Probability, Logic, and Cognition

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Attempts to define "reasoning"

- *"[. . . R]easoning is a mental process that produces new representations from old ones. Of course, not all such processes qualify as reasoning"* (Rips, 2002, p. 363).
- *"[O]ne may be rational in terms of achieving personal goals (rationality$_1$) without being rational in the sense of conforming to a normative system such as logic (rationality$_2$)"* (Evans, Newstead, & Byrne, 1993, p. X). *"When most psychologists talk about "reasoning", they mean an explicit, sequential thought process of some kind, consisting of propositional representations. . . . The psychologists' use of th[is] term—which is linked with their endorsement of rationality$_2$—is much closer to what a philosopher would call theoretical reasoning"* (Evans et al., 1993, p. 15).
- *"There are three main varieties of reasoning: calculation, deduction, and induction"* (Johnson-Laird & Byrne, 1991, p. 2).

Attempts to define "reasoning"

- *"[... Reasoning is a mental process that produces new representations from old ones. Of course, not all such processes qualify as reasoning"* (Rips, 2002, p. 363).
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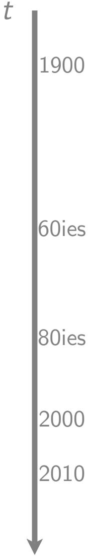
A (very) short history of logic and probability in Ψ



A (very) short history of logic and probability in Ψ

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1900 — Judgment and decision making (*probability* & *utility*)

60ies — Bounded rationality

80ies — Heuristics and biases

2000 — Adaptive toolbox

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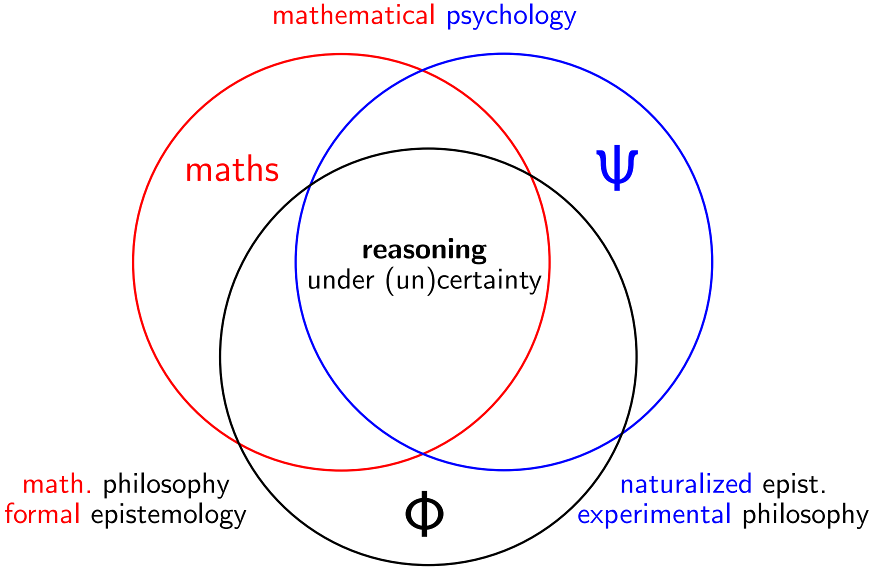
Disciplines



Interaction of formal and empirical work (Pfeifer, 2011, 2012b)

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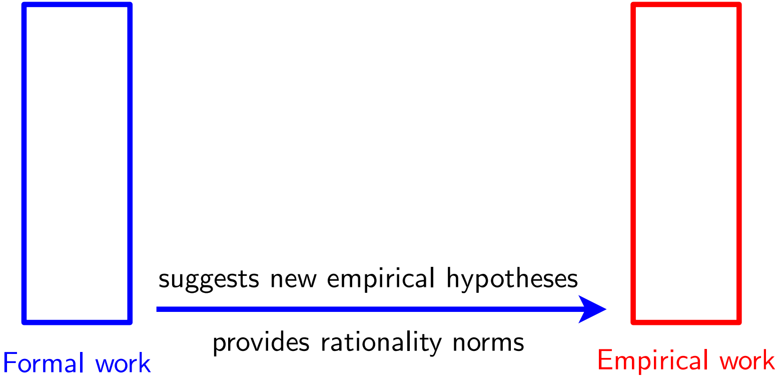

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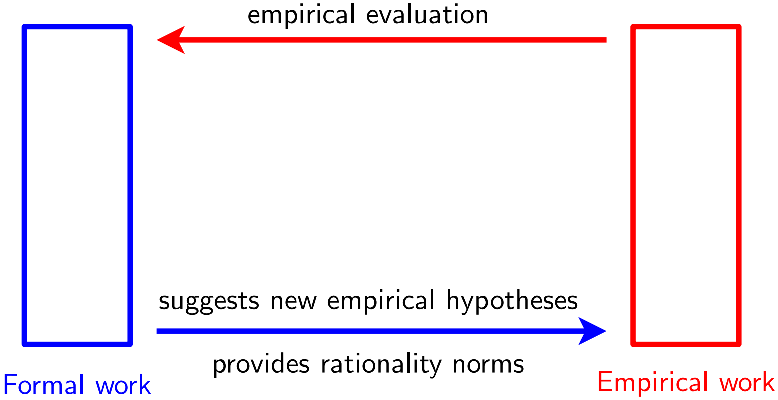

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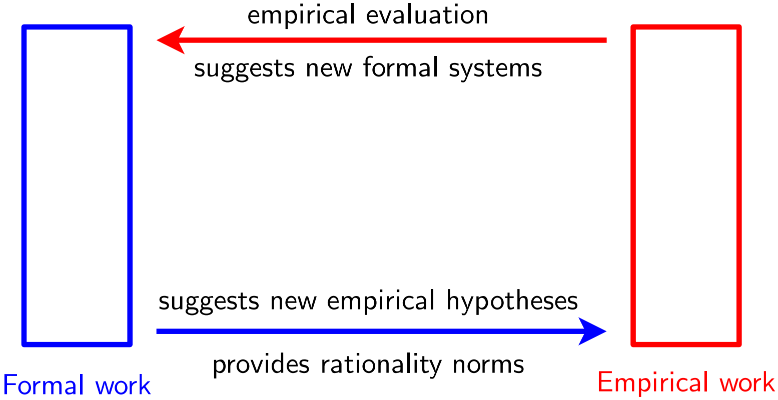

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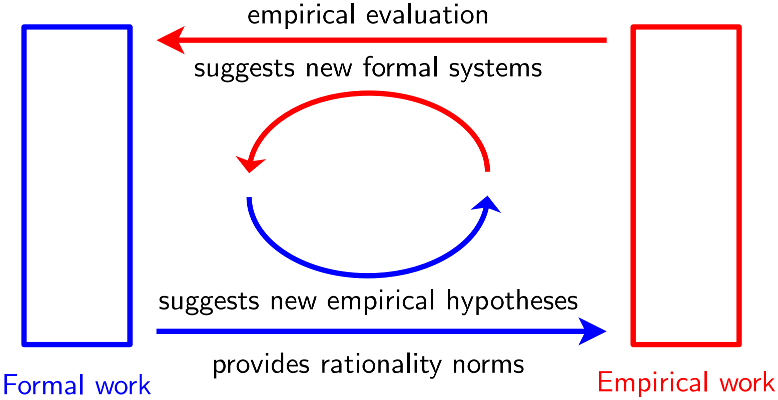

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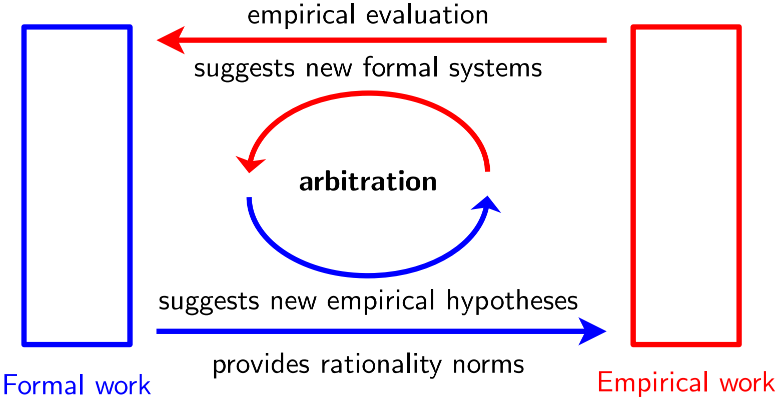

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Truth tables

Negation:

A	not-A $\neg A$
T	F
F	T

Samples of other connectives:

A	B	A and B $A \wedge B$	A or B $A \vee B$	If A, then B $A \supset B$	A iff B $A \equiv B$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

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Mental Model Theory

"Individuals minimize the load on working memory by tending to construct mental models that represent explicitly only what is true, and not what is false." (Johnson-Laird, 1999, p. 116)

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- Step 2: Integration of individual models into a single one

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- Step 3: Validation. Is it possible with alternative models that all premises are true but the conclusion is false? If YES, stop: The inference is not valid.

