

Why Adverse Selection need not be adverse ?

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- 1 Introduction
- 2 Why do people buy insurance?
- 3 What drives demand for insurance?
- 4 How much of population losses is compensated by insurance?
- 5 Which regime is most beneficial to society?
- 6 Conclusions

Background

Adverse selection:

If insurers cannot charge **risk-differentiated** premiums, then:

- higher risks buy more insurance, lower risks buy less insurance,
- raising the **pooled** price of insurance,
- lowering the demand for insurance,

usually portrayed as a bad outcome, both for **insurers** and for **society**.

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In practice:

Policymakers often see merit in restricting insurance risk classification

- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

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In practice:

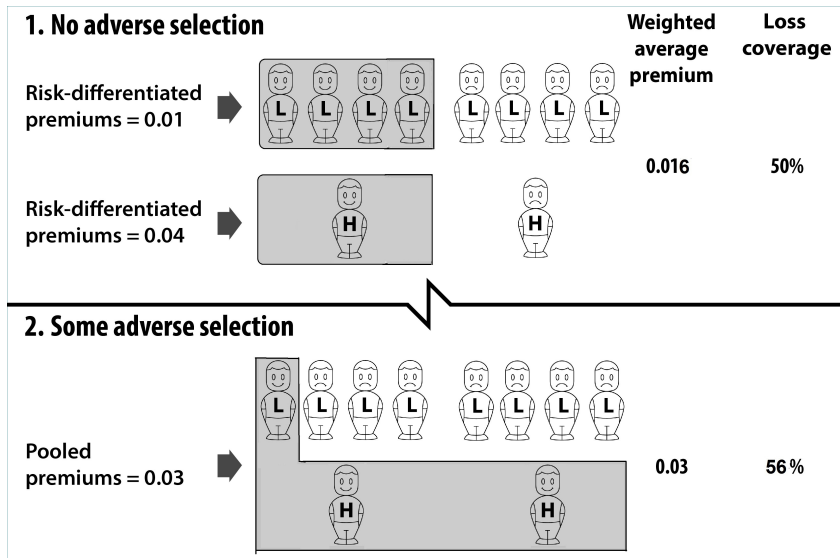
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Question:

How can we reconcile theory with practice?

Motivating example



Agenda

We ask:

- **Why** do people buy insurance?

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We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance (with and without risk classification)?
- **Which** regime is most beneficial to society?

We find:

Social welfare is maximised by maximising **loss coverage**.

Definition (Loss coverage)

Expected population losses compensated by insurance.

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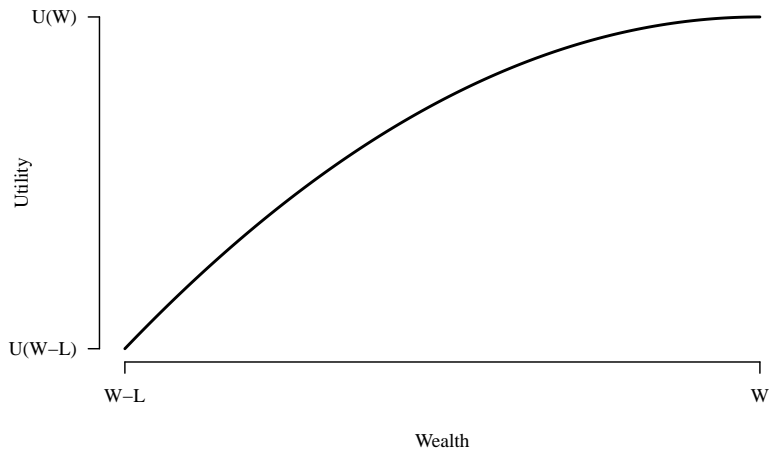
Why do people buy insurance?

