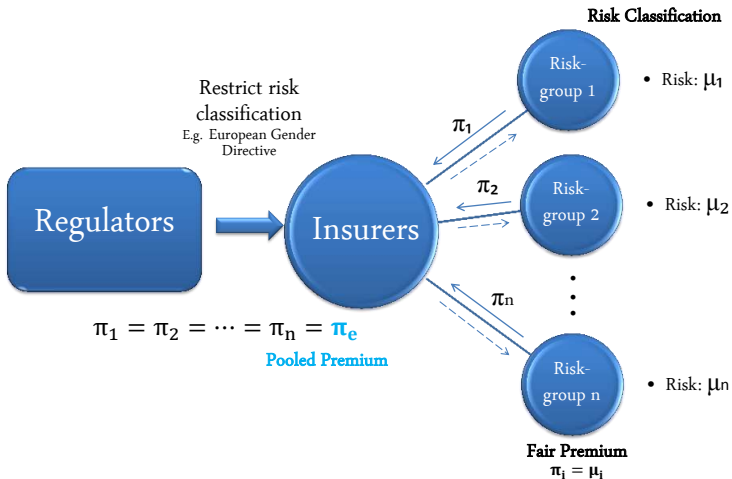
- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage
- Summary and Further research
- References

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage
- Summary and Further research
- References

# Background

## How insurance works and risk classification scheme



# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- **Adverse Selection**
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
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- References

# Adverse Selection

- $0, \pi_1, \pi_2, \pi_3, \pi_e, \dots, \pi_7, \pi_8, \dots, \pi_n, 1.$

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## Typical definition

Purchasing decision is positively correlated with losses  
-Chiappori and Salanie (2000) “Positive Correlation Test”



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- Empirical results are mixed and vary by market.

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- Empirical results are mixed and vary by market.

Life Insurance	Cawley and Philipson (1999)	X
Auto Insurance	Chiappori and Salanie (2000) Cohen (2005)	X O
Annuity	Finkelstein and Poterba (2004)	O
Health Insurance	Cardon and Hendel (2001)	X

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## Definition

$$\text{Adverse Selection (AS)} = \frac{\text{expected claim per policy}}{\text{expected loss per risk}} \quad (1)$$

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$$\text{Adverse Selection Ratio: } \mathcal{S} = \frac{\text{AS at pooled premium } \pi_e}{\text{AS at risk-differentiated premiums}} \quad (2)$$

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$> 1 \Rightarrow$  **Adverse Selection.**

# Example

## Example

- A population of 1000
- Two risk groups
  - ▶ 200 high risks with risk 0.04
  - ▶ 800 low risks with risk 0.01
- No moral hazard



# Example

Full risk classification

# Example

## Full risk classification

	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums ( <b>differentiated</b> )	0.01	0.04	0.016
Numbers insured	400	100	500
Adverse Selection Ratio (S)			1

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**No adverse selection.**

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## Restriction on risk classification-Case 1

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	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums ( $\pi_e$ )	0.02	0.02	0.02
Numbers insured	300(400)	150(100)	450(500)
Adverse Selection Ratio (S)			1.25 > 1

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	Low risks	High risks	Aggregate
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**Moderate adverse selection**

# Example

## Restriction on risk classification-Case 2

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	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums ( $\pi_e$ )	0.02154	0.02154	0.02154
Numbers insured	200(400)	125(100)	325(500)
Adverse Selection Ratio (S)			1.3462 > 1



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**Heavier adverse selection**

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**Heavier adverse selection**

**Adverse selection suggests pooling is always bad. But is it?**

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- **Loss Coverage**
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage
- Summary and Further research
- References

# Loss Coverage

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$$\text{Loss Coverage (LC)} = \frac{\text{insured expected losses}}{\text{population expected losses}} \quad (3)$$

$$\begin{aligned} \text{Loss Coverage Ratio: } C &= \frac{\text{LC at a pooled premium } \pi_e}{\text{LC at at risk-differentiated premium } \pi_i} \quad (4) \\ &> 1, \text{ **Favorable!**} \end{aligned}$$

# Example

No restriction on risk classification

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No restriction on risk classification

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Risk	0.01	0.04	0.016
Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums ( <b>differentiated</b> )	0.01	0.04	0.016
Numbers insured	400	100	500
Insured losses	4	4	8
LC ( $\pi_i$ ) = LC ( $\pi_e$ )			8/16
Loss coverage ratio (C)			1

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No restriction on risk classification

	Low risks	High risks	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1000
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Break-even premiums (differentiated)	0.01	0.04	0.016
Numbers insured	400	100	500
Insured losses	4	4	8
LC ( $\pi_i$ ) = LC ( $\pi_e$ )			8/16
Loss coverage ratio (C)			1

