A probabilistic logic incorporating posteriors of hierarchic graphical models

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Motivation and background

Fusion of factual knowledge and complex posteriors

The „Most Probable Sentences” problem

Applications

Summary

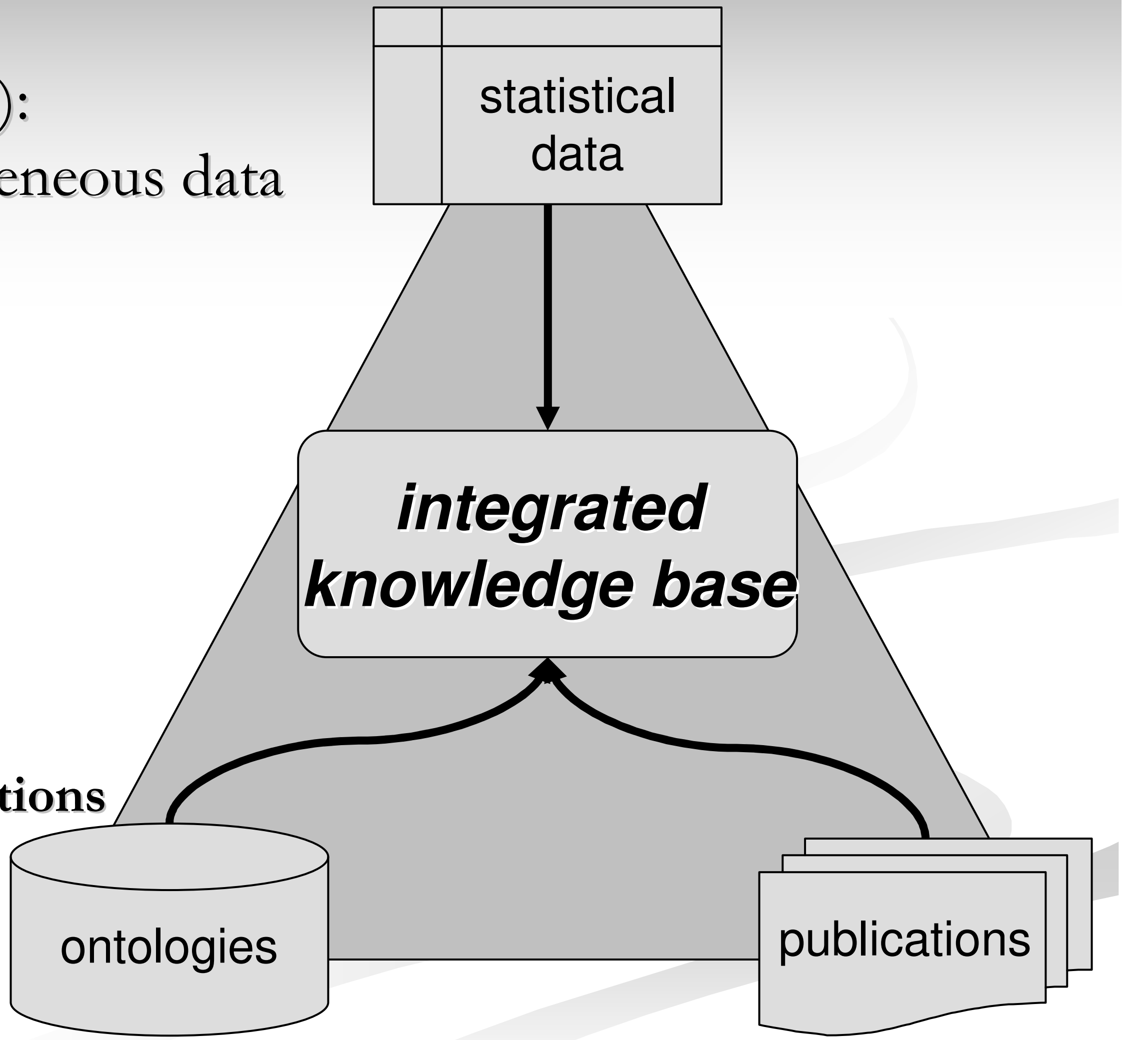
Motivation

Background (e.g. biomedicine):
Rapidly accumulating heterogeneous data

Uncertain (statistical) sources
- **Clinical observations**
- Gene activity measurements
- Expert-defined models

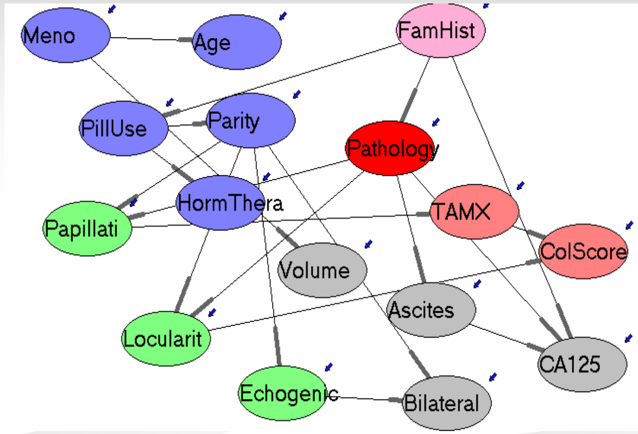
Factual knowledge
- Domain ontologies
- **Natural-language publications**

Goal:
- Fusion



The model class: Bayesian networks

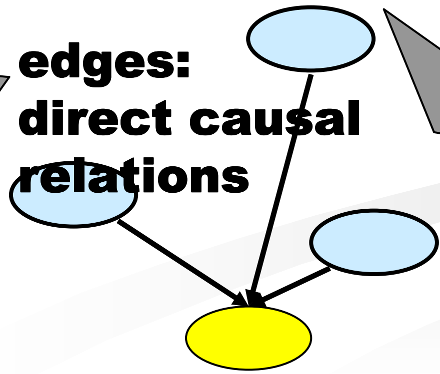
- directed acyclic graph (DAG)
  - nodes – domain entities
  - edges – direct probabilistic relations
- conditional probability models P(X|Pa(X))
- interpretations:



	effective representation of the distribution

$$P(X_1,..X_N)=\prod_{i=1}^{N} P(X_i \mid Pa(X_i))$$

DAG structure: dependency map (d-separation)



edges: direct causal relations

Bayesian statistics and inference

Knowledge representation:

- set of models (feature values)
- distribution over them

Learning (predictive inference):

$$P(G \mid D) = \frac{P(D \mid G) \times P(G)}{P(D)}$$

Parametric inference:

application: feature learning

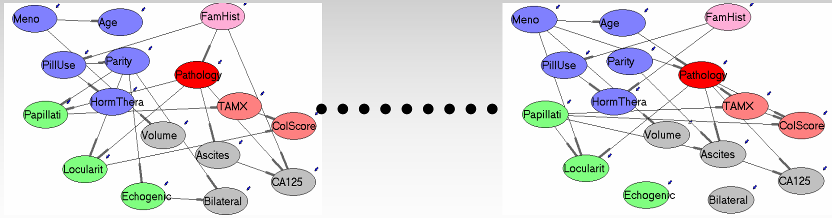
$$P(F = f) = \sum_{G:F^G = f} P(G)$$

- Practical methods: MCMC (Markov Chain Monte Carlo) sampling

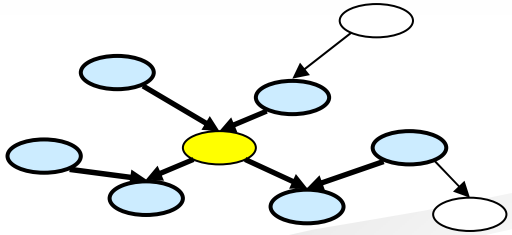
Interesting Bayesian network features

Levels of model features/posteriors:

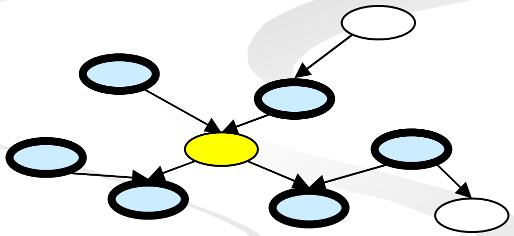
- full structures/DAGs



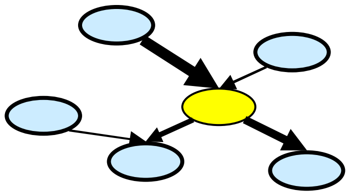
- Markov Blanket Graphs (MBGs)
  - (1) parents of the node,
    (2) its children,
    (3) parents of the children



- Markov Blankets (MBs)
  - the set of nodes which probabilistically isolate the target from the rest of the model



- Markov Blanket Membership (MBM)
- directed edges



Motivations:

- simpler (lower-level) features are easier to learn

Basics of MCMC methods I.

Goal:

- approximating the full-scale summation/integral with an average over DAGs

- DAG-MCMC algorithm:
  - random walk in the space of DAGs
  - evaluating the feature for the visited models
  - approximating the distribution with the sample / calculating average

$$P(F = f) = \sum_{F^G = f} P(G)$$

Basics of MCMC methods II.

- Ordering-based MCMC algorithm
  - Random walk in the space of variable orderings

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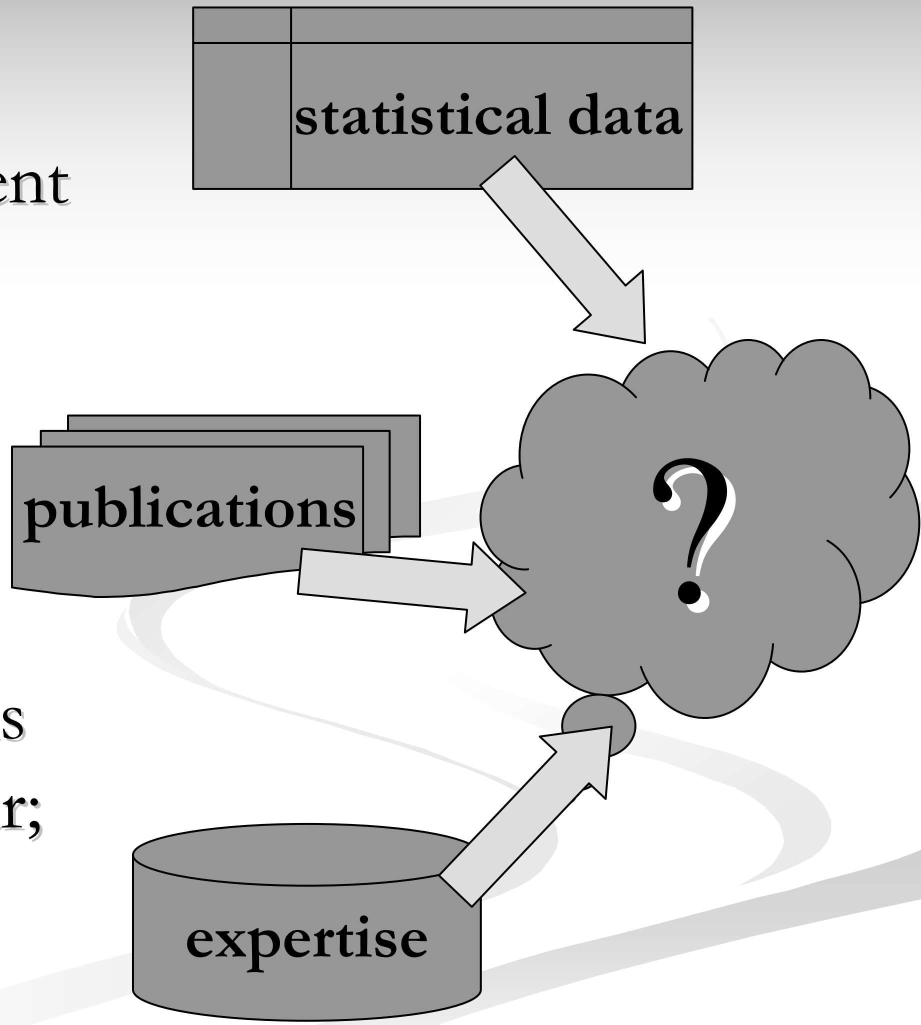
Fusion of factual knowledge and complex posteriors