Assumptions

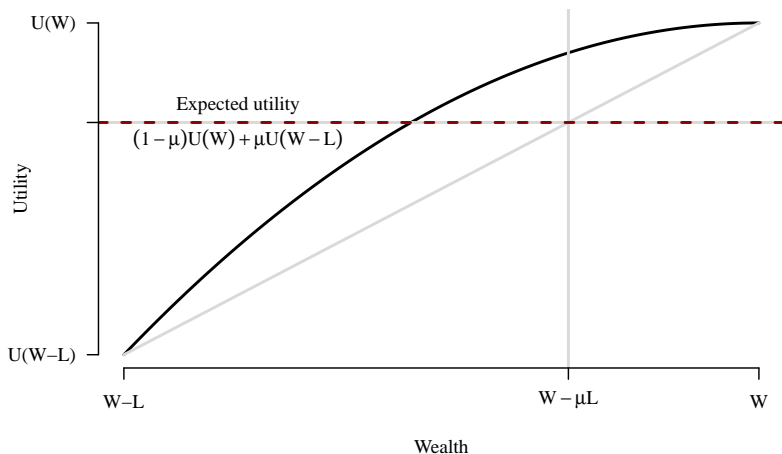
Consider an individual with

- an initial wealth W ,
- exposed to the risk of loss L ,
- with probability μ ,
- utility of wealth $U(w)$, with $U'(w) > 0$ and $U''(w) < 0$,
- an opportunity to insure at premium rate π .

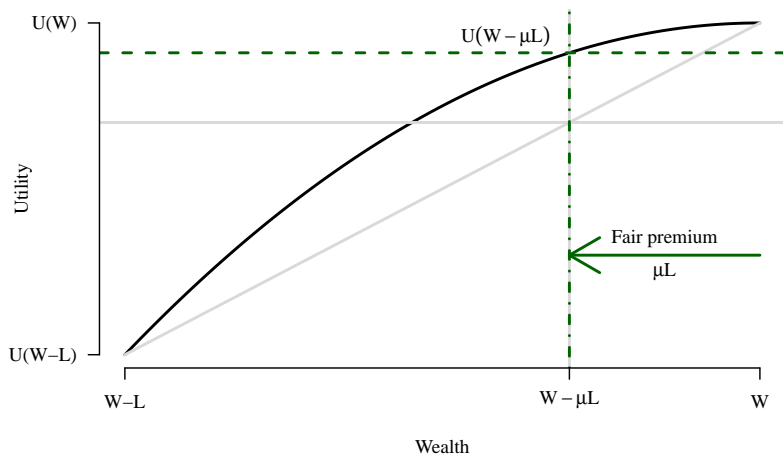
Utility of wealth



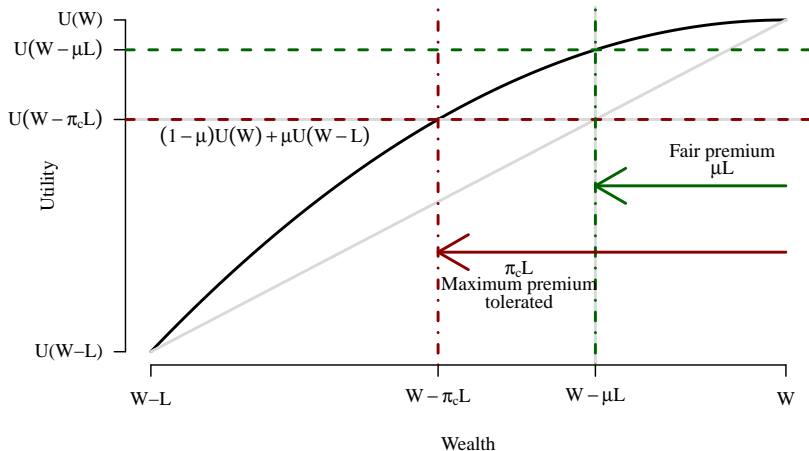
Expected utility: Without insurance



Expected utility: Insured at fair actuarial premium



Maximum premium tolerated: π_c



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Modelling demand for insurance

Simplest model:

If everybody has exactly the same W , L , μ and $U(\cdot)$, then:

- All will buy insurance if $\pi < \pi_c$.
- None will buy insurance if $\pi > \pi_c$.