**No adverse selection.**

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Break-even premiums ( $\pi_e$ )	0.02	0.02	0.02
Numbers insured	300(400)	150(100)	450(500)
Insured losses	3	6	9
LC ( $\pi_e$ )			9/16
LC ( $\pi_i$ )			8/16
Loss coverage ratio ( $C$ )			1.125 > 1

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Numbers insured	300(400)	150(100)	450(500)
Insured losses	3	6	9
LC ( $\pi_e$ )			9/16
LC ( $\pi_i$ )			8/16
Loss coverage ratio ( $C$ )			1.125 > 1

**Moderate adverse selection ( $S = 1.25$ ) but favorable loss coverage.**

# Example

## Restriction on risk classification-Case 2



# Example

## Restriction on risk classification-Case 2

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Total population	800	200	1000
Expected population losses	8	8	16
Break-even premiums ( $\pi_e$ )	0.02154	0.02154	0.02154
Numbers insured	200(400)	125(100)	325(500)
Insured losses	2	5	7
LC ( $\pi_e$ )			7/16
LC ( $\pi_i$ )			8/16
Loss coverage ratio ( $C$ )			0.875 < 1

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Loss coverage ratio ( $C$ )			<b>0.875 &lt; 1</b>

**Heavier adverse selection ( $S = 1.3462$ ) and worse loss coverage.**

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Loss coverage ratio ( $C$ )			<b>0.875 &lt; 1</b>

**Heavier adverse selection ( $S = 1.3462$ ) and worse loss coverage.  
Loss coverage might be a better measure!**

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage
- Summary and Further research
- References

# Demand Function

## Definition

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## Definition

Demand elasticity:  $\epsilon(\mu, \pi) = -\frac{\partial d(\mu, \pi)}{d(\mu, \pi)} / \frac{\partial \pi}{\pi}$  i.e. sensitivity of demand to premium changes.



# Demand Function

## Iso-elastic demand function

$$\epsilon(\mu, \pi) = \lambda, \text{ i.e. constant} \quad (5)$$

# Demand Function

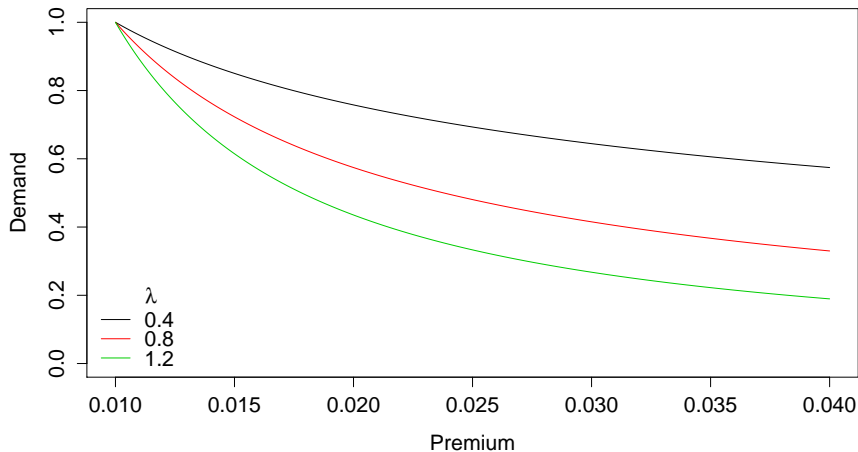
## Iso-elastic demand function

$$\epsilon(\mu, \pi) = \lambda, \text{ i.e. constant} \quad (5)$$

$$d(\mu, \pi) = \tau \left[ \frac{\pi}{\mu} \right]^{-\lambda}. \quad (6)$$

# Iso-elastic demand function

$\tau = 1, \mu = 0.01, \lambda = 0.4, 0.8$  and  $1.2$



# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- **Equilibrium Premium**
- Results on adverse selection and loss coverage
- Summary and Further research
- References

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$$d(\mu_i, \pi_e) = \tau_i \left[ \frac{\pi_e}{\mu_i} \right]^{-\lambda_i}, \quad i = 1, 2$$

If  $\lambda_1 = \lambda_2 = \lambda$ ,

$$\pi_e = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^\lambda + \alpha_2 \mu_2^\lambda}, \quad (8)$$

where

$$\alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, \quad i = 1, 2 \quad (9)$$

(Fair-premium demand-share)

# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- **Results on adverse selection and loss coverage**
- Summary and Further research
- References