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Principle of truth

"Each mental model of a set of assertions represents a possibility given the truth of the assertions, and each mental model represents a clause in these assertions only when it is true in that possibility." (Johnson-Laird & Byrne, 2002, p. 653)

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Example 1:

There is a heart or there is <u>no</u> triangle ($\heartsuit \vee \neg \triangle$).

Principle of truth

Example 1:

There is a heart or there is not a triangle ($\heartsuit \vee \neg \triangle$).

Truth table

$\heartsuit$	$\triangle$	$\heartsuit \vee \neg \triangle$
T	T	T
T	F	T
F	T	F
F	F	W

Principle of truth

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Principle of truth

Example 1:

There is a heart or there is not a triangle ($\heartsuit \lor \neg\triangle$).

Truth table			Mental Models	
$\heartsuit$	$\triangle$	$\heartsuit \lor \neg\triangle$		
T	T	T	$\heartsuit$	
T	F	T	$\heartsuit$	$\neg\triangle$
F	T	F		
F	F	T		$\neg\triangle$

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F	T	F		
F	F	T		$\neg\triangle$

three mental models

the set of all three models represents the whole sentence

Principle of truth

Example 2:

If there is a heart, then there is a triangle ($\heartsuit \supset \triangle$).

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Mental models

$\heartsuit$	$\triangle$

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Truth table			Mental models	
$\heartsuit$	$\triangle$	$\heartsuit \supset \triangle$		
T	T	T	$\heartsuit$	$\triangle$
T	F	F		
F	T	T	. . .	
F	F	T		

"..." denotes the "mental footnote" (implicit mental model).

Principle of truth

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If there is a heart, then there is a triangle ($\heartsuit \supset \triangle$).

Truth table			Mental models	
$\heartsuit$	$\triangle$	$\heartsuit \supset \triangle$		
T	T	T	$\heartsuit$	$\triangle$
T	F	F		
F	T	T	$\neg\heartsuit$	$\triangle$
F	F	T	$\neg\heartsuit$	$\neg\triangle$

Explicit mental model

Mental Model Theory

Example 3:

There is a heart if and only if there is a triangle ($\heartsuit \equiv \triangle$).

Mental Model Theory

Example 3:

There is a heart if and only if there is a triangle (♡ ≡ △).

Truth table			Mental models	
♡	△	♡ ≡ △		
T	T	T	♡	△
T	F	F		
F	T	F	. . .	
F	F	T		

Implicit mental model

Mental Model Theory

Example 3:

There is a heart if and only if there is a triangle ($\heartsuit \equiv \triangle$).

Truth table			Mental models	
$\heartsuit$	$\triangle$	$\heartsuit \equiv \triangle$		
T	T	T	$\heartsuit$	$\triangle$
T	F	F		
F	T	F		
F	F	T	$\neg\heartsuit$	$\neg\triangle$

Explicit mental model

Inference, Example 1: Modus Ponens

If there is a heart, then there is a triangle.	$\heartsuit \supset \triangle$
There is a heart.	$\heartsuit$
There is a triangle.	$\triangle$

Inference, Example 1: Modus Ponens

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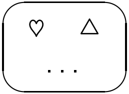
Premise 1:



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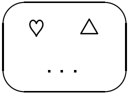
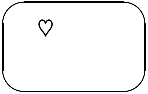


Premise 2:



Inference, Example 1: Modus Ponens

If there is a heart, then there is a triangle.	$\heartsuit \supset \triangle$
There is a heart.	$\heartsuit$
There is a triangle.	$\triangle$

Premise 1:

$\heartsuit$ $\triangle$

...

Premise 2:

$\heartsuit$

Integrated model:

$\heartsuit$ $\triangle$

$\triangle$ can directly be read off.

Inference, Example 2: Modus Tollens

If there is a heart, then there is a triangle.	$\heartsuit \supset \triangle$
There is not a triangle.	$\neg\triangle$
There is not a heart.	$\neg\heartsuit$

Inference, Example 2: Modus Tollens

If there is a heart, then there is a triangle.	$\heartsuit \supset \triangle$
There is not a triangle.	$\neg\triangle$
There is not a heart.	$\neg\heartsuit$

Premise 1:

$\heartsuit \quad \triangle$

...

Inference, Example 2: Modus Tollens

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...

Premise 2:

$\neg\triangle$

Inference, Example 2: Modus Tollens

If there is a heart, then there is a triangle.	$\heartsuit \supset \triangle$
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Premise 1:

$\heartsuit \quad \triangle$

$\ldots$

Premise 2:

$\neg\triangle$

Integrated model:

$\heartsuit \quad \triangle$

$\neg\heartsuit \quad \triangle$

$\neg\heartsuit \quad \neg\triangle$

"Fleshing out" adds difficulty!

Inference: Not log. valid arguments

Examples:

- Affirming the Consequent ($\heartsuit \supset \triangle$, $\triangle$, therefore: $\heartsuit$)

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Explanation: People who (mistakenly) interpret these argument forms as logically valid, interpret the conditional premise (mistakenly) as a biconditional ($\heartsuit \equiv \triangle$)

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$\heartsuit \quad \triangle$

$\ldots$

Mental models: Predictions

- The difficulty of inferences increases with the number of mental models (working memory load).

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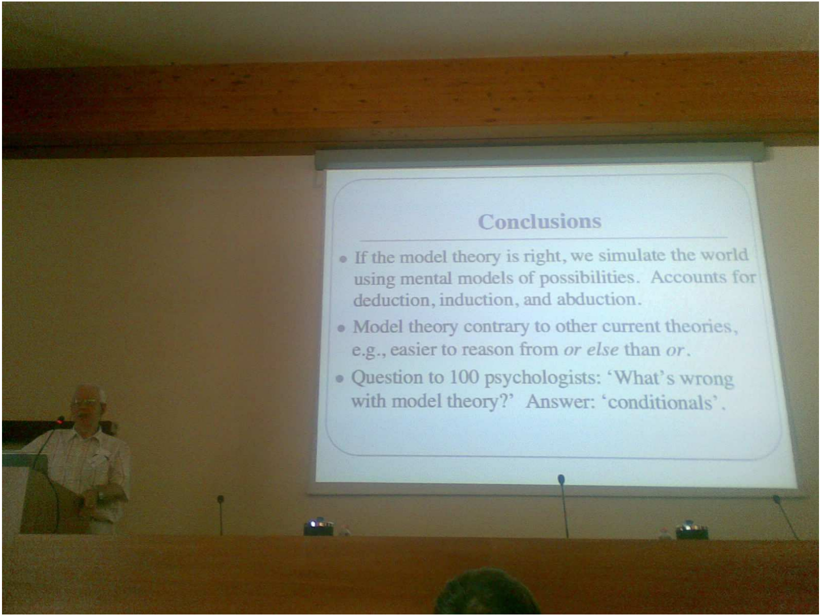
- Reasoning is a process involving representing, integrating and validating mental models; the search for inconsistencies requires time.