- uncertain (statistical) inference
  - clinical/gene measurement data
  - statistics of publications
- factual knowledge
  - ontologies
  - meta-data of publications (authors, publication year; concept occurrence)



Earlier works – first-order probabilistic logic

J. Y. Halpern. 1990. An analysis of first-order logics of probability.

- distribution over possible worlds
- distribution over possible objects

Practical approaches:

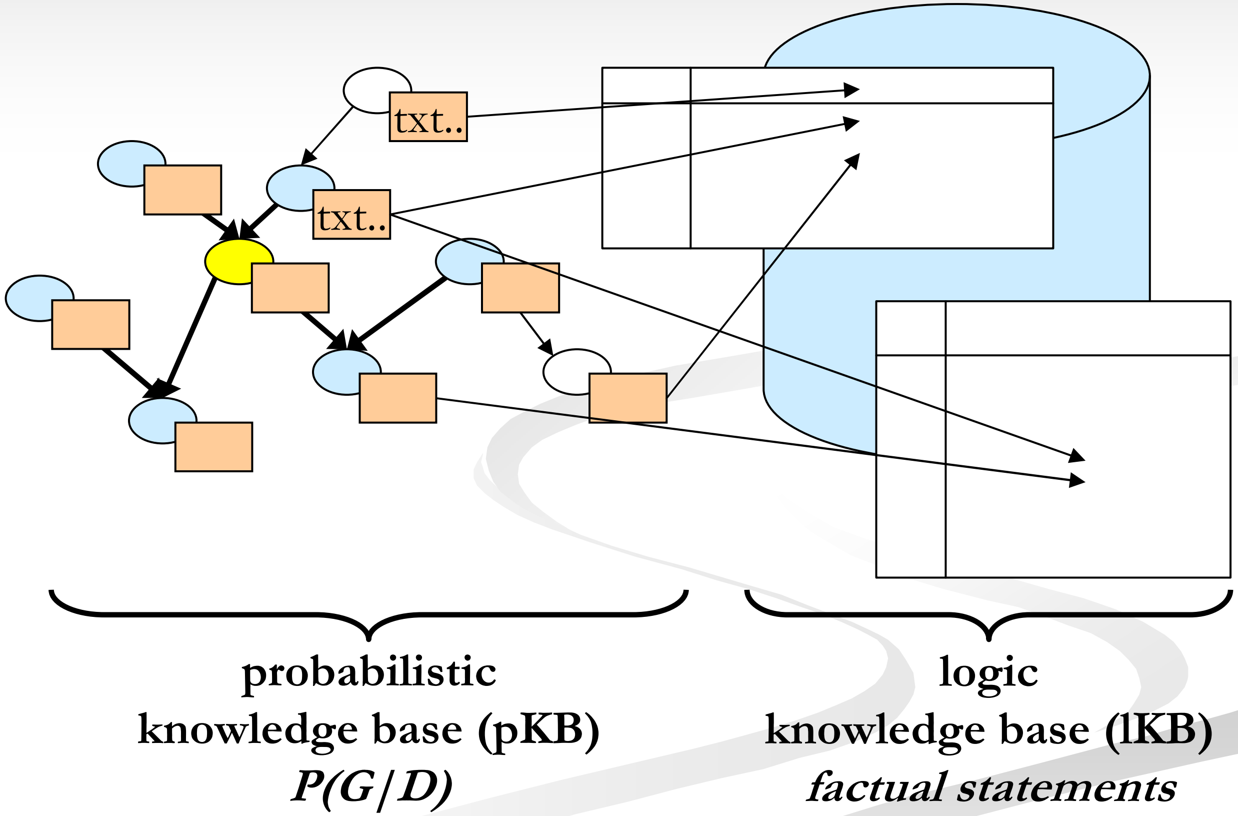
- Knowledge-based model construction
  M. P. Wellman, J. S. Breese and R. P. Goldman. 1992. From knowledge bases to decision models.
- Relational Bayesian networks
  M. Jaeger. 1997. Relational Bayesian networks.
- Bayesian logic programs
  K. Kersting, and L. de Raedt. 2000. Bayesian logic programs.
- Stochastic logic programs
  S. H. Muggleton. 2001. Stochastic logic programs.
- Bayesian logic
  B. Milch, B. Marthi, and S. Russell. 2004. Blog: Relational modeling with unknown objects.
- Markov logic
  P. Domingos and M. Richardson. 2006. Markov logic networks.

- Overview
  Nicos Angelopoulos and James Cussens. 2006. Bayesian learning of Bayesian networks with informative priors.

Probabilistic Annotated Bayesian Network Knowledge Bases (PABN-KBs)

Model elements:

- Bayesian networks
  - uncertain part
  - probabilistic relations
- Textual/xml annotations
  - basic description of entities
  - mapping model elements to „outer" knowledge sources
  - Factual knowledge bases logical relations among objects



The FOPL language

Language elements:

- Bayesian networks – possible worlds
- distribution over models
- factual knowledge sources

- predicates:
  - inherited from the factual part
  - dependence (structural) relations of model elements (nodes)
- logic operators
  - $\wedge, \vee, \neg, \exists, \forall$
- semantics (probability of a sentence):

$$p(\alpha \mid \mathcal{K}) = E_{p(M \mid \mathcal{K})}[\alpha^M] = \sum_{G:M(G) \in \mathcal{M}(K^l)} \alpha^{M(G)} p(G \mid \mathcal{K})$$

A BN oriented FOPL language

Predicates about structural relations of nodes

- directed edge
- directed path
- MBM – Markov blanket membership

- parental set
- MB – Markov blanket set
- MBG – Markov blanket graph

A BN oriented FOPL KB

KB elements:

- set of Bayesian networks
- prior distribution over them

- annotations: nodes → ontology entities

  concepts in articles

- publication repository
- ontologies like GO

FOPL query examples

Basics:

„What is the Markov blanket of variable 'X'?"

Involving annotations:

„What is the probability that the Markov blanket of 'X' will contain variables from a certain class?"

Involving the logic knowledge base:

„What is the probability that every concept in the Markov blanket of 'X' appears in one publication?"

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Feature subset selection and its generalizations I.

Feature subset selection (FSS)

- A relevant subset of features
- Two main approaches:
    1. Wrappers (score function)
    2. Filters (conditional distribution of target variables)

Feature subset selection and its generalizations II.

Feature Subgraph Selection (FSG)

- Identification of the relevant subgraph :
  - Relevant subset of features
  - Dependency between them

Most Probable Sentence (MPS)

- Not enough data to select a feature with a dominant posterior
- Multiple selection : K best features

The „Most Probable Sentences" problem

Given: a set of sentences of interest – target set

Task: find the N highest-scoring (those of highest probability) ones

E.g.: *„find the N most probable MBM sets of variable X"*

Search-and-estimate schemes

Exhaustive enumeration of DAGs:

- for each possible DAG: evaluate target sentences
- calculate the probability of each sentence on-the-fly (sum the probability of the models in which the sentence is true)

