There is more to a paradox than credence

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Abstract

Besides the usual business of solving paradoxes, there has been recent philosophical work on their essential nature. Lycan characterises a paradox as "an inconsistent set of propositions, each of which is very plausible". Building on this definition Paseau offers a numerical measure of paradoxicality of a set of principles: a function of the degrees to which a subject believes the principles considered individually (all typically high), and of the degree to which the subject believes the principles considered together (typically low). We argue (a) that Paseau's measure fails to score certain paradoxes properly, and (b) that this failure is not due to the particular measure but rather that any any such function just of credences fails to adequately capture paradoxicality. Our analysis leads us to conclude that Lycan's definition also fails to capture the notion of paradox.

Keywords: Paradox, Measure

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1 Introduction

Classical analyses of paradoxes by Quine (1976), Sainsbury (1988) and others concern, among other things, the question of what exactly a paradox *is*. In those works, the answer is along these lines: what makes a paradox paradoxical is the reasoning from apparently true principles via prima facie correct inferences to a conclusion that seems unacceptable or even contradictory.

A common further thought (suggested by Sainsbury, for example, but not worked out in detail) is that paradoxes come in degrees. A low degree of paradoxicality means that we are in an only mildly puzzling situation, while a high degree of paradoxicality means that we are faced with a real mind bender.

We agree entirely with Sainsbury (1988, p. 2) on this. The Barber paradox is easily dissolved on realising that there simply cannot be a barber who shaves all and only those who do not shave themselves. The Liar, on the other hand, yields to no such simple solution: there is no easily rejected component among the basic ingredients required to generate the paradox.

One might be tempted to say that the Barber is not really a paradox at all; then this pair will not support the intuition that paradoxes come in degrees. The natural response is that, surely, one could come up with another genuine paradox that is nevertheless easier than the Liar. Examples are bound to be controversial, but a case might be made for the Sorites: That it is a substantial paradox is evidenced by the number and diversity of attempts at solution, but in each case one can see that the solution is somewhat plausible. Natural languages are complex phenomena, but there is at least a fact of the matter about usage, however messy, and so one can see, for example, the epistemicist's case. The semantic notions involved in the Liar and kindred paradoxes ("true", "valid"), initially appear much more precise (and in any case can be formally simulated), so it is more puzzling that they turn out to be problematic.

A related thought is that nothing is *truly* paradoxical: understanding why, say, the twins 'paradox' is no such thing is a route to better understanding of relativity; hence Schutz (1985): "Students should realise that all 'paradoxes' are really mathematically ill-posed problems" [p. 26]. This too seems to us an unhelpful way to proceed, at least for more philosophical problems. In the case of Russell's paradox, for example, the problem didn't seem to be a matter of precision: rather it was the fault of an antecedently plausible comprehension principle. When it comes to presently unsolved paradoxes, like the Liar, it is hard to see exactly what the problem *could* be. Rather than saying that there are no *true* paradoxes, then, it is more useful to embrace Schutz's pedagogical point by saying that the harder a paradox is, the more we learn by its resolution (cf. Lewis, 1984, p. 221).

So the intuition seems very plausible - some paradoxes are more challenging than others. This intuition raises an immediate question: Can it be formalised? Intuitively, we find this plausible too. If something is rankable, then the ranking ought to be formalisable. Moreover, it is also plausible that if these things can be formally ranked, then the scores ought to be *numerically* quantifiable. On the other hand, one would expect subtleties. For example, it might be hasty to suppose that that the order is total - why shouldn't there be some incommensurability involved? But then one ought to be able to formally articulate that incommensurability. And so on: If paradoxicality comes in degrees, then degrees of paradox must be measurable; at least, it will be significant if they are *not*, so the project of quantifying paradoxicality is worth investigating.

Noting that quantitative modelling of inexact phenomena comes with many caveats, Paseau (2013, p. 17) suggests that it might be *useful* to have a measure that is approximately right. Paseau doesn't give specific examples of possible uses, but this further claim also seems plausible.