Mental models: Predictions

- The difficulty of inferences increases with the number of mental models (working memory load).
- The difficulty of inferences decreases, if less explicit mental models are required.
- Reasoning is a process involving representing, integrating and validating mental models; the search for inconsistencies requires time.
- Errors occur, if:
  - not all alternatives are represented
  - inconsistencies are overlooked



(photo source: Niki Pfeifer)

Deductive proof

Premise 1: If p, then q.	$p \supset q$
Premise 2: If p, then r.	$p \supset r$
Conclusion: If p, then both, q and r.	$p \supset (q \wedge r)$

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	Formula	Justification
(1)	$p \supset q$	*Premise 1*
(2)	$p \supset r$	*Premise 2*

- Goal: Try to infer the conclusion ($p \supset (q \wedge r)$), only from the premises and the (valid) inference rules.

Deductive proof

Premise 1: If p, then q.	$p \supset q$
Premise 2: If p, then r.	$p \supset r$
Conclusion: If p, then both, q and r.	$p \supset (q \wedge r)$

	Formula	Justification
(1)	$p \supset q$	*Premise 1*
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(3)	p	*Conditional Proof (Assumption)*

- Conditional Proof (Assumption). Goal: $p \supset (q \wedge r)$.
  Subgoal 1: q, r.
  Subgoal 2: $q \wedge r$.

Deductive proof

Premise 1: If p, then q.	$p \supset q$
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	Formula	**Justification**
(1)	$p \supset q$	*Premise 1*
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(3)	p	*Conditional Proof (Assumption)*
(4)	q	*Modus ponens:* (1)+(3)

- Modus Ponens: applied to (1) and (3).

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(3)	p	*Conditional Proof (Assumption)*
(4)	q	*Modus ponens:* (1)+(3)
(5)	r	*Modus ponens:* (2)+(3)

- Modus Ponens: applied to (2) and (3).
  Subgoal 1 (q, r) completed.

Deductive proof

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(3)	p	*Conditional Proof (Assumption)*
(4)	q	*Modus ponens:* (1)+(3)
(5)	r	*Modus ponens:* (2)+(3)
(6)	$q \wedge r$	*Conjunction Rule:* (4)+(5)

- Conjunction Rule applied to (4) and (5).
  Subgoal 2 ($q \wedge r$) completed.

Deductive proof

Premise 1: If p, then q.	$p \supset q$
Premise 2: If p, then r.	$p \supset r$
Conclusion: If p, then both, q and r.	$p \supset (q \wedge r)$

	Formula	Justification
(1)	$p \supset q$	*Premise 1*
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(3)	p	*Conditional Proof (Assumption)*
(4)	q	*Modus ponens:* (1)+(3)
(5)	r	*Modus ponens:* (2)+(3)
(6)	$q \wedge r$	*Conjunction Rule:* (4)+(5)
(7)	$p \supset (q \wedge r)$	*Conditional Proof:* (3)-(6)

- We derived the conclusion.
  Therefore, the argument is logically valid.

Features of Deductive Proofs

- Each step is justified exclusively by the premises or valid inference rules

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- Each step is justified exclusively by the premises or valid inference rules
- No reference to truth values or meaning, thus purely syntactically
- Process principles:
  - Translation of the natural language argument into logical language (What belongs to the "logical form/skeleton"?)
  - Top-down, bottom-up (Goals, Subgoals)
  - Pattern matching: Recognition of applicability of inference rules

Mental Rule theories

Assumptions:

- Human reasoning apparatus is built up with with a set of formal rules

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- Human reasoning apparatus is built up with with a set of formal rules
- Ability of constructing a mental proof by use of formal rules
- Ability of pattern matching: understanding of which rules are applicable

Two strategies:

- Bottom up: derive everything that follows directly by application of the formal inference rules
- Top down: determine and prove subgoals from which the conclusion may be reached

Mental Rule theories: 3 error types

- Comprehension errors (mis-representing premises, wrong pattern matching, . . .)

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Mental Rule theories: 3 error types

- Comprehension errors (mis-representing premises, wrong pattern matching, . . .)
- Coordination errors (mistaken sub-goals, . . .)
- Processing errors (attention, WM, . . .)

Mental Rule theories

- The more steps a mental proof requires, the harder the reasoning task will be

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- The more steps a mental proof requires, the harder the reasoning task will be
- Clear description of the reasoning process (production system)
- Less problems to explain multiple premise inferences (building many mental models $\Longrightarrow$ WM overload)
- Problems: Which rules are built in? What exactly is represented? How can suppression effects be explained (Byrne, 1989)?
- ...

Mental rules/models: Summary

- Mental rule theories (Rips, 1994; Braine & O'Brien, 1998)
  - psychological fragment of proof-theory
  - formal rules
  - reasoning is constructing a mental proof
  - pattern matching, top down and bottom up strategies
- Mental model theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 2002)
  - psychological fragment of model theory
  - truth tables
  - reasoning is constructing, combining and evaluating mental models

Problems of the old paradigm

- unable to deal with degrees of belief
- unable to deal with nonmonotonicity
- interpreting natural language conditionals by the material conditional $(\cdot \supset \cdot)$ is highly problematic

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Truth tables & Ramsey test

Negation:

A	not-A $\neg A$
T	F
F	T

Samples of other connectives:

A	B	A and B $A \wedge B$	A or B $A \vee B$	If A, then B $A \supset B$	A iff B $A \equiv B$	B given A $B\|A$
T	T	T	T	T	T	T
T	F	F	T	F	F	F
F	T	F	T	T	F	void
F	F	F	F	T	T	void

"If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; . . . We can say they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered void" (Ramsey, 1929/1994, footnote, p. 155).

Truth tables & Ramsey test

Negation:

A	not-A $\neg A$
T	F
F	T

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- rationality framework: coherence based probability logic framework

Coherence based probability logic

- Coherence
  - de Finetti, and $\{$Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Walley, $\ldots\}$
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $P(B|A)$ by

$$\frac{P(A \wedge B)}{P(A)} \quad \text{and assume that} \quad P(A) > 0$$

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- Probability logic
  - uncertain argument forms
  - deductive consequence relation

E.g.: Probabilistic modus ponens (e.g., Hailperin, 1996; Pfeifer & Kleiter, 2006a)

Modus ponens	Probabilistic modus ponens	
	(Conditional event)	*(Material conditional)*
If A, then C	$p(C\|A) = x$	$p(A \supset C) = x$
A	$p(A) = y$	$p(A) = y$
C	$xy \leq p(C) \leq xy + 1 - x$	$\max\{0, x + y - 1\} \leq p(C) \leq x$

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Example: Probabilistic modus ponens (e.g., Hailperin, 1996)

Modus ponens	Probabilistic modus ponens	
	(Conditional event)	*(Material conditional)*
If A, then C	$p(C\|A) = .90$	$p(A \supset C) = .90$
A	$p(A) = .50$	$p(A) = .50$
C	$.45 \leq p(C) \leq .95$	$.40 \leq p(C) \leq .90$

...where the consequence relation ("———") is deductive.

Example: Modus ponens

From $\boxed{A}$ and $\boxed{\text{If } A\text{, then } B}$ infer $\boxed{B}$

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From $\boxed{P(A) = x}$, $\boxed{P(B|A) = y}$ and $\boxed{P(B|\neg A) = q}$
infer $\boxed{P(B) = xy + (1 - x)q}$

Proprieties of arguments

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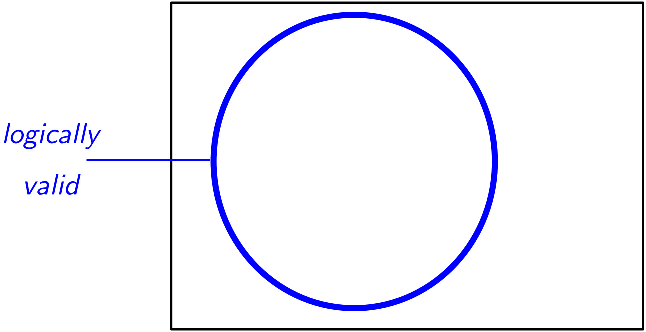
- An argument is *p*-valid if and only if the uncertainty of the conclusion of a valid inference cannot exceed the sum of the uncertainties of its premises (where “uncertainty of X” is defined by $1 - P(X)$) (Adams, 1975).
- An argument is probabilistically informative if and only if it is possible that the premise probabilities constrain the conclusion probability. I.e., if the coherent probability interval of its conclusion is not necessarily equal to the unit interval $[0, 1]$ (Pfeifer & Kleiter, 2006a).