Theoretical solution used for testing

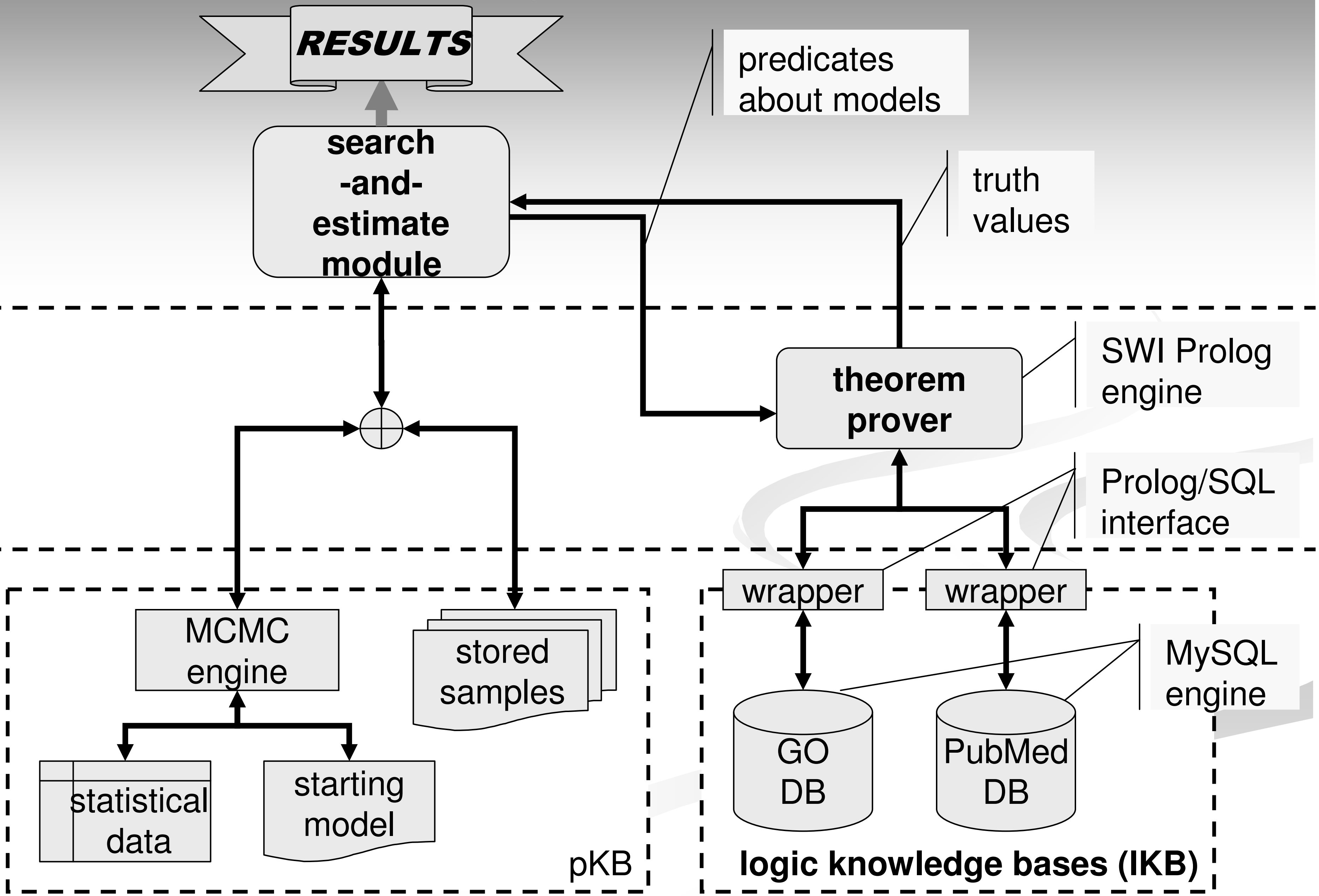
MCMC sampling

- MCMC random walk over DAGs
- for each visited DAG:
  find true sentences / groundings
- update their probabilities:

$$P(S) = \frac{\#(G : KB^G \mapsto S)}{\#(G)}$$

```
listMPS = [];
while( ! MCMC.hasConverged() ){
      model = MCMC.nextModel();
      listNewS = PLEngine.evaluatePredicate(query, model);
      listMPS.insert(listNewS);
}
listMPS.orderBy(prob);
listMPS.truncate(N);
return listMPS;
```

The implemented framework



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Application domain – I. Rheumatoid arthritis

Statistical data:

- clinical observations
  - age, …
  - gender
  - received cures
- gene measurements: single nucleotide polymorphisms (SNP)

Logic knowledge:

- SNP database

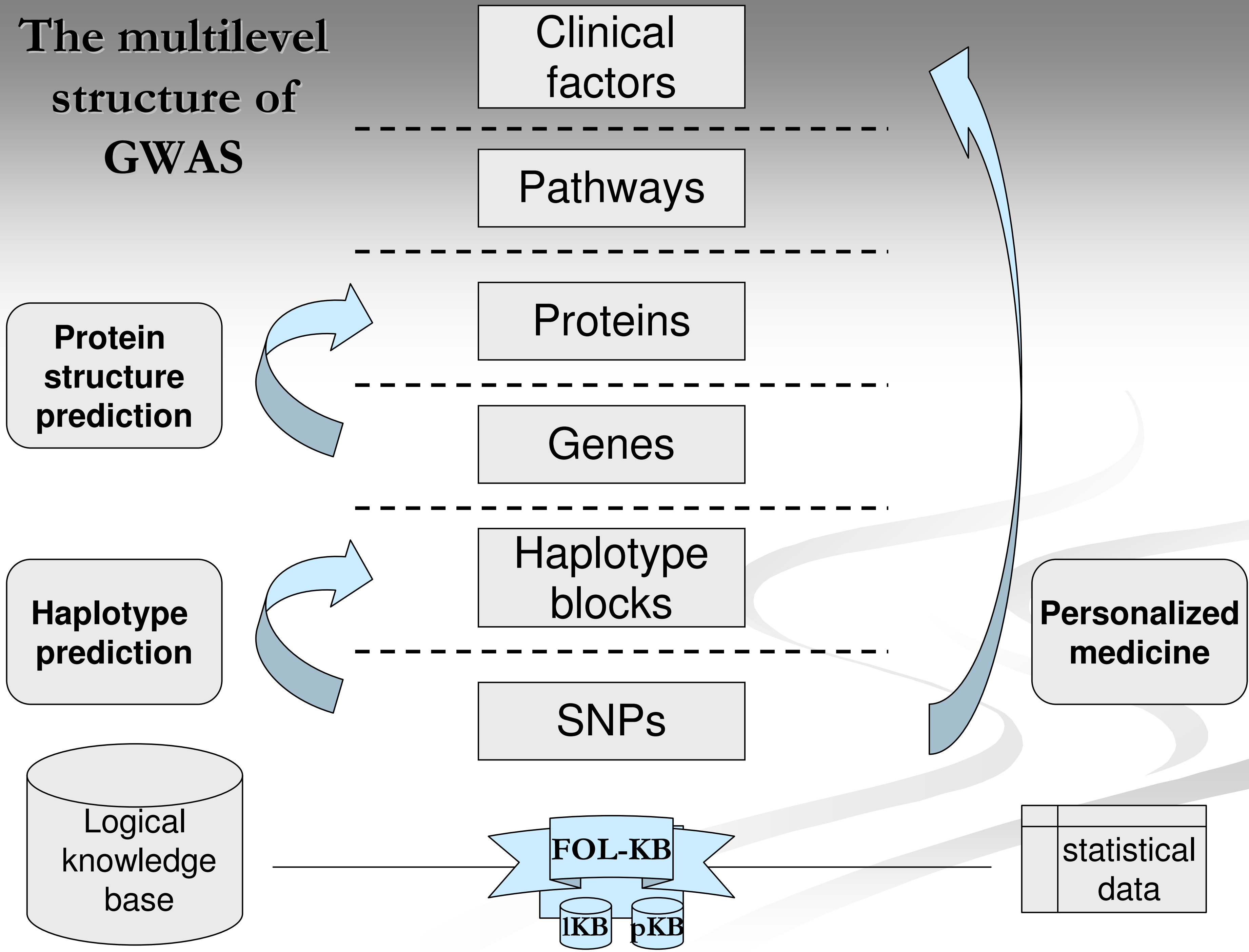
Application domain – II. Genome Wide Association Studies

- moderate number of clinical variables (in the range of 50)
- hundreds of genotypic SNP variables for each patient
- thousands of gene expression measurements

E.g.: Asthma

  - Complex disease mechanism
  - Half of the patients do not respond well to current treatments
  - Unknown pathways in the asthmatic process

The multilevel structure of GWAS



FOPL query examples

„What is the probability that a given SNP 'X' influences certain encoding genes Y_1,Y_2 *that have an effect on certain symptoms of asthma* S_1,S_2,S_3*?"*

„What is the probability that a SNP directly influences the structure of a certain protein, which modifies the „pathway" (the process of the disease), which in turn results in a change of the phenotype (some clinical variable)?"

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Goal:

Fusion of expertise, data and factual/textual domain knowledge within a first-order logic

Implemented:

Bayesian fusion of a complex posterior over BNs (causal models) and domain literature

Future work

- Extending the model representation
- Hierarchic Bayesian networks
- Describing priors by graph-grammars