Maximum Entropy and Inductive Logic II

Jürgen Landes

Spring School on Inductive Logic

Canterbury, 20.04.2015 - 21.04.2015

Outline

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- Rational subjective Beliefs
- Finite propositional language L
- Variables $v_1, \ldots, v_n$
- Sentences of L, SL
- No funny business, self-reference, truth predicates, self-fulfilling

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Probabilities

- $P : SL \to [0, 1]$
- Set of probability functions $\mathbb{P}$
- Possible world, states

$$\omega = v_1 \wedge v_2 \wedge \neg v_3 \wedge \ldots \wedge v_{n_1} \wedge \neg v_n$$

- $$P(\varphi) = \sum_{\substack{\omega \in \Omega \\ \omega \models \varphi}} P(\omega).$$

- A probability function $P \in \mathbb{P}$ is uniquely determined by its values on possible worlds, $\langle P(\omega) \; : \; \omega \in \Omega \rangle$.

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Inference Processes

- Knowledge K leads to $\mathbb{E} \subseteq \mathbb{P}$.
- Formally, an *inference process* is a map from a set of probability functions (here $\mathbb{E}$) to the set of probability functions.
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Obviously right

- Adopt the function, $P^\dagger$, which solve this optimisation problem

$$\text{maximise: } -\sum_{\omega\in\Omega} P(\omega)\log(P(\omega))$$
$$\text{subject to: } P \in \mathbb{E} \ .$$

- Shannon Entropy: $H(P) = -\sum_{\omega\in\Omega} P(\omega)\log(P(\omega))$.
- Maximum Entropy Inference Process (MaxEnt):

$$\{P^{+}\} = \arg\sup_{P\in\mathbb{E}} H(P)$$

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Desideratum: Open-mindedness

- $P^{+}(\omega) > 0$, if there exists a $P \in \mathbb{E}$ such that $P(\omega) > 0$.

Desideratum: Language Invariance

- L' generated by $v_1, v_2, \ldots, v_n, v_{n+1}$
- and the same knowledge and the same patient? For all $\varphi \in SL$

$$P'(\varphi) = P^+(\varphi) \ .$$

- Repeat argument for even larger languages.

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1. Recap
2. The Classical Derivation
   - Desiderata
   - The Classical Result
3. A Modern Approach
   - Decision Making
   - Justification
4. Universes
5. Objections, your Honor!
   - Bug or Feature?
   - Issues of Language
   - Risqué

Renaming

- Names should not matter.

Irrelevance

- Knowledge entirely irrelevant to the problem in hand can he ignored.
- Languages: L_1 with variables $v_1, \ldots, v_s$, L_2 with variables $v_{s+1}, \ldots, v_n$.
- 2 Bodies of Knowledge: K_1 formulated within L_1, K_2 formulated with in L_2
- For all $\varphi \in L_1$ (problem at hand)

$$IP(\{v_1, \ldots, v_n\}, K_1)(\varphi) = IP(\{v_1, \ldots, v_n\}, K_1 \cup K_2)(\varphi) \ .$$

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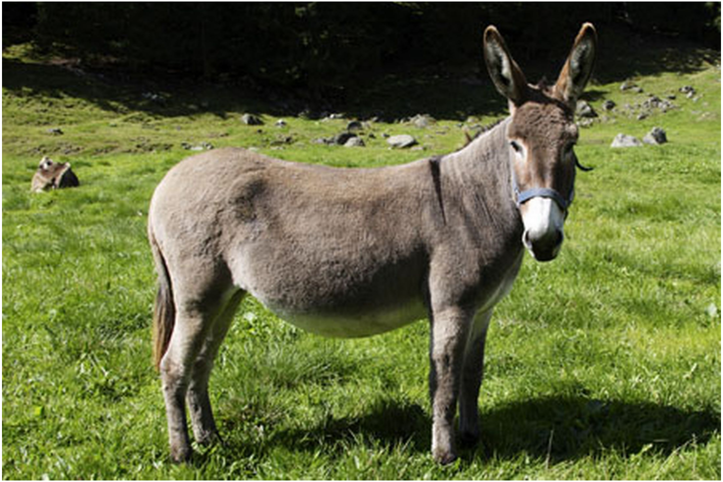

Obstinacy

- Learning something one already beliefs should not make any difference.
- Given two consistent bodies of knowledge, K_1, K_2 (on the same language)
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Relativisation

- Suppose you know the probability of φ.
- That is, for all $P \in \mathbb{E}$ there exists some $c \in [0, 1]$ such that $P(\varphi) = c$.
- If $\omega^- \models \varphi$, then $IP(\omega^-)$ should not depend on your knowledge about the $\neg\varphi$-worlds.