Log. valid–prob. informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)



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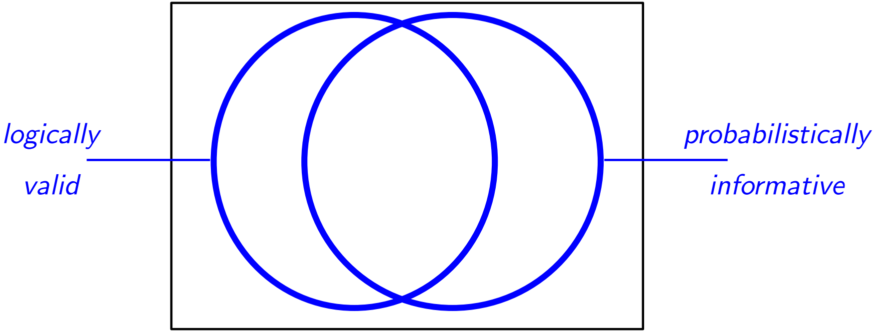

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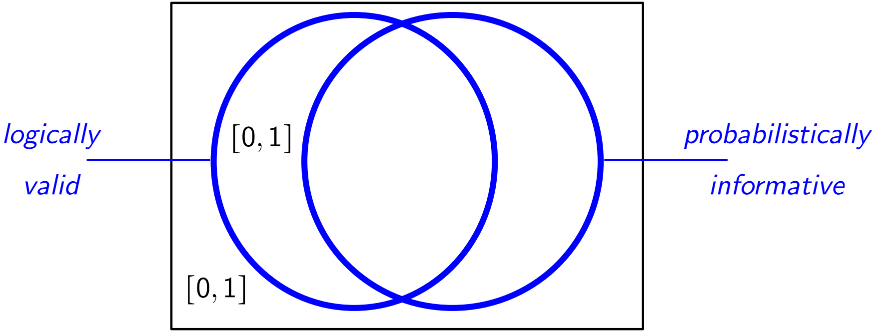

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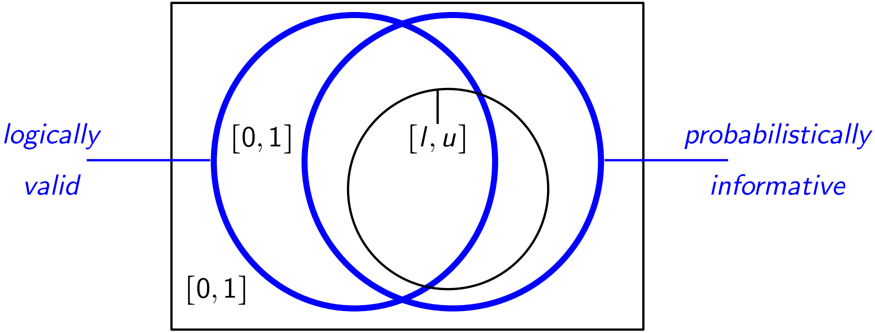

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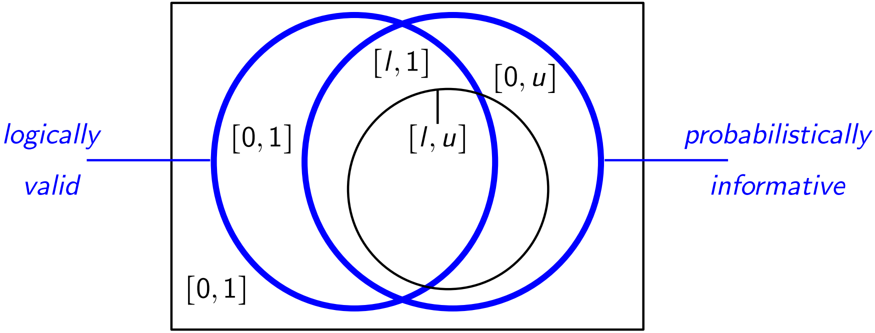

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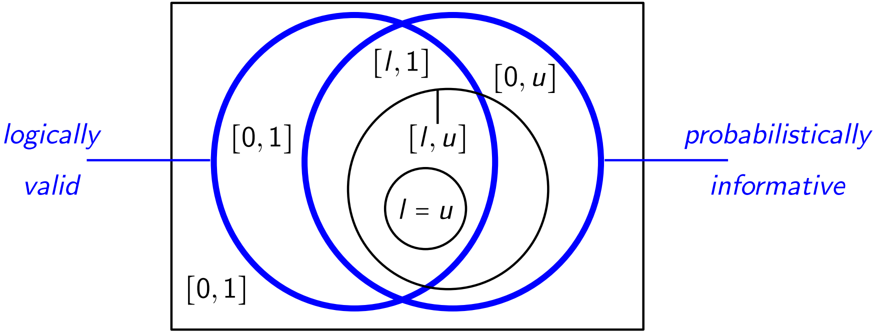

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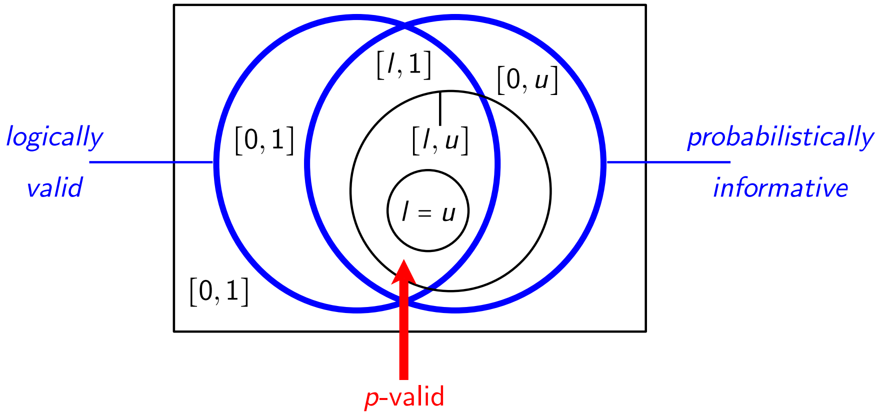

Mental probability logic II

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Sample paradoxes of the material conditional

(Paradox 1)	(Paradox 2)
B	Not: A
If A, then B	If A, then B

Sample paradoxes of the material conditional

(Paradox 1)

$$\frac{B}{\text{If } A\text{, then } B}$$

(Paradox 2)

$$\frac{\text{Not: } A}{\text{If } A\text{, then } B}$$

(Paradox 1)

$$\frac{B}{A \supset B}$$

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$$\frac{\neg A}{A \supset B}$$

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Paradoxes of the material conditional, e.g.,

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Inf. vers. of t. paradoxes (Pfeifer (2014). *Studia Logica*; Pfeifer and Douven (2014). *R*ev. Phil. Psy.)

From $\Pr(B) = 1$ and $A \wedge B \equiv \bot$ infer $\Pr(B \mid A) = 0$ is coherent.

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...a special case of the cautious monotonicity rule of System P (Gilio, 2002).

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Probabilistic truth table task (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

$$
\begin{array}{rcl}
P(A \wedge C) & = & x_1 \\
P(A \wedge \neg C) & = & x_2 \\
P(\neg A \wedge C) & = & x_3 \\
P(\neg A \wedge \neg C) & = & x_4 \\
\hline
P(\text{If } A, \text{ then } C) & = & ?
\end{array}
$$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} P(A \wedge C) & = & x_1 \\ P(A \wedge \neg C) & = & x_2 \\ P(\neg A \wedge C) & = & x_3 \\ P(\neg A \wedge \neg C) & = & x_4 \\ \hline P(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Conclusion candidates:

- $P(A \wedge C) = x_1$
- $P(C|A) = x_1/(x_1 + x_2)$
- $P(A \supset C) = x_1 + x_3 + x_4$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} P(A \wedge C) & = & x_1 = .25 \\ P(A \wedge \neg C) & = & x_2 = .25 \\ P(\neg A \wedge C) & = & x_3 = .25 \\ P(\neg A \wedge \neg C) & = & x_4 = .25 \\ \hline P(\text{If } A, \text{ then } C) & = & ? \end{array}$$

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\hline
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Conclusion candidates:

- $P(A \wedge C) = x_1 = .25$
- $P(C|A) = x_1/(x_1 + x_2) = .50$
- $P(A \supset C) = x_1 + x_3 + x_4 = .75$

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$$\begin{array}{rcl} P(A \wedge C) & = & x_1 \\ P(A \wedge \neg C) & = & x_2 \\ P(\neg A \wedge C) & = & x_3 \\ P(\neg A \wedge \neg C) & = & x_4 \\ \hline P(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Main results:

- More than half of the responses are consistent with $P(C|A)$
- Many responses are consistent with $P(A \wedge C)$

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Main results:

- More than half of the responses are consistent with $P(C|A)$
- Many responses are consistent with $P(A \wedge C)$
- Generalized version: Interpretation shifts to $P(C|A)$ (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011a, *Journal of Experimental Psychology: LMC*)

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$
\begin{array}{rcl}
P(A \wedge C) & = & x_1 \\
P(A \wedge \neg C) & = & x_2 \\
P(\neg A \wedge C) & = & x_3 \\
P(\neg A \wedge \neg C) & = & x_4 \\
\hline
P(\text{If } A, \text{ then } C) & = & ?
\end{array}
$$

Main results:

- More than half of the responses are consistent with $P(C|A)$
- Many responses are consistent with $P(A \wedge C)$
- Generalized version: Interpretation shifts to $P(C|A)$ (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011a, *Journal of Experimental Psychology: LMC*)

Key feature:

- Reasoning under complete probabilistic knowledge

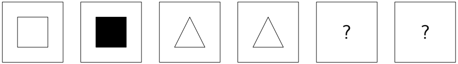
Experiment

Motivation

- probabilistic truth table task with incomplete probabilistic knowledge
- Is the conditional event interpretation still dominant?
- Are there shifts of interpretation?

Example: Task 5 (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



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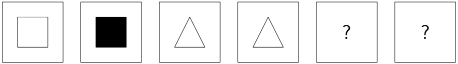

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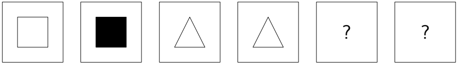

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

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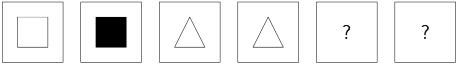

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Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer:

at least

	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6	

at most

	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6	

(please tick the appropriate boxes)

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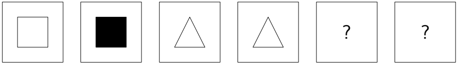

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Cond. event: *at least* 1 out of 5 and *at most* 3 out of 5

at least

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

at most

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

(please tick the appropriate boxes)

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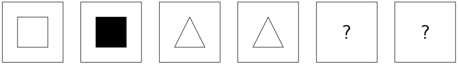

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Conjunction: *at least* 1 out of 6 and *at most* 3 out of 6

at least	*at most*
☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6	☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6
out of ☐1 ☐2 ☐3 ☐4 ☐5 ☐6	out of ☐1 ☐2 ☐3 ☐4 ☐5 ☐6

(please tick the appropriate boxes)

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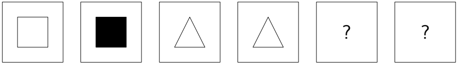

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.
Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Mat. cond.: *at least* 2 out of 6 and *at most* 4 out of 6

at least								*at most*							
	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6		☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6	out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

(please tick the appropriate boxes)

Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

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Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Sample

- 20 Cambridge University students
- 10 female, 10 male
- between 18 and 27 years old (mean: 21.65)
- no students of mathematics, philosophy, computer science, or psychology

Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Results

- Overall (340 interval responses)
  - 65.6% consistent with conditional event
  - 5.6% consistent with conjunction
  - 0.3% consistent with material conditional

Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

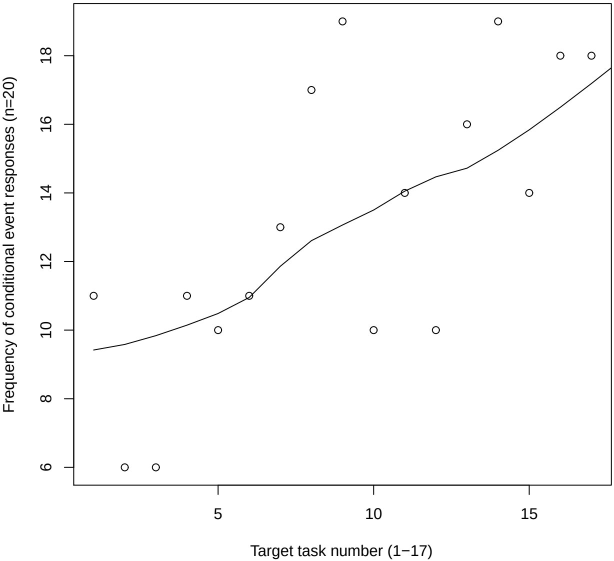
Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Results

- Overall (340 interval responses)
  - 65.6% consistent with conditional event
  - 5.6% consistent with conjunction
  - 0.3% consistent with material conditional
- Shift of interpretation
  - First three tasks: 38.3% consistent with conditional event
  - Last three tasks: 83.3% consistent with conditional event
  - Strong correlation between conditional event frequency and item position ($r(15) = 0.71$, $p < 0.005$)

Increase of cond. event resp. ($n_1 = 20$) (Pfeifer, 2013a, *Thinking & Reasoning*)



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Aristotle's Theses

AT #1: $\neg(\neg A \to A)$

AT #2: $\neg(A \to \neg A)$

Aristotle's Theses

AT #1: $\neg(\neg A \rightarrow A)$

$$\neg(\neg A \supset A)$$

AT #2: $\neg(A \rightarrow \neg A)$

$$\neg(A \supset \neg A)$$

Aristotle's Theses

AT #1: $\neg(\neg A \rightarrow A)$

$$\neg(\neg A \supset A) \quad \equiv \quad \neg A \wedge \neg A \quad \equiv \quad \neg A$$

AT #2: $\neg(A \rightarrow \neg A)$

$$\neg(A \supset \neg A) \quad \equiv \quad A \wedge A \quad \equiv \quad A$$

Aristotle's Theses: Prob. log. predictions (Pfeifer, 2012a, *The Monist*)

AT #1: $\neg(\neg A \rightarrow A)$

- $P(\neg(\neg A \supset A)) = P(\neg A)$

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- $P(A|\neg A) = 0$, its negation: $P(\neg A|\neg A) = 1$

Aristotle's Theses: Prob. log. predictions (Pfeifer, 2012a, *The Monist*)

AT #1: $\neg(\neg A \to A)$

- $P(\neg(\neg A \supset A)) = P(\neg A)$
- $P(A|\neg A) = 0$, its negation: $P(\neg A|\neg A) = 1$

AT #2: $\neg(A \to \neg A)$

- $P(\neg(A \supset \neg A)) = P(A)$
- $P(\neg A|A) = 0$, its negation: $P(\neg\neg A|A) = P(A|A) = 1$

Experiment 1: Abstract version, Aristotle's Thesis #1

The letter "A" denotes a sentence, like "It is raining".

There are sentences, where you can infer only on the basis of their logical form, whether they are guaranteed to be false or guaranteed to be true. For example:

- "A and not-A" is guaranteed to be false.
- "A or not-A" is guaranteed to be true.

There are sentences, where you cannot infer only on the basis of their logical form, whether they are true or false. The sentence "A" ("It is raining."), for example, can be true but it can just as well be false: this depends upon whether it is actually raining.

Evaluate the following sentence (please tick exactly one alternative):

It is not the case, that: If not-A, then A.

The sentence in the box is guaranteed to be false	□
The sentence in the box is guaranteed to be true	□
One cannot infer whether the sentence is true or false	□

Experiment 1: Abstract version, Aristotle's Thesis #2

The letter "A" denotes a sentence, like "It is raining".

There are sentences, where you can infer only on the basis of their logical form, whether they are guaranteed to be false or guaranteed to be true. For example:

- "A and not-A" is guaranteed to be false.
- "A or not-A" is guaranteed to be true.

