Maximum Entropy and Inductive Logic I

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Spring School on Inductive Logic

Canterbury, 20.04.2015 - 21.04.2015

Outline

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Background

- Ballpark: Rational Beliefs
- What does an agent rationally belief?
- As opposed to: When do *we say* that an agent beliefs X?
- Imagine: You are the agent.
- Normative formal approach

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- Patient, migraines, doctor
- Given the doctor's background knowledge,
- patient complaining of certain symptoms,
- exhibiting certain traits:
- what should the doctor believe?
- Eventually, what should the doctor do?
- On the basis of *imperfect information*.

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Carnap's Principle

- Carnap1947: A rational agent ought to take all available evidence into account when forming beliefs.
- Reasonable.
- So, the doctor has to take *all* background knowledge and the patient's individual properties into account.
- Ignoring some information is a "No no".
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- Reasonable, ... *really?*

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Our Language

- Finite propositional language L
- Variables $v_1, \ldots, v_n$
- Connectives $\wedge, \vee, \neg, \rightarrow, \leftarrow, \leftrightarrow$
- Sentences of L, SL
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- No funny business, self-reference, truth predicates, self-fulfilling

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Probabilities

- Probabilistic framework
- Probability function, P
- Subjective degrees of belief – Dutch Book
- $P : SL \to [0, 1]$
- $P(\tau) = 1$ for all tautologies $\tau \in SL$.
- $P(\varphi \vee \theta) = P(\varphi) + P(\theta)$, if $\models \neg(\varphi \wedge \theta)$.
- $\varphi \models \theta$ implies $P(\varphi) \leq P(\theta)$.
- $P(\varphi \vee \theta) = P(\varphi) + P(\theta) - P(\varphi \wedge \theta)$.
- Set of probability functions $\mathbb{P}$

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Worlds

- Possible world, states

$$\omega = v_1 \wedge v_2 \wedge \neg v_3 \wedge \ldots \wedge v_{n_1} \wedge \neg v_n$$

- Set of possible worlds Ω.
- Proposition $F \subseteq \Omega$.
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$$P(\varphi) = \sum_{\substack{\omega \in \Omega \\ \omega \models \varphi}} P(\omega).$$

- A probability function $P \in \mathbb{P}$ is uniquely determined by its values on possible worlds, $\langle P(\omega) \ : \ \omega \in \Omega \rangle$.

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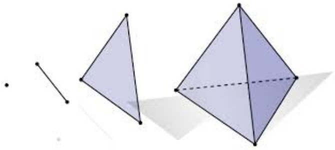
Knowledge

- Think of $\mathbb{P}$ as some set of functions.
- Spanned by the possible worlds ω.
- It can be represented by a simplex.
- Dimension equal to the number of possible worlds/states.
- Doctor's background knowledge K.
- Identify K with a set $\mathbb{E} \subseteq \mathbb{P}$



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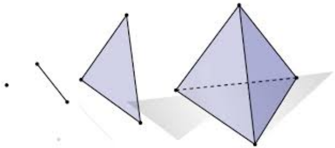

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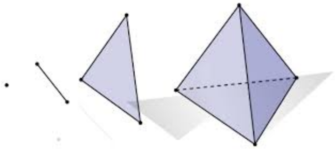

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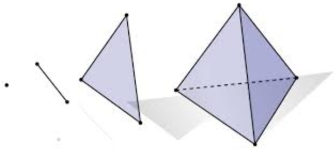

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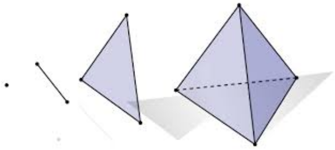

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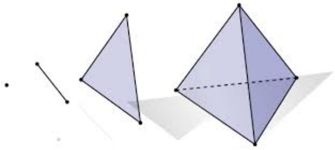

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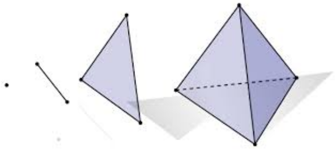

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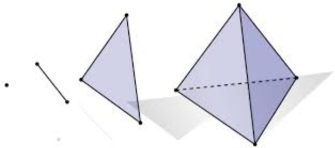

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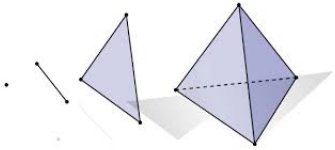

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Watt's Assumption

- The set $\mathbb{E}$ contains all the doctor's knowledge.
- If you don't formalise *all* knowledge, then you should not be surprised by our answer.
- Garbage in, garbage out.
- This course is *not* about the highly relevant and non-trivial problem of obtaining $\mathbb{E}$ from K.
- We will assume that $\mathbb{E} \neq \emptyset$.