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Heterogeneity:

- Individuals are **homogeneous** in terms of underlying risk.
- But they can be **heterogeneous** in terms of **risk-aversion**.

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Source of Randomness:

An individual's utility function: $U_\gamma(w)$, where parameter γ is drawn from random variable Γ with distribution function $F_\Gamma(\gamma)$.

Demand is a survival function

Standardisation

As certainty equivalent is invariant to positive affine transformations, we assume $U_\gamma(W) = 1$ and $U_\gamma(W - L) = 0$ for all γ .

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Condition for buying insurance:

Given a premium π , an individual will buy insurance if:

$$\underbrace{U_\gamma(W - \pi L)}_{\text{With insurance}} > \underbrace{(1 - \mu) U_\gamma(W) + \mu U_\gamma(W - L)}_{\text{Without insurance}} = (1 - \mu).$$

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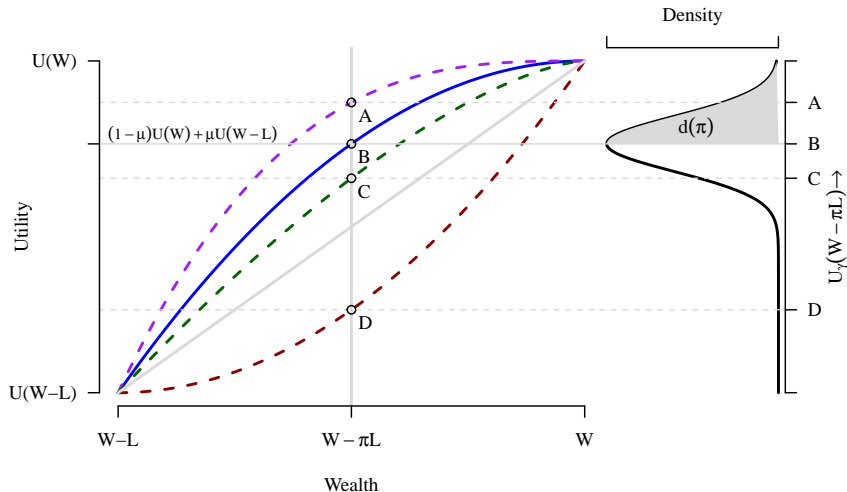
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Demand as a survival function:

Given a premium π , insurance demand, $d(\pi)$, is the survival function:

$$d(\pi) = \mathbb{P}[U_\Gamma(W - \pi L) > 1 - \mu].$$

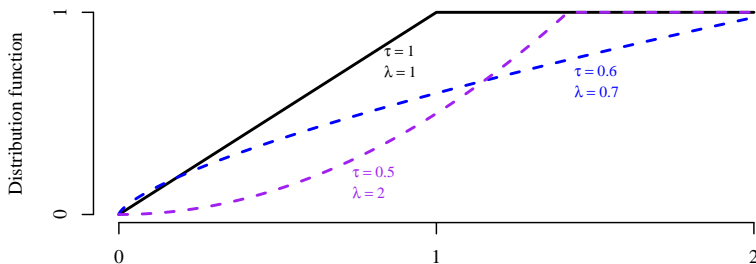
Demand is a survival function



Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

$$F_\Gamma(\gamma) = \mathbf{P}[\Gamma \leq \gamma] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau \gamma^\lambda & \text{if } 0 \leq \gamma \leq (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}. \end{cases}$$