# Results on adverse selection

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## Adverse Selection Ratio

$$S = \frac{\pi e}{\alpha_1 \mu_1 + \alpha_2 \mu_2}. \quad (10)$$

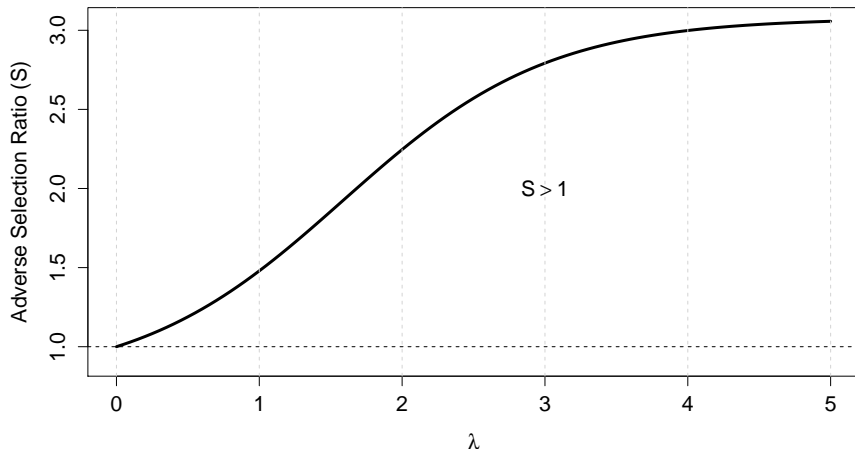
$$\alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, i = 1, 2$$

(Fair-premium demand-share)

# Results: Adverse Selection Ratio (S)

$$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04$$

## Adverse selection ratio plot



# Results on loss coverage

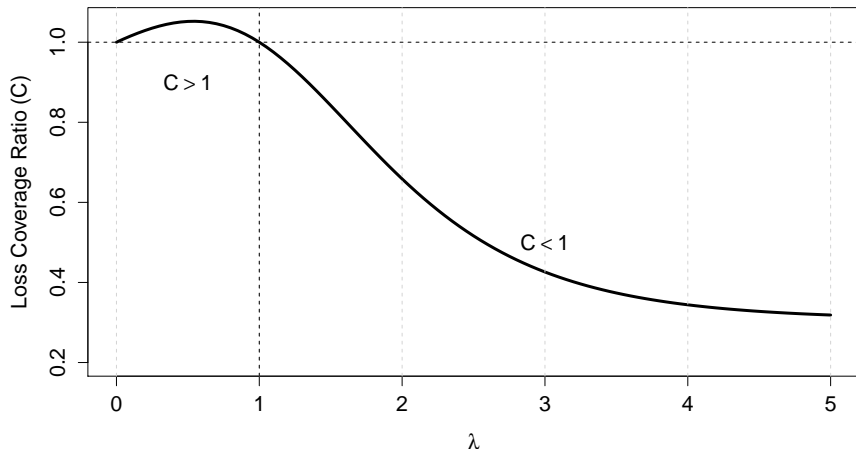
## Loss Coverage Ratio

$$C = \frac{1}{\pi e^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}. \quad (11)$$

# Results: Loss Coverage Ratio (C)

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## Loss coverage ratio plot

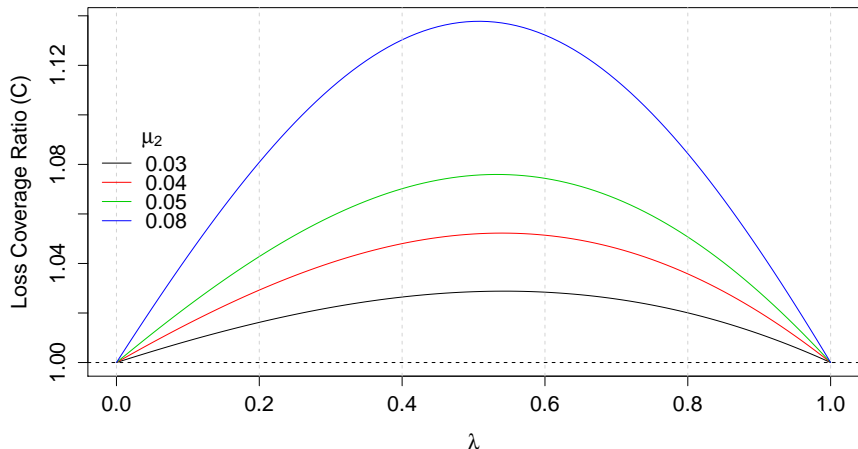




# Results: Loss Coverage Ratio (C)

$p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.03, 0.04, 0.05, 0.08$

## Loss coverage ratio plot



# Table of contents

- Background
  - ▶ How does insurance work?
  - ▶ Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - ▶ Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage
- **Summary and Further research**
- References

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# Summary

- When there is restriction on risk classification, a **pooled premium**  $\pi_e$  is charged across all risk-groups.
- There will always be adverse selection  $\Rightarrow$  Adverse selection may not be a good measure.
- Loss coverage is an alternative metric.
- **Adverse selection is not always a bad thing!**  
**A moderate level of adverse selection can increase loss coverage.**

# Further Research

- Other/more general demand e.g.  $d(\mu, \pi) = \tau e^{1 - (\frac{\pi}{\mu})^\lambda}$ .
- Loose restriction on demand elasticities.
- Partial restriction on risk classification.



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# Questions?

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Thank you!