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Independence

- If K does not contain any information which makes v_1, v_2 conditionally dependent on v_3, then v_1, v_2 should be conditionally independent given v_3.
- If $K = \{P(v_3) = \gamma, P(v_1|v_3) = \frac{\alpha}{\gamma}, P(v_2|v_3) = \frac{\beta}{\gamma}\}$ $(P(\gamma) > 0)$, then

$$IP(K)(v_1 \wedge v_2|v_3) = \frac{\alpha}{\gamma}\frac{\beta}{\gamma} \ .$$

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Continuity

- If $\mathbb{E}'$ can be obtained from $\mathbb{E}$ by moving or deforming $\mathbb{E}$ a bit, then
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- For all sentences $\varphi \in SL$: $IP(\mathbb{E}')(\varphi) \approx IP(\mathbb{E})(\varphi)$.

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Alena & Jeff

Theorem

If $\mathbb{E}$ is closed, convex and non-empty, IP satisfies Renaming, Irrelevance, Obstinacy, Relativisation, Independence and Continuity, then IP is MaxEnt.
MaxEnt satisfies language invariance and open-mindedness and is internal.

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Continuity - Bristolean

- One considers the decision problem of setting degrees of belief
- and wonders which beliefs are *best*.
- A *utility function u* is used to measure the goodness / badness / utility of a belief function.
- Determine which beliefs have the "best utility".
- The usual caveats for decision making apply: Non-causal, act-state independence, etc.
- Decision theoretic norm still undetermined.

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Logarithmic Utility

- If ω is the true world, then the utility is $u(\omega, P^+) = \log(P^+(\omega))$.
- Expected utility for $P \in \mathbb{E}$ is: $\sum_{\omega\in\Omega} P(\omega)\log(P^+(\omega))$.
- If $\mathbb{E}$ is closed, convex and non-empty, maximise worst case expected utility:

$$\arg\sup_{P^+\in\mathbb{P}} \inf_{P\in\mathbb{E}} \sum_{\omega\in\Omega} P(\omega)\log(P^+(\omega)) = \{P^\dagger\} \quad .$$

- The latest rage is to understand u as an accuracy measure.
- Measure closeness to the truth.

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Utility Theory

- If you do have a utility function and a Decision Theoretic Norm,
- then you can show that a particular inference process is optimal with respect to the above.
- Justifications in terms of *common-sense* principles hinge on the common-sensicality of the principles.
- Justifications in terms of utility functions *appear* much more objective.
- However, one has to give a story explaining where the utility function and the DTN come from.

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Ensembles, populations

- Consider an entire population M with $|M|$ members.
- K consists of statements of the form
- 32.2% of patients complain of symptom φ
- $$|\{x \in M \mid x \text{ complains about } \varphi\}| \approx \frac{32.2}{100}|M|$$
- Consider all populations M of fixed large size $|M|$ which satisfy the *above*.
- Then almost all *such* populations M satisfy

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Alena & Jeff 2 – one last justification

Theorem

For all $\delta > 0$ there exists some natural number M_0 such that for all sentences $\varphi \in SL$ and all fixed sizes of populations $|M| \geq M_0$ the proportion of populations M of fixed size $|M| \geq M_0$ which satisfy

$$\left| \frac{|\{x \in M \mid x \text{ complains about } \varphi\}|}{|M|} - P^{\dagger}(\varphi) \right| < \delta$$

is greater or equal than $1 - \delta$.

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Equivocation, come what may

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- $P_{=}(\omega) = \frac{1}{|\Omega|}$, $\{P_{=}\} = \arg\sup_{P\in\mathbb{E}} H(P)$.
- If $P_{=} \in \mathbb{E}$, then $P^{\dagger} = P_{=}$.
- Entropy strictly decreases along the rays originating from $P_{=}$.
- Entropy maximiser different from $P_{=}$ face $P_{=}$.
- If $\mathbb{E} = \{P \in \mathbb{P} \mid 0 \leq P(v_1) \leq 0.5\}$, then $P^{\dagger}(v_1) = 0.5$.
- Can this be right?

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Outline

1. Recap
2. The Classical Derivation
   - Desiderata
   - The Classical Result
3. A Modern Approach
   - Decision Making
   - Justification
4. Universes
5. Objections, your Honor!
   - Bug or Feature?
   - Issues of Language
   - Risqué

Dependence

- No knowledge, $\mathbb{E} = \mathbb{P}$.
- Possible worlds: red and blue.
- $P^{\dagger}(red) = \frac{1}{2}$. Okay.
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- $L = \langle v_1, v_2, \ldots, v_n \rangle$, $L' = \langle v_1, v_2, \ldots, v_n, v_{n+1} \rangle$.
- Knowledge only concerns $v_1, \ldots, v_n$.
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$$IP(L)(\varphi) = IP(L')(\varphi)$$

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MaxEnt_W

- MaxEnt_W: An agent ought to equivocate (sufficiently) between the basic possibilities that she can express.
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- Equivocating over a density level ($\geq$ 1) leads to centre of mass, regardless of the level.
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That's it. Thank you! Questions? – Progic Tomorrow

Logarithmic Utility

- Intuitive human sensations tend to be logarithmic functions of the stimulus. – Jaynes
- Savage
- Jon's L1 – L4