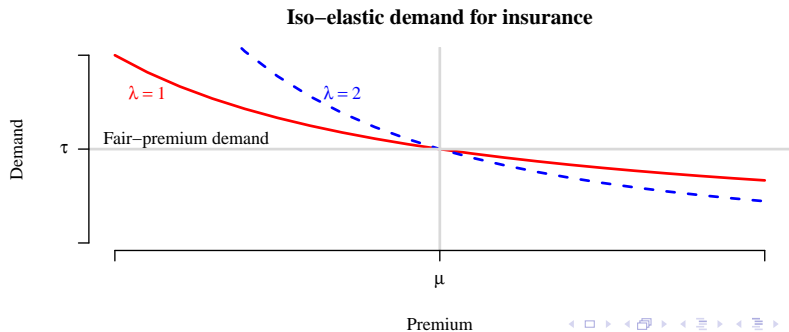
(Kumaraswam's double bounded distribution)



Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

$$d(\pi) = \mathbb{P}[U_\Gamma(W - \pi L) > 1 - \mu] \approx \tau \left(\frac{\mu}{\pi}\right)^\lambda$$

$$\Rightarrow \epsilon(\pi) = \left| \frac{\frac{\partial d(\pi)}{d(\pi)}}{\frac{\partial \pi}{\pi}} \right| = \lambda \quad (\text{constant elasticity} \Rightarrow \text{Iso-elastic demand}).$$



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Risk classification

Assume all have same $W = L = 1$ and constant demand elasticity λ .

Risk-groups

Suppose the population can be divided into 2 risk-groups, with:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: p_1 and p_2 ;
- fair premium demand: $d_1(\mu_1) = \tau_1$ and $d_2(\mu_2) = \tau_2$, i.e.

$$d_i(\pi) = \tau_i \left(\frac{\pi}{\mu_i} \right)^{-\lambda}, \quad i = 1, 2;$$

- premiums offered: π_1 and π_2 (note that $\pi_1 = \pi_2$ is allowed).

Equilibrium

Case 1: Risk-differentiated premium

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Observations:

If risk-differentiated premiums are allowed,

- One possible equilibrium is achieved when $\pi_1 = \mu_1, \pi_2 = \mu_2$.
- No losses for insurers.
- No (actuarial/economic) adverse selection.

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Loss coverage (Population losses compensated by insurance):

Loss coverage: $p_1 \tau_1 \mu_1 + p_2 \tau_2 \mu_2$.

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Equilibrium:

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If $\lambda_1 = \lambda_2 = \lambda$,

$$\pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^\lambda + \alpha_2 \mu_2^\lambda}, \text{ where } \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, i = 1, 2$$

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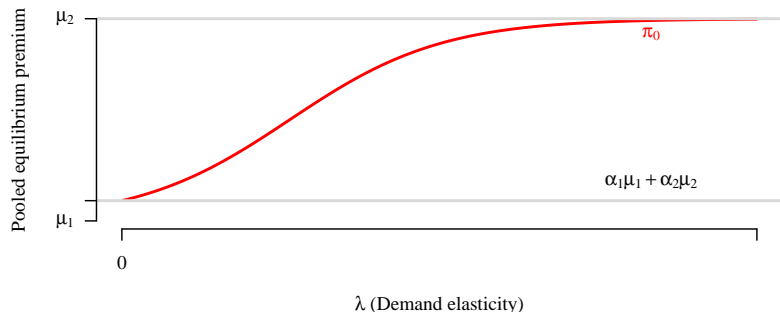
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Observation:

No losses for insurers! \Rightarrow No (actuarial) adverse selection.

Case 2: Pooled premium



Observation:

Pooled equilibrium is greater than average premium charged under full risk classification: $\pi_0 > \alpha_1\mu_1 + \alpha_2\mu_2 \Rightarrow$ (Economic) adverse selection.

Loss coverage ratio

Loss coverage under pooled premium:

$$\text{Loss coverage} = \sum_{i=1}^2 p_i d_i(\pi_0) \mu_i.$$

Loss coverage ratio

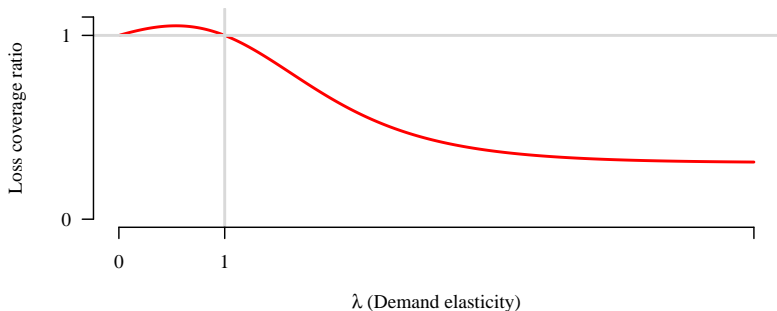
Loss coverage under pooled premium:

$$\text{Loss coverage} := \sum_{i=1}^2 p_i d_i(\pi_0) \mu_i.$$

Loss coverage ratio:

$$\begin{aligned} C &= \frac{\text{Loss coverage for pooled premium}}{\text{Loss coverage for risk-differentiated premium}}, \\ &= \frac{\sum_{i=1}^2 p_i d_i(\pi_0) \mu_i}{\sum_{i=1}^2 p_i d_i(\mu_i) \mu_i}, \\ &= \frac{1}{\pi_0^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}. \end{aligned}$$

Loss coverage ratio



- $\lambda < 1 \Rightarrow$ Pooled premium \succ Full risk classification.
- $\lambda > 1 \Rightarrow$ Pooled premium \prec Full risk classification.
- Empirical evidence suggests $\lambda < 1$ in many insurance markets.

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Social welfare

Definition (Social welfare)

Social welfare, S , is the sum of all individuals' expected (standardised) utilities:

$$\begin{aligned}
 S &= E [Q U_{\Gamma}(W - \Pi L)] + (1 - Q) [(1 - X) U(W) + X U(W - L)], \\
 &= \sum_i \left[\underbrace{d_i(\pi_i) U_i^*(W - \pi_i L)}_{\text{Insured population}} + \underbrace{(1 - d_i(\pi_i)) \{(1 - \mu_i) U(W) + \mu_i U(W - L)\}}_{\text{Uninsured population}} \right] p_i,
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where $U_i^*(W - \pi_i L)$ is the expected utility of i -th risk-group's insured population.

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where $U_i^*(W - \pi_i L)$ is the expected utility of i -th risk-group's insured population.

Linking social welfare to loss coverage

Assuming $L\pi_i \approx 0$, so that $U(W) = U_i^*(W - \pi_i L)$, gives:

$$S = \text{Positive multiplier} \times \underbrace{\sum_i p_i d_i(\pi_i) \mu_i}_{\text{Loss Coverage}} + \text{Constant}.$$

Loss coverage provides a good proxy (which depends only on observable data) for social welfare (which depends on unobservable utilities).

Result: Maximising loss coverage maximises social welfare.

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Restricting risk classification increases loss coverage if $\lambda < 1$.

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Restricting risk classification increases loss coverage if $\lambda < 1$.

Loss coverage is an observable proxy for social welfare.

References

- HAO, M., MACDONALD, A.S., TAPADAR, P. & THOMAS, R.G. (2016). Insurance loss coverage and social welfare. *Submitted*, <https://kar.kent.ac.uk/54235/>.
- HAO, M., MACDONALD, A.S., TAPADAR, P. & THOMAS, R.G. (2016). Insurance loss coverage under restricted risk classification: The case of iso-elastic demand. *ASTIN Bulletin*, **46**, 265–291.
- MACDONALD, A.S. & TAPADAR, P. (2010). Multifactorial disorders and adverse selection: epidemiology meets economics. *Journal of Risk and Insurance*, **77**, 155–182.
- THOMAS, R.G. (2009). Demand elasticity, risk classification and loss coverage: when can community rating work?. *ASTIN Bulletin*, **39**, 403–428.
- THOMAS, R.G. (2008). Loss coverage as a public policy objective for risk classification schemes. *Journal of Risk and Insurance*, **75**, 997–1018.