There are sentences, where you cannot infer only on the basis of their logical form, whether they are true or false. The sentence "A" ("It is raining."), for example, can be true but it can just as well be false: this depends upon whether it is actually raining.

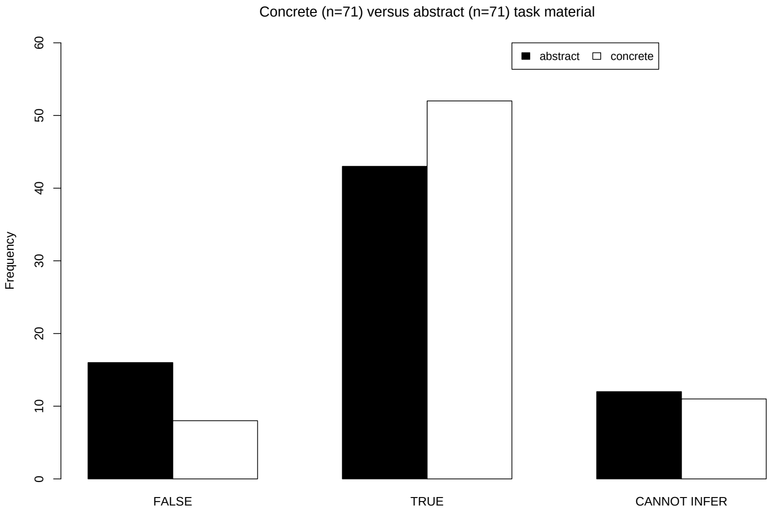
Evaluate the following sentence (please tick exactly one alternative):

It is not the case, that: If A, then not-A.

The sentence in the box is guaranteed to be false	☐
The sentence in the box is guaranteed to be true	☐
One cannot infer whether the sentence is true or false	☐

Experiment 1: Sample (Pfeifer, 2012a, *The Monist*)

- $N = 141$
- all psychology students (University of Salzburg)
- 91% third semester
- 78% female
- median age: 21 (1st Qu. $=$ 20, 3rd Qu. $=$23)



Concrete (n=71) versus abstract (n=71) task material
abstract
concrete
Frequency
0
10
20
30
40
50
60
FALSE
TRUE
CANNOT INFER

Scope ambiguities (Pfeifer, 2012a, *The Monist*)

(W) Negating the conditional: $\neg \underbrace{(A \rightarrow \neg A)}_{\text{wide scope}}$

(N) Negating the consequent: $(A \rightarrow \neg \underbrace{\neg A)}_{\text{narrow scope}}$

Scope ambiguities (Pfeifer, 2012a, *The Monist*)

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(W) and (N) are well defined for $\wedge$ and $\supset$.

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(W) Negating the conditional: $\neg \underbrace{(A \rightarrow \neg A)}_{\text{wide scope}}$

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(W) and (N) are well defined for $\wedge$ and $\supset$. Conditional events, $B|A$, are usually negated by (N), $P(\neg B|A)$.

Experiment 2: Design (Pfeifer, 2012a, *The Monist*)

Between participants: Explicit ($n_1 = 20$) vs. implicit negation ($n_2 = 20$)
Within participants: 12 Tasks

Task	Name	Argument form
1	Aristotle's Thesis 1	$\neg(A \to \neg A)$
2	Negated Reflexivity	$\neg(A \to A)$
3	Aristotle's Thesis 2	$\neg(\neg A \to A)$
4	Reflexivity	$A \to A$
5	Contingent Arg. 1	$A \to B$
6	Contingent Arg. 2	$\neg(A \to B)$
7-10	4 Probabilistic truth-table tasks	
11	Paradox 1	from B infer $A \to B$
12	Neg. Paradox 1	from B infer $A \to \neg B$

Experiment 2: Predictions (Pfeifer, 2012a, *The Monist*)

Argument form	Scope			
		wide	narrow	
	$\cdot\vert\cdot$	$\cdot\supset\cdot$	$\cdot\supset\cdot$	$\cdot\wedge\cdot$
$\neg(A \rightarrow \neg A)$	T	CT	T	T
$\neg(A \rightarrow A)$	F	F	CT	CT
$\neg(\neg A \rightarrow A)$	T	CT	T	T
$A \rightarrow A$	T	T	T	CT
$A \rightarrow B$	CT	CT	CT	CT
$\neg(A \rightarrow B)$	CT	CT	CT	CT
from B infer $A \rightarrow B$	U	H		U
from B infer $A \rightarrow \neg B$	U	H		L

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Experiment 2: Predictions $\cdot | \cdot$ against wide scope of $\cdot \supset \cdot$

Argument form		Scope		
		wide	narrow	
	$\cdot|\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$
$\neg(A \rightarrow \neg A)$	T	CT	T	T
$\neg(A \rightarrow A)$	F	F	CT	CT
$\neg(\neg A \rightarrow A)$	T	CT	T	T
$A \rightarrow A$	T	T	T	CT
$A \rightarrow B$	CT	CT	CT	CT
$\neg(A \rightarrow B)$	CT	CT	CT	CT
from B infer $A \rightarrow B$	U	H		U
from B infer $A \rightarrow \neg B$	U	H		L

Note: CT=can't tell, T=true, F=false,

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Experiment 2: Predictions $\cdot|\cdot$ against narrow scope of $\cdot \supset \cdot$

Argument form		Scope		
		wide	narrow	
	$\cdot\vert\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$
$\neg(A \rightarrow \neg A)$	T	CT	T	T
$\neg(A \rightarrow A)$	F	F	CT	CT
$\neg(\neg A \rightarrow A)$	T	CT	T	T
$A \rightarrow A$	T	T	T	CT
$A \rightarrow B$	CT	CT	CT	CT
$\neg(A \rightarrow B)$	CT	CT	CT	CT
from B infer $A \rightarrow B$	U	H		U
from B infer $A \rightarrow \neg B$	U	H		L

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Experiment 2: Sample (Pfeifer, 2012a, *The Monist*)

- $N = 40$ (University of Salzburg)
- no psychology students
- individual tested
- 50% female
- median age: 22 (1st Qu. $=$ 21, 3rd Qu. $=$23)

Experiment 2: Results (Pfeifer, 2012a, *The Monist*)

Argument form		Scope wide	Scope narrow		Responses in percent T	F	CT
	$\cdot \vert \cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$	T	F	CT
$\neg(A \to \neg A)$	T	CT	T	T	78	18	5
$\neg(A \to A)$	F	F	CT	CT	10	88	2
$\neg(\neg A \to A)$	T	CT	T	T	80	13	8
$A \to A$	T	T	T	CT	93	3	5
$A \to B$	CT	CT	CT	CT	0	13	88
$\neg(A \to B)$	CT	CT	CT	CT	20	3	78
from B infer $A \to B$	U	H		U	40	0	60
from B infer $A \to \neg B$	U	H		L	5	30	65

Note: CT=can't tell, T=true, F=false,

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Experiment 2: Results (Pfeifer, 2012a, *The Monist*)

Argument form		Scope wide	Scope narrow		Responses in percent T	F	CT
	$\cdot\mid\cdot$	$\cdot\supset\cdot$	$\cdot\supset\cdot$	$\cdot\wedge\cdot$	T	F	CT
$\neg(A\to\neg A)$	T	CT	T	T	78	18	5
$\neg(A\to A)$	F	F	CT	CT	10	88	2
$\neg(\neg A\to A)$	T	CT	T	T	80	13	8
$A\to A$	T	T	T	CT	93	3	5
$A\to B$	CT	CT	CT	CT	0	13	88
$\neg(A\to B)$	CT	CT	CT	CT	20	3	78
from B infer $A\to B$	U	H		U	40	0	60
from B infer $A\to\neg B$	U	H		L	5	30	65

Note: CT=can't tell, T=true, F=false,

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Aristotelian Syllogisms

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- Aristotelian syllogisms:
  - either too strict (universal, $\forall$) or too weak (existential, $\exists$) quantifiers
  - not a language for uncertainty / vagueness