Paseau draws a comparison with confirmation theory: The proposal of various exact confirmation measures, even though none is agreed on universally, is on the whole beneficial to confirmation theory. We agree with Paseau on this point but we would like to draw a wider comparison. Simply making precise the intuition that beliefs come in degrees has proven to be a highly fertile idea for the philosophy of science and for epistemology. The notion that beliefs come in degrees has not only had a major impact in philosophy, but has also led to a staggering number of applications in computer science, in particular in artificial intelligence. As in the case of belief, it does not strike us as outlandish to hope that the development of successful measures of paradox will improve our understanding of paradoxes, which may lead to as yet unforeseen applications.

At the very least, it is useful to know about others' rankings: Although a ranking of paradoxes is intuitively plausible, our intuitions are not always in agreement with those of others. An informal survey in our department revealed that there is considerable disagreement on whether the Liar or the Sorites is more challenging (though it was agreed that both constitute substantial paradoxes). A formal measure of paradoxicality approximately reflecting individuals' intuitions would give some insight into this difference of opinion.

One might even hope that measuring paradoxes would help to solve them. Taking a paradox to consist of a set of claims (for example: Generalisations are supported by their instances; support for one generalisation attaches also to its logical equivalents; observation of a white shoe does *not* support "All ravens are black"), a *solution* to the paradox consists of some other set, revised in some way (perhaps allowing that observing non-black non-ravens *does* support "All ravens are black", which is a standard solution to the Ravens Paradox). An adequate measure of paradoxicality will assign a solution set a low paradoxicality score. Hence, the score of putative solutions of paradoxes informs reasoners whether a putative solution is (close to) a solution of the paradox. Unfortunately, the score will not yield any clues as to whether there *is* a better solution, and of course there might be merely local minima, but a formal tool would at least enable reasoners to check that the paradoxicality score was decreasing.

A priori, then, we find it plausible that it is both possible, and useful, to develop an exact measure of paradox. This is precisely what is done by Paseau (2013): the paradoxicality of a set of principles, for a subject, is a function of the degrees to which the subject believes the principles considered individually (all typically high), and of the degree to which the subject believes the principles considered together (which is typically low).

Unfortunately, however, the proposed measure is not even approximately right; we demonstrate this below. We further demonstrate that *no* measure of this kind will work. The basic problem is that Paseau's measure is a function only of a subject's credences. We can identify a pair of setups that are, so to speak, credence-isomorphic (in which case the function will have to score them the same) but such that one is a paradox and the other is not.

Since we find the basic idea of quantifying paradoxicality *prima facie* neither implausible nor of merely intrinsic interest, we find it significant that this implementation fails. But irrespective of whether one finds the whole project plausible or worthwhile, reflecting on *why* this implementation fails is useful, for the following reason.

For comparison, take the idea that some universities are better than others. Again, this is pretty plausible, and also worth considering. Turning this into a numerical measure is where it gets tricky, as illustrated by the Shanghai ranking of universities worldwide. That the latter is problematic is well established - see Billaut et al. (2009). But it is useful to reflect on *why* the numerical score is problematic *when the initial intuition was fine*. The fact that measures that use only publication venue, citation count and impact factor don't tell the whole story is informative about the nature of value of research.

A parallel applies here: Our analysis shows that, in a certain sense to be clarified below, there is more to severity of paradox than the degrees of belief that go into Paseau's measure. This then refutes the characterisation offered by Lycan (2010) of the nature of paradox, which uses only those same ingredients in a non-quantitative form: Lycan seeks to simplify the account of what a paradox *is*, and so characterises it as "an inconsistent set of propositions, each of which is very plausible". But again, we have two sets of propositions which are, so to speak, plausibility-isomorphic, so that one set would constitute a paradox if and only if the other does, on Lycan's account. But it is clear that one set constitutes a paradox and the other does not. So a paradox cannot be simply a set of inconsistent but individually plausible propositions.

Thus by reflecting on this attempt to *measure* paradox, we get insight into what a paradox *is*.

In Section 2 we outline Paseau's measure. Then we argue that the proposed measure is not even approximately right: some paradoxes are scored implausibly low (Section 3). That being the case, adjustments might be made. However, *no* function of the credences in the principles under investigation alone can track paradoxicality. In more detail, we will show that there are two sets of principles with matching credences where one set is paradoxical and the other is not. Thus, no function depending only on the credences of propositions can track paradoxicality (Section 4). We consider two ways in which Paseau might reply in Sections 5 and 6, and argue that they are unpersuasive. Pending an understanding of "plausible" that distinguishes it from probability or credence, this also undermines Lycan's simplification. Having established that there is more to paradox than credence, we briefly consider what more is needed, and see that adding syntactical form to credence in a certain way is still too weak a framework to capture paradoxicality (Section 7).

2 Paseau's measure

Paseau's recipe for approximately measuring paradoxicality is as follows: Start with the set $\{p_1, ..., p_N\}$ of *principles* that make up the paradox. Principles can include premises (two stones don't make a heap; if two stones don't make a heap then three stones don't make a heap...), rejected conclusions (a million stones *do* make a heap), and more general factors (e.g. rules of inference). A subject has credences π_{p_i} in individual principles p_i and collective credence $\pi_{\{p_1,...,p_N\}}$ in the conjunction $\bigwedge_i p_i$. (Our notation differs slightly from Paseau's. As we will be interested in more than one set of principles we use "π_{p_i}" denote the subject's credence in p_i and not simply "π_i".)

The guiding idea is that a paradox is worse for a subject the more they believe each principle individually but cannot accept them all together. From the π_{p_i} and $\pi_{\{p_1,...,p_N\}}$ the aim is to extract a degree of paradoxicality between 0 and 1 such that paradoxicality increases (1) as the credences in the principles (considered separately) go up, (2) as the credence in the set as a whole goes down, and (3) as it becomes harder to see which principle to blame (that is, as the credences in the principles become more equal). These translate into three measures:

$$m_1 = max\{0, 1 - \Sigma_{i=1}^{N}(1 - \pi_{p_i})\}$$

$$m_2 = max\{0, 1 - \pi_{\{p_1,...,p_N\}} - \Sigma_{i=1}^{N}(1 - \pi_{p_i})\}$$

$$m_3 = 1 - max\{\pi_{p_i} - \pi_{p_{i+1}} : 1 \leq i \leq N - 1\}$$

These are then combined via some function $P(m_1, m_2, m_3)$ to give the final paradoxicality score. Since, for example, a set of principles in each of which one has no credence at all would not generate a paradox, Paseau proposes the boundary conditions $P(m_1, m_2, 0) = P(m_1, 0, m_3) = P(0, m_2, m_3) = 0$. An example would be $m_1^a m_2^b m_3^c$, and, as a rule of thumb, Paseau uses the product measure $a = b = c = 1$.

Lycan mentions related, but slightly different, ideas (Lycan, 2010, p. 621): "There are actually two matters of degree involved: the disparity in plausibility between a putative culprit proposition and the other, more plausible propositions in the set, and the average degree of plausibility all around". The first of these suggests an alternative m_3, the second an alternative m_1; Lycan's approach to m_2 is absorbed into the thought that any paradox involves *inconsistent* propositions. We focus on Paseau's formulation here, and later on the general idea, which covers Lycan's alternatives.

As a test case, consider the *short sorites*, which concerns the argument:

- 1 stone does not make a heap.
- For $1 \leq n \leq 10^6$: if n stones do not make a heap, then $n + 1$ stones do not make a heap.
- Therefore 10^6 stones do not make a heap.

But 10^6 stones *do* make a heap. Paradox! The full set of principles includes the first premise, the universally quantified conditional, the negation of the conclusion, and rules of inference that go under the heading *classical logic*:

p_1: 1 stone does not make a heap.

p_2: The conjunction over $1 \leq n \leq 10^6 - 1$, if n stones do not make a heap, then $n + 1$ stones do not make a heap.

p_3: 10^6 stones *do* make a heap.

p_4: Classical logic.[1]

Credence in every one of these principles will be very nearly 1. That will make m_1 very nearly 1, since each of the four principles subtracts only a small increment. The credence in the conjunction of the principles will be very nearly 0, since p_1 to p_3 give a classical contradiction, so m_2 is very nearly 1. And the credences are (at least very nearly) equal, so m_3 is very nearly 1. So the paradoxicality score $m_1 m_2 m_3$ is high, as it should be.

3 Against Paseau's measure

We now argue that the proposed measure gives a very low m_1 for some paradoxes. Given the product measure, this means that those paradoxes are erroneously given a low score. In fact, the current proposal gives $m_1 = 0$

[1] The part of classical logic required for the paradox is fairly minimal: modus ponens, universal instantiation and transitivity of consequence; different solutions make different alterