

# A New Causal Power Theory

Univ of Kent  
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# Causal Power Theories

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- S Wright (1934) The method of path coefficients. *Ann Math Stat*, 5, 161-215.
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- C Glymour & P Cheng (1998) Causal mechanism and probability. Oaksford and Chater (Eds.) *Rational models of cognition*. Oxford.
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- L Hope and K Korb (2005) An Information-theoretic causal power theory. *Australian AI Conference*, pp. 805-811. Springer.

# Causal Power

Causal Power  
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## What is *causal power*?

*The power of some event to bring about  
(prevent) another event.*

### Examples

- The power of anticoagulants to prevent death from heart attack.
- The power of exercise to prevent heart attacks.
- The power of a doctor's advice to exercise to bring about exercise.
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One important point:

*Causal power is always relative to a reference class.*

- The power of the pill to prevent pregnancy
  - Amongst women
  - Amongst men
- The power of extra exercise to prevent heart attacks.
  - Amongst middle-aged couch potatoes
  - Amongst athletes
  - Amongst teenagers

Usually the reference class is implicit.

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We should like to develop an explicit quantitative measure of causal power, generalizing (improving on) our intuitive judgments.

- Stochastic causality comes in degrees (“effect size” in medicine)
- Potentially allowing for precise judgments of causal attribution
  - hence, the interest of cog psych
- Clarifying the explanatory import of causal Bayesian networks

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# Path Models

Most theories of causal power are based on binary networks (Cheng, Glymour, Hiddleston).

The first theory, Wright (1934), uses standardized linear Gaussian models: path models.

## Desideratum 1

Causal power theory should apply to **any** kind of causal Bayesian network – linear, binomial, multinomial.



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## Path Models

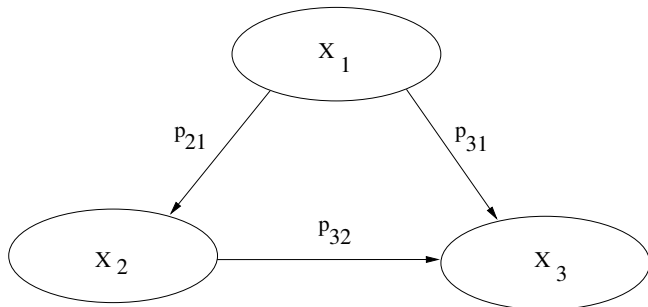
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# Path Models



r	1	2	3
1	1		
2	$r_{12}$	1	
3	$r_{13}$	$r_{23}$	1

# Path Models

## Theorem (Explained Variation)

*Path coefficients are equal to the square root of the variation in the child variable attributable to the parent.*

i.e.,

$$\sum_i p_{ji}^2 = 1$$

- As a consequence of standardization
- Requires a residual term  $U$  with coefficient  $p_{ju}$

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# Wright's Decomposition Rule

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Wright developed a graphical rule for relating (observed) correlations with path coefficients (i.e., relating probability and causality).

*Fundamental idea: correlation results from causal influence along certain paths between variables.*

## Definition (Admissible Path)

$\Phi_k$  is an **admissible path** between  $X_i$  and  $X_j$  iff it is an undirected path connecting  $X_i$  and  $X_j$  s.t. it does not go against the direction of an arc *after* having gone forward.



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This can be thought of as 3 rules in 1 for defining paths supporting causal influence:

- 1 Directed chains support causal influence
- 2 Common ancestors support causal influence between descendants
- 3 Common descendants don't support causal influence between ancestors

*(This prefigures Pearl's d-separation rules.)*

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To assess the strength of causal influence along an admissible path:

## Definition (Valuation)

The valuation of a path is

$$v(\Phi_k) = \prod_{lm} p_{lm} \text{ for all } X_m \rightarrow X_l \in \Phi_k$$

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## Theorem (Wright's Decomposition Rule)

*The correlation  $r_{ij}$  between variables  $X_i$  and  $X_j$ , where  $X_i$  is an ancestor of  $X_j$ , can be rewritten as:*

$$r_{ij} = \sum_k v(\Phi_k)$$

*where  $\Phi_k$  is an admissible path between  $X_i$  and  $X_j$  and  $v(\cdot)$  is a valuation of that path.*

## Wright's Decomposition Rule

This gives a direct relation between path coefficients and correlations:

$$r_{12} = \rho_{21}$$

$$r_{13} = \rho_{31} + \rho_{21}\rho_{32}$$

$$r_{23} = \rho_{32} + \rho_{21}\rho_{31}$$

We can solve for the  $\rho_{ij}$ :

$$\rho_{21} = r_{12}$$

$$\rho_{31} = \frac{r_{13} - r_{23}r_{12}}{1 - r_{12}^2}$$

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# Wright's Power Theory

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## Wright's implicit causal power theory:

The causal power of  $C$  for  $E$  is:

$$CP(C, E) = \sum_k \prod_{lm} p_{lm} \quad \begin{array}{l} \text{for all } X_m \rightarrow X_l \in \Phi_k \\ \text{for all } \Phi_k = C \rightarrow \dots \rightarrow E \end{array}$$

**NB:** This is *implicit* in Wright's treatment; Wright had no explicit causal power theory.

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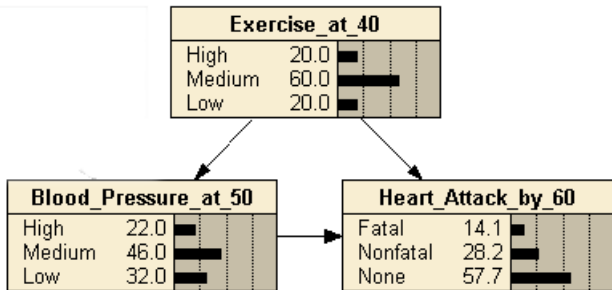
## Heart Attack Example

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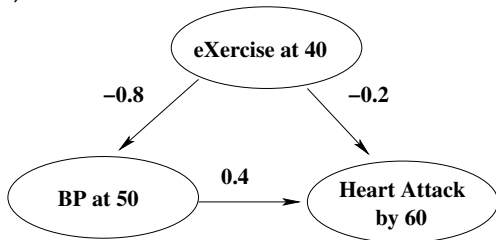
So: What is the causal power of BP for HA?

Note:

- Backpath BP  $\leftarrow$  X  $\rightarrow$  HA
- Messy interaction btw BP and X upon HA

## Heart Attack Example

Consider the linear approximation (dropping the messy interaction):



$$r_{BP,HA} = \rho_{BP,X}\rho_{HA,X} + \rho_{HA,BP} = 0.56$$

The Wright causal power of BP for HA

- Discounts the backpath  $BP \leftarrow X \rightarrow HA$
- Equals 0.4

# Wright's Power Theory

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- Relates *variables*  $C$  and  $E$ , not their *values*
  - To relate values, we should need to discretize variable ranges in some way
- Wright's theory has been very successful
- Wright's theory is compatible with current Bayesian network theory

## Desideratum 2

Causal power theory should generalize Wright's power theory.

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# Modern Causal Power Theory

Cheng & Glymour

The Cheng (1997) and Glymour & Cheng (1998) PC Theory applies to binary variables taking particular values,  $C = c$  and  $E = e$ , given assumptions:

- $\exists$  a direct causal connection  $C \rightarrow E$
- $C$  is independent of any other cause of  $E$
- $C$  does not interact with any other cause of  $E$
- Probabilistic relevance:  
$$\Delta P = p(e|c) - p(e|\neg c) \neq 0$$
- Spurious causes must be eliminated
  - e.g., replaced by common causes

*(Echoing Salmon on SR explanation and Suppes on probabilistic causation)*

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# “Power PC” Theory

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## Definition (Causal Power)

For positive  $\Delta P$  (generative cause), the power of  $c$  to bring about  $e$ :

$$p_c = \frac{\Delta P}{1 - P(e|\neg c)}$$

Idea:  $\Delta P$  directly is not a fair measure of  $p_c$

- since there is a background rate  $P(e|\neg c)$
- $\Delta P$  should be relativized to the remainder
  - those cases that would have been  $\neg e$  but for  $c$

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## Definition (Preventive Causal Power)

For negative  $\Delta P$  (preventive cause), the power of  $c$  to stop  $e$ :

$$\overline{p}_c = \frac{-\Delta P}{P(e|\neg c)}$$

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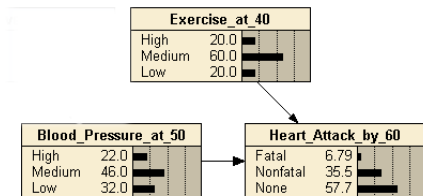
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Reconstruct variables as binary; delete arc between X and BP; eliminate messy interaction btw X and BP

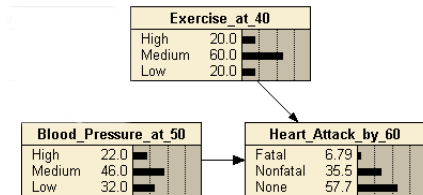
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The prob that high BP will kill someone, given survival o/w **relative to the model**

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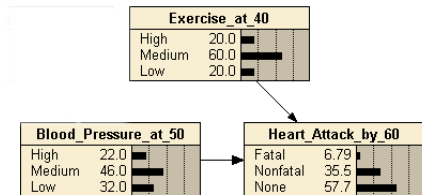
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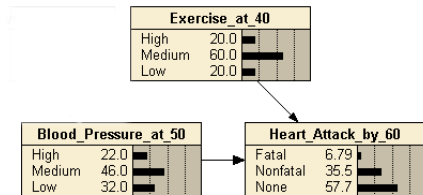
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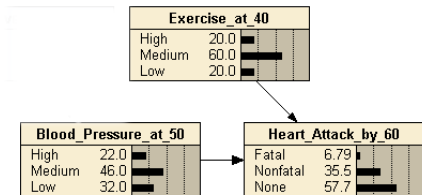
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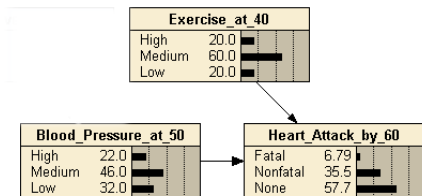
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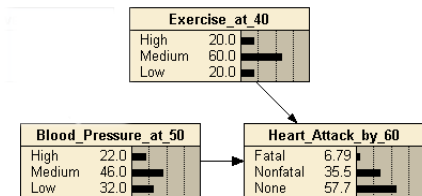
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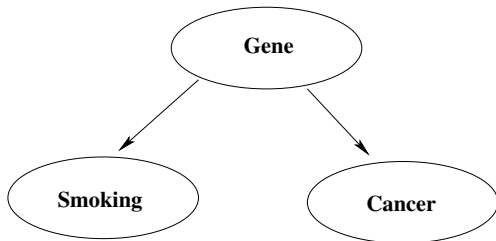
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# Fisher Model



MI travels up and down any Wrightian path, including back paths; causal influences clearly don't (outside of EPR problems).

# Intervention

## An intervention upon $V \in g$

- Alters the distribution over  $V$  in  $g$
- From outside the system, outside  $g$

*Causal Bayesian networks are ideal for representing interventions, augmenting  $g$  by adding an intervention variable  $I$ , yielding the augmented  $g^*$ .*

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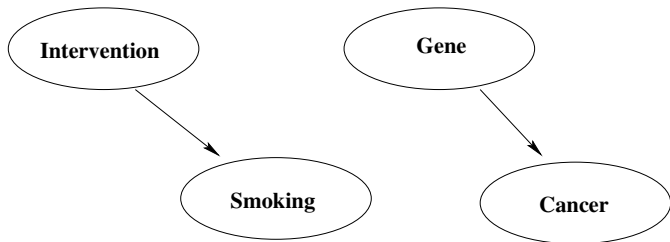
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## Fisher Model II

We shall use perfect (overwhelming) interventions to measure causal power



*which was, of course, Fisher's idea!*

# Causal Information

Idea

## Use MI, but *asymmetrically*

- By first intervening perfectly upon  $C$  and only then measuring MI

We get the asymmetrical dependence of  $E$  upon  $C$ , when  $C$  is set to a fixed distribution.

This automates Wright's power theory,

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Questions related to the various CI measures:

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*As in randomized experimental designs*

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*What is the greatest possible influence of C for E? How strongly could lowering BP impact on heart attack outcomes?*

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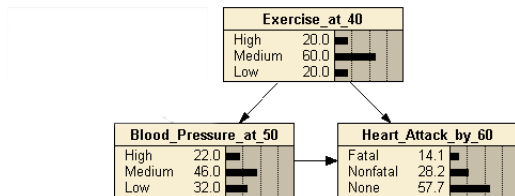
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# Heart Attack Example

MI vs CI



$$\begin{aligned}
 MI(BP, HA) &= \sum_{c \in C, e \in E} P(c, e) \log \frac{P(c, e)}{P(c)P(e)} \\
 &= 0.28 \\
 CI(BP, HA) &= 0.13
 \end{aligned}$$

- The difference is due to the interventional elimination of the backpath through X

# Heart Attack Example

CI causal power

Causal Power  
Theory

Wright's Theory

PC Theory

Causal Information

Two CI causal powers for fatal heart attack:

$$CI(c, e) = p(e|c) \log \frac{p(e|c)}{p(e)}$$

- $CI(\text{high BP, fatal HA}) = 0.23 \log \frac{0.23}{0.0679} = 0.405$
- $CI(\text{low BP, fatal HA}) = 0.052 \log \frac{0.052}{0.0679} = -0.02$

# Heart Attack Example

Cheng

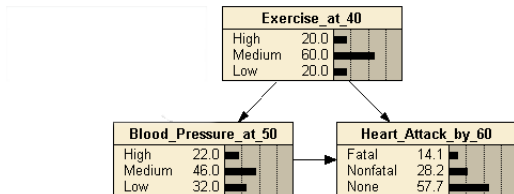
Causal Power  
Theory

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PC Theory

Causal Information

What happens to Cheng's PC Theory when we apply it to the original model?



The reintroduction of backpath and interaction

- $p_c = \Delta P / [1 - P(HA | \neg BP)] = 0.16$   
– a decline of 20%

This shows significant errors in attempting to apply PC Theory.

## PC Theories

Causal Power  
Theory

Wright's Theory

PC Theory

Causal Information

	Variables	Structures	Causality
Wright	Linear	Open	Transitive
Cheng/Glymour	Binary	Noisy-OR Isolated Causes	Transitive
CI	Various	Open	Various (Interactions, thresholds)

## Desiderata

- 1 Causal power theory should apply to **any** kind of causal Bayesian network – linear, binomial, multinomial.
- 2 Causal power theory should generalize Wright's power theory.
- 3 Causal power theory should apply both to variables and their values.
- 4 Causal power theory should allow for non-transitive and interactive relations.



# CI Summary

- CI reports the expected code length needed to report the value of  $E$  given the value of  $C$  in  $g^*$ 
  - This can be converted back into the language of probabilities
- CI satisfies all of our desiderata, unlike any known alternative
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