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Example Knowledge

- Symptom S_1 is a very good indicator of condition C.
- 10% of migraines are triggered by stress.
- In 23% of cases in which patients complain about symptom S_2, migraines will spontaneously cease within half a day.

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Condensing Knowledge

- Idea:
- Input knowledge K
- Output belief function $P^+ \in \mathbb{P}$
- Adopt this function P^+ for decision making.
- P^+ reflects the doctor's knowledge K.
- We are not fabricating knowledge out of thin air.
- We do the best we can, given limited information.
- Some information is lost in this process.

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Conditionalisation

- What about the properties of the patient?
- Yes, what about them??
- I have not forgotten about them.
- Properties $\psi_1, \ldots, \psi_r$
- $\psi := \psi_1 \wedge \ldots \wedge \psi_r$
- Consider conditional probability ($P^+(\psi) > 0$)

$$P^+(\varphi|\psi) := \frac{P^+(\varphi \wedge \psi)}{P^+(\psi)} .$$

- Roughly, assume that ψ is true.

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- Formally, an *inference process* is a map from a set of probability functions (here $\mathbb{E}$) to the set of probability functions.
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Outline

Inductive Logic

- How strongly do uncertain premises entail a conclusion?
- Uncertain premises ...
- conclusion ...
- entail ...

$$\underbrace{P^*(\varphi_1) \in I_1, P^*(\varphi_2) \in I_2, \ldots, P^*(\varphi_k) \in I_k}_{\text{Knowledge}} \underbrace{\approx}_{\text{Entailment}} \underbrace{\psi^?}_{\text{Conclusion}}$$

- $? = P^\dagger(\psi)$, a single real number.

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Outline

Obviously right – Tomorrow!

- Adopt the function, $P^{\dagger} = P^{+}$, which solve this optimisation problem

$$\text{maximise: } -\sum_{\omega\in\Omega} P(\omega)\log(P(\omega))$$

$$\text{subject to: } P \in \mathbb{E} \quad .$$

Intuitively right

- Pick a “middling” function in $\mathbb{E}$.
- How formalise “middling”?
-
- Pick the centre of $\mathbb{E}$!
- Okay, yes, but how does one work out the centre?

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At the tip of a finger

The centre

- Centre of Mass
- Point of Balance
-

$$P^{+}_{CoM} := \frac{\int_{P\in\mathbb{E}} P\ dp}{\int_{P\in\mathbb{E}}\ dp} \ .$$

- Centre of Mass inference process.

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You

- ....

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- $P^{+} \in \mathbb{E}$
- ...
- ...
- if $\mathbb{E}$ is not empty
- convex
- and closed.

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Careful, careful

- Open-mindedness
- Do not rule anything out, which you consider possible.
- If there exists a probability function $P \in \mathbb{E}$ with $P(\omega) > 0$, then $P^+(\omega) > 0$.

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Open-mindedness is Possible

- For all such ω, pick an $P_\omega \in \mathbb{P}$
- and also pick an $r_\omega \in (0,1)$ such that $\sum r_\omega = 1$.
- Then the convex combination $\sum r_\omega P_\omega$ is a probability function. ($\mathbb{P}$ is convex.)
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Language Invariance

- What if we started with $v_1, v_2, \ldots, v_n, v_{n+1}$
- the same knowledge and the same patient?
- Apply the above recipe and obtain P'.
- For a sentence in the original language $\varphi \in SL$, please complete the below

$$P'(\varphi) = \ ?$$

- Shocker: Centre of Mass is not language invariant!

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That's it. Thank you! Questions?