Causal inference in graphical models with latent variables. From theory to practice.

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Causality and Probability in the Sciences
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Outline

1. Introduction
2. Causal Inference
3. Learning
4. Parametrisation

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Introduction

Subtasks of causal modeling with latent variables:

- Structure learning from data:
  - observational
  - experimental
- Learning parameters
- Probabilistic inference
- Causal inference

Problem

No integral approach for all these tasks in the presence of latent variables.

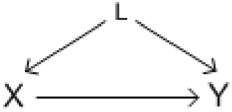
- Causal inference :
  semi-Markovian causal models
- Structure learning from observational data:
  ancestral graph models

Our solution:

- use ancestral graph models for learning
- transform the result into a semi-Markovian causal model

Assumptions

- data generated by an underlying faithful causal DAG
- some variables are latent to the user
- sufficient data and learning algorithms make no errors
- no latent common cause of 2 variables with an immediate causal connection
- no selection bias



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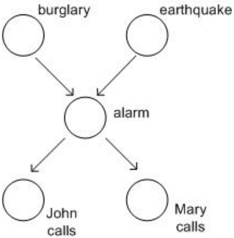
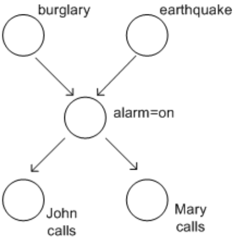
Probabilistic vs Causal Inference

underlying DAG



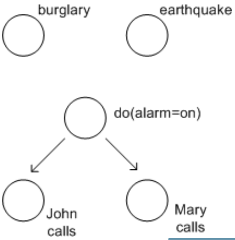
after observation

- instantiate the observed variables
- propagate



after manipulation

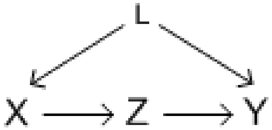
- replacer old causes
- instantiate
- propagate



With latent variables

Causal inference becomes more complicated:

- **replace old causes**
- instantiate
- propagate

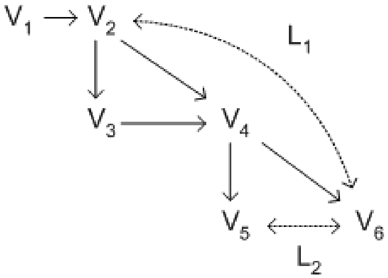


$P(Y = y|do(X = x))$: manipulate variable X and study the effect on Y.

Representation for causal inference

semi-Markoviens causal models (SMCM)

- directed edge represents an autonomous causal relation
- bi-directed edge represents a latent common cause
- importance: every models with arbitrary latent variables can be transformed into a SMCM
- a joint probability distribution: e.g. $P(V_1, \ldots, V_6)$



Inference in SMCMs

causal inference algorithm exists (Tian & Pearl), but:

- no efficient parametrisation
- no probabilistic inference algorithm
- no structural learning algorithm

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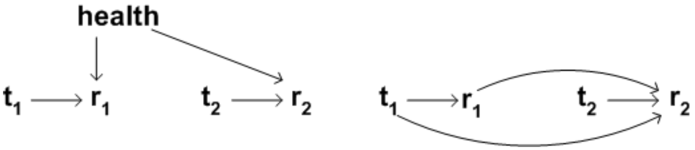
Representation for learning

The class of DAGs is not complete under marginalisation.

I.e., a DAG of the observable variables can not exactly represent all the independences between the variables.



Maximal ancestral graphs (MAG)

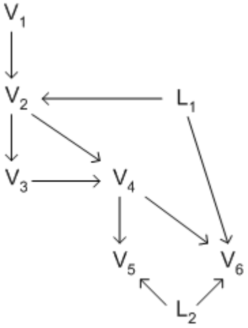
Maximal ancestral graphs without conditionning

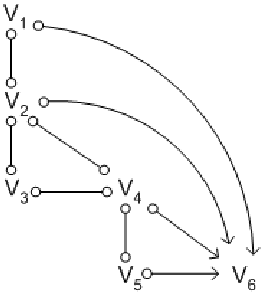
- directed edges have an ancestral meaning $\neq$ causal: there is a causal path in the underlying DAG
- bi-directed edges: latent common causes
- max. 1 edge between 2 variables: ancestral relations absorb latent common causes
- maximal: every absent edge represents an independence

Markov equivalence class

complete partial ancestral graph (CPAG):

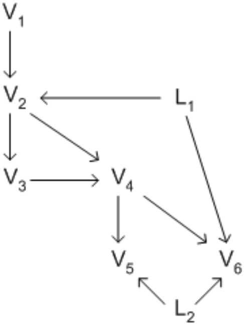
- Fast Causal Inference (FCI)
- Rules to orient edges
- 3 possible edge marks: o, -, >

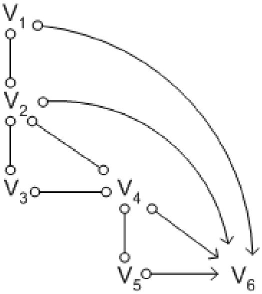




3 possible explanations for each edge:

- causal relation V_1, V_2
- latent variable V_2, V_6
- "inducing path" between V_1 et V_6
  - V_1 can not be separated from V_6 by using observed variables
  - due to the maximality of the models FCI finds an edge





CPAG $\longrightarrow$ SMCM

Perform experiments to differ between cases:

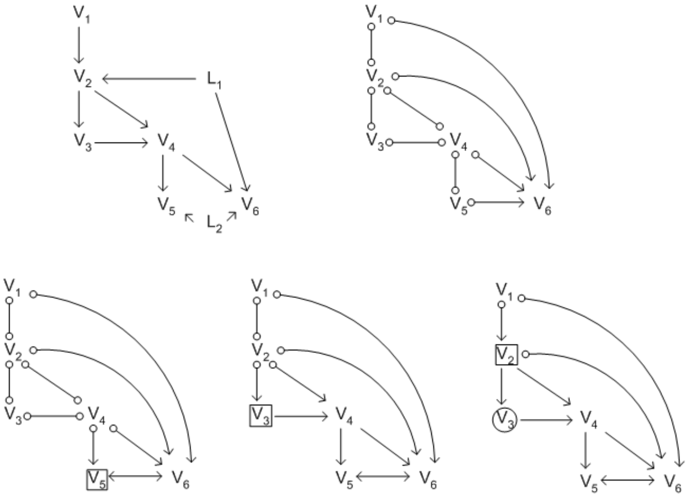
- Type 1: resolve $o\!\!\longrightarrow$
- Type 2: resolve $o\!\!-\!\!o$
- Remove i-false edges

CPAG $\longrightarrow$ SMCM (Type 1)

Type 1: resolve $A \multimap B$

- $exp(A) \not\rightsquigarrow B$: $A \leftrightarrow B$
- $exp(A) \rightsquigarrow B$:
  - $\nexists$ pot.dir. path $A \dashrightarrow B$ of length ≥ 2: $A \longrightarrow B$
  - $\exists$ pot.dir. path $A \dashrightarrow B$ of length ≥ 2:
    block each pot.dir. path by conditioning on a set D
    - $exp(A)|D \rightsquigarrow B$: $A \longrightarrow B$
    - $exp(A)|D \not\rightsquigarrow B$: $A \leftrightarrow B$

Type 1: examples



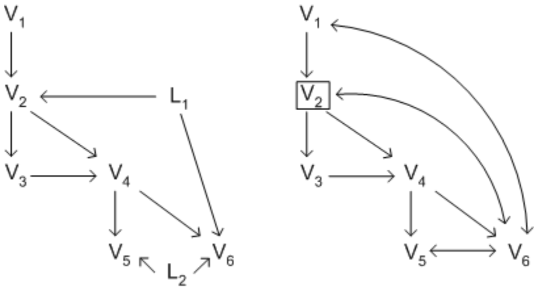
CPAG $\rightarrow$ SMCM (Type 2)

Type 2: resolve o—o

- Easily transformed to Type 1

Remove i-false edges

- recognize the edges $A \leftrightarrow B$ and $A \rightarrow B$ that are possible created due to an inducing path
- block each inducing path between A, B with experiments E
- block each other path between A, B by condit. on a set D
- $exp(E)|D$: $A \not\perp\!\!\!\perp B$: confounding edge is real
  $A \perp\!\!\!\perp B$: remove i-false edge



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Parametrisation for discrete variables

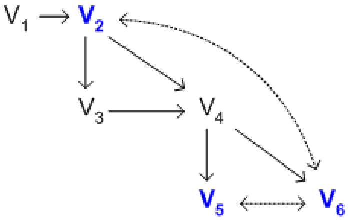
Represent the SMCM with a DAG

- this DAG is an I-map of SMCM
- i.e. all the indep. found in the DAG are also in the SMCM
- use this DAG for:
  - learning parameters
  - probabilistic inference
- for causal inference:
  - use the SMCM for the structure
  - use the DAG for the parameters

Some definitions

C-component

In a SMCM, the set of observable variables that are connected by a bi-directed path belong to the same **c-component** (from "confounded component").



Parametrisation: procedure

Given a SMCM and a topological order, the PR-representation is obtained as follows:

- The nodes are V, the observable variables of the SMCM.
- The directed edges of the SMCM remain.
- The bi-directed edges in the SMCM are replaced:

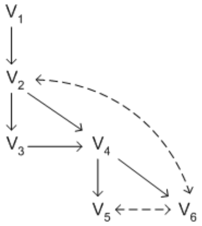
  Add a directed edge from node V_i to node V_j iff:
  - V_i is a topological predecessor in the same c-component
  - a parent of a topological predecessor in the same c-component
  - except if there was already an edge between nodes V_i and V_j.

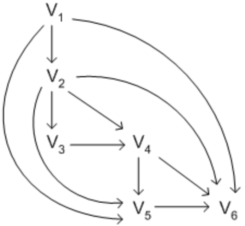
Parametrisation: example

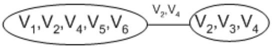
Add an edge $V_i \rightarrow V_j$ iff:

- V_i is a topological predecessor in the same c-component
- a parent of a topological predecessor in the same c-component

with topological order $V_2 < V_5 < V_6$:







Conclusion

From theory to practice.

Somewhat closer, still a lot of assumptions to relax:

- take into account:
  - possible errors in the learning algorithms
  - cost of experiments and impossible experiments
  - rules for marking edges after each experiment
- allow:
  - selection bias
  - confounding between variables with an immediate causal connection
- resolve practical issues such as finding pot. inducing paths and pot. directed paths