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- Long history in psychology (starting with Störring (1908))
- Aristotelian syllogisms:
  - either too strict (universal, $\forall$) or too weak (existential, $\exists$) quantifiers
  - not a language for uncertainty / vagueness
- Developing coherence based probability logic semantics for Aristotelian syllogisms

Syllogistic types of propositions and figures (see, e.g. Pfeifer, 2006a)

Name of Proposition Type	*PL formula*
Universal affirmative (A)	$\forall x(Sx \supset Px) \wedge \exists x Sx$
Particular affirmative (I)	$\exists x(Sx \wedge Px)$
Universal negative (E)	$\forall x(Sx \supset \neg Px) \wedge \exists x Sx$
Particular negative (O)	$\exists x(Sx \wedge \neg Px)$

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	Figure name			
	1	2	3	4
Premise 1	*MP*	*PM*	*MP*	*PM*
Premise 2	*SM*	*SM*	*MS*	*MS*
Conclusion	*SP*	*SP*	*SP*	*SP*

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	Figure name			
	1	2	3	4
Premise 1	*MP*	*PM*	*MP*	*PM*
Premise 2	*SM*	*SM*	*MS*	*MS*
Conclusion	*SP*	*SP*	*SP*	*SP*

256 possible syllogisms, 24 Aristotelianly-valid, 9 require $\exists x Sx$

Traditionally valid syllogisms (see, e.g., Pfeifer, 2006a, Figure 2)

	Explicit existence assumptions		Implicit existence assumptions	
Figure I	AAA	Barbara	AAI	Barbari
	AII	Darii	EAO	Celaront
	EAE	Celarent		
	EIO	Ferio		
Figure II	AEE	Camestres	AEO	Camestrop
	AOO	Baroco	EAO	Cesaro
	EAE	Cesare		
	EIO	Festino		
Figure III	AII	Datisi	AAI	Darapti
	EIO	Ferison	EAO	Felapton
	IAI	Disamis		
	OAO	Bocardo		
Figure IV	AEE	Camenes	AAI	Bramantip
	EIO	Fresison	AEO	Camenop
	IAI	Dimaris	EAO	Fesapo

Example: Modus Barbara

All philosophers are mortal.
All members of the Vienna Circle are philosophers.

All members of the Vienna Circle are mortal.

Modus Barbara

(A)	All M are P
(A)	All S are M
(A)	All S are P

Modus Barbara

(A)	All M are P
(A)	All S are M
(A)	All S are P

(A)	$\forall x(Mx \supset Px)$	$(\wedge \exists x Mx)$
(A)	$\forall x(Sx \supset Mx)$	$(\wedge \exists x Sx)$
(A)	$\forall x(Sx \supset Px)$	

Modus Barbara

(A)	All M are P
(A)	All S are M
(A)	All S are P

(A)	$\forall x(Mx \supset Px)$	$(\wedge \exists x Mx)$
(A)	$\forall x(Sx \supset Mx)$	$(\wedge \exists x Sx)$
(A)	$\forall x(Sx \supset Px)$	

	Figure name			
	1	2	3	4
Premise 1	MP	PM	MP	PM
Premise 2	SM	SM	MS	MS
Conclusion	SP	SP	SP	SP

...transitive structure of Figure 1

Example: Modus Barbari

$$\frac{\begin{array}{l}\text{All } M \text{ are } P \\ \text{All } S \text{ are } M\end{array}}{\text{At least one } S \text{ is } P}$$

$$\frac{\begin{array}{lll}\forall x(Mx \supset Px) & \wedge & \exists x Mx \\ \forall x(Sx \supset Mx) & \wedge & \exists x Sx\end{array}}{\exists x(Sx \wedge Px)}$$

The probability heuristics model (Chater & Oaksford, 1999; Oaksford & Chater, 2009)

Definitions of the basic sentences:

	Quantified statement	Prob. interpretation
(A)	All S are P	$p(P \mid S) = 1$
(E)	No S is P	$p(P \mid S) = 0$
(I)	Some S are P	$p(P \mid S) > 0$
(O)	Some S are not-P	$p(P \mid S) < 1$

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(E)	No S is P	$p(P\|S) = 0$
(I)	Some S are P	$p(P\|S) > 0$
(O)	Some S are not-P	$p(P\|S) < 1$
	Most S are P	$1 - \Delta < p(P\|S) < 1$
	Few S are P	$0 < p(P\|S) < \Delta$

. . . where Δ is small

The probability heuristics model: Probabilistic syllogisms

- Assumption: Conditional independence between the end terms (i.e., S and P) given the middle term (i.e., M):

$$p(S \wedge P|M) = p(S|M)p(P|M)$$

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- Sample reconstruction of Modus Barbara (assumed implicitly $p(S) > 0$, $p(M) > 0$):

$$\begin{array}{rc}
\text{(A)} & p(P|M) = 1 \\
\text{(A)} & p(M|S) = 1 \\
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Note, that we do not assume $p(S) > 0$ and $p(M) > 0$ in the coherence framework. Moreover, if $p(S|M) = 0$, then $p(S \wedge P|M) = 0$.

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Note, that we do not assume $p(S) > 0$ and $p(M) > 0$ in the coherence framework. Moreover, if $p(S|M) = 0$, then $p(S \wedge P|M) = 0$. Then, the premises are satisfied but $0 \leq p(P|S) \leq 1$ is coherent. Thus, Modus Barbara does not hold.

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The coherence perspective on syllogisms

(joint work with G. Sanfilippo & A. Gilio)

Towards a probabilistic semantics

CondEv-Formalization:

All S are P:	$p(P\|S) = 1$
Almost-all S are P:	$p(P\|S) \gg .5$
Most S are P:	$p(P\|S) > .5$
At least one S is P:	$p(P\|S) > 0$

Existential import: Different options

- Positive probability of the conditioning event, e.g.:

$$\text{All } S \text{ are } P\text{: } p(S) > 0$$

- $p(S|M) > 0$ (and $p(M|P) > 0$) (Dubois, Godo, López de Màntaras, & Prade, 1993)

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- Replacing the first premise by a logical constraint, e.g.:

$$\frac{\begin{array}{c}\models (M \supset P)\\ p(M|S) = 1\end{array}}{p(P|S) = 1}$$

- Strengthening the antecedent of the first premise, e.g.:

$$\frac{\begin{array}{c}p(P|S \wedge M) = 1\\ p(M|S) = 1\end{array}}{p(P|S) = 1}$$

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- Conditional event EI: Positive probability of the conditioning event, given the disjunction of all conditioning events (Gilio, Pfeifer, & Sanfilippo, submitted):

$$\frac{\begin{array}{c}p(P|M) = 1\\ p(M|S) = 1\\ p(S|S \vee M) > 0\end{array}}{p(P|S) = 1}$$

- $p(S|S \vee M) > 0$ neither implies $p(S) > 0$ nor $p(S|M) > 0$

Probabilistic Figure 1, conditional event EI

Premises		E.I.	Conclusion
$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
x	y	t	$[z', z'']$
x	y	0	[0, 1]

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1	1	$t > 0$	[1, 1]	(Modus Barbara)
1	y	$t > 0$	$[y, 1]$	
.9	1	1	[.9, .9]	
.9	1	.5	[.8, 1]	
.9	1	.2	[.5, 1]	
.9	1	.1	[0, 1]	

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.9	1	.1	$[0, 1]$	
1	$]0, 1]$	$t > 0$	$]0, 1]$	(Modus Darii)

If $p(S|S \vee M) > 0$, then

$$z' = \max\left\{0, xy - \frac{(1-t)(1-x)}{t}\right\}$$

$$z'' = \min\left\{1, (1-x)(1-y) + \frac{x}{t}\right\}.$$

(Theorem 3 of Gilio, Pfeifer, and Sanfilippo (submitted). *Transitive reasoning with imprecise probabilities.*)

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The Tweety problem

The Tweety problem (picture© by L. Ewing, S. Budig, A. Gerwinski; http://commons.wikimedia.org)

The Tweety problem (picture© by ytse19; http://mi9.com/flying-tux_35453.html)

System P: Rationality postulates for nonmonotonic reasoning (Kraus, Lehmann, & Magidor, 1990)

Reflexivity (axiom): $\alpha \mid\!\sim \alpha$

Left logical equivalence:

from $\vDash \alpha \equiv \beta$ and $\alpha \mid\!\sim \gamma$ infer $\beta \mid\!\sim \gamma$

Right weakening:

from $\vDash \alpha \supset \beta$ and $\gamma \mid\!\sim \alpha$ infer $\gamma \mid\!\sim \beta$

Or: from $\alpha \mid\!\sim \gamma$ and $\beta \mid\!\sim \gamma$ infer $\alpha \vee \beta \mid\!\sim \gamma$

Cut: from $\alpha \wedge \beta \mid\!\sim \gamma$ and $\alpha \mid\!\sim \beta$ infer $\alpha \mid\!\sim \gamma$

Cautious monotonicity:

from $\alpha \mid\!\sim \beta$ and $\alpha \mid\!\sim \gamma$ infer $\alpha \wedge \beta \mid\!\sim \gamma$

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$\alpha \mid\!\sim \beta$ is read as If α, $\underbrace{\textit{normally}}_{?}$ β

Probabilistic version of System P (Gilio (2002); Table 2 Pfeifer and Kleiter (2009))

Name	*Probability logical version*
Left logical equivalence	$\models(E_1 \equiv E_2), P(E_3\|E_1) = x \therefore P(E_3\|E_2) = x$
Right weakening	$P(E_1\|E_3) = x, \models(E_1 \supset E_2) \therefore P(E_2\|E_3) \in [x, 1]$
Cut	$P(E_2\|E_1 \wedge E_3) = x, P(E_1\|E_3) = y$
	$\therefore P(E_2\|E_3) \in [xy, 1 - y + xy]$
And	$P(E_2\|E_1) = x, P(E_3\|E_1) = y$
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Cautious monotonicity	$P(E_2\|E_1) = x, P(E_3\|E_1) = y$
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	$\therefore P(E_3\|E_1 \vee E_2) \in [xy/(x+y-xy), (x+y-2xy)/(1-xy)]$
Transitivity	$P(E_2\|E_1) = x, P(E_3\|E_2) = y \therefore P(E_3\|E_1) \in [0, 1]$
Contraposition	$P(E_2\|E_1) = x \therefore P(\neg E_1\|\neg E_2) \in [0, 1]$
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. . . probabilistically non-informative

The Tweety problem (Pfeifer, 2012b)

$\mathfrak{P}1$	$P[\text{Fly}(x) \mid \text{Bird}(x)] = .95.$	*(Birds can normally fly.)*
$\mathfrak{P}2$	Bird(Tweety).	*(Tweety is a bird.)*
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$\mathfrak{P}3$	Penguin(Tweety).	*(Tweety is a penguin.)*
$\mathfrak{P}4$	$P[\text{Fly}(x)\|\text{Penguin}(x)] = .01.$	*(Penguins normally can't fly.)*
$\mathfrak{P}5$	$P[\text{Bird}(x)\|\text{Penguin}(x)] = .99.$	*(Penguins are normally birds.)*
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Example 1: (Cautious) monotonicity

- In logic
  from $\boxed{A \supset B}$ infer $\boxed{(A \wedge C) \supset B}$
- In probability logic
  from $\boxed{P(B|A) = x}$ infer $\boxed{0 \leq P(B|A \wedge C) \leq 1}$

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  But: from $P(A \supset B) = x$ infer $x \leq P((A \wedge C) \supset B) \leq 1$

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- Cautious monotonicity (Gilio, 2002)
  from $P(B|A) = x$ and $P(C|A) = y$
  infer $\max(0, (x + y - 1)/x) \leq P(C|A \wedge B) \leq \min(y/x, 1)$

Example task: Monotonicity (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

exactly 72% wear a black suit.

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About the guests at a prom we know the following:

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Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit,
given they are at this prom and wear glasses?

Example task: Cautious monotonicity (Pfeifer & Kleiter, 2003)

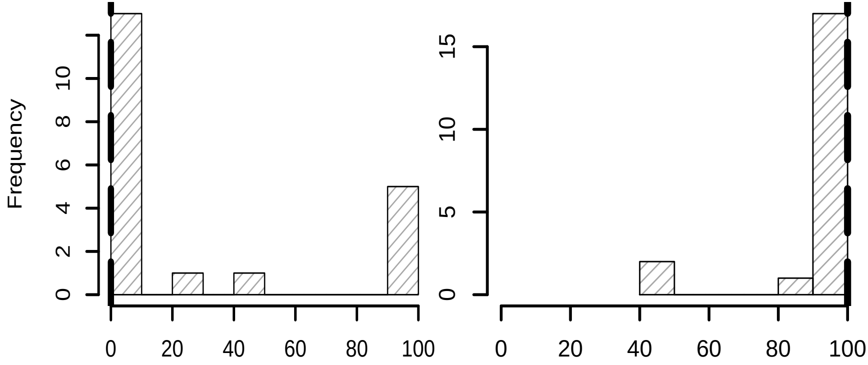
About the guests at a prom we know the following:

exactly 72% wear a black suit.
exactly 63% wear glasses.

Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit,
given they are at this prom and wear glasses?

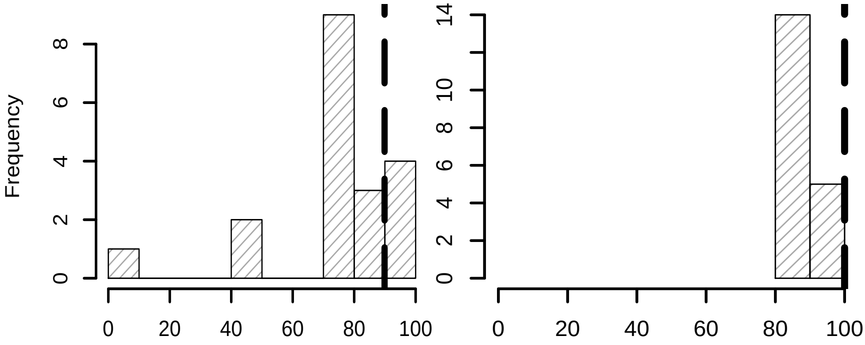
Results – Monotonicity (Example Task 1; Pfeifer and Kleiter (2003))



lower bound responses upper bound responses

$(n_1 = 20)$

Results – Cautious monotonicity (Example Task 1; Pfeifer and Kleiter (2003))



lower bound responses upper bound responses

$(n_2 = 19)$

Example 2: Contraposition

- In logic
  from $\boxed{A \supset B}$ infer $\boxed{\neg B \supset \neg A}$

  from $\boxed{\neg B \supset \neg A}$ infer $\boxed{A \supset B}$

Example 2: Contraposition

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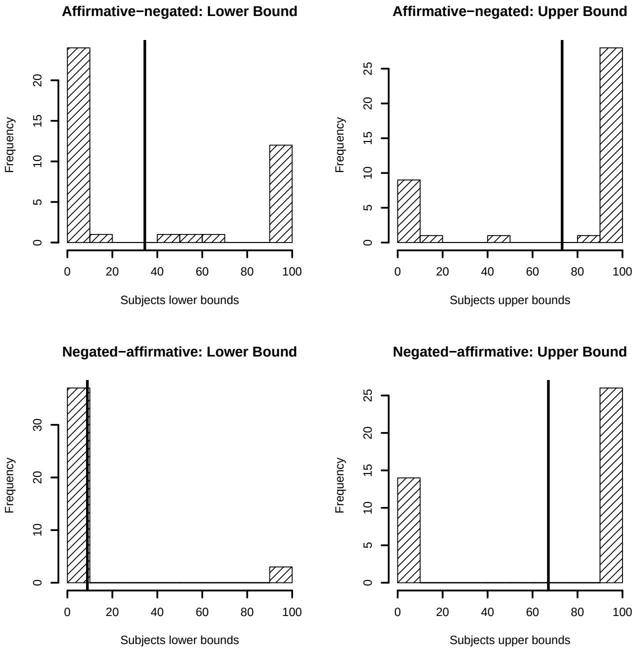
- But

$$P(A \supset B) = P(\neg B \supset \neg A)$$

Results Contraposition ($n_1 = 40$, $n_2 = 40$; Pfeifer and Kleiter (2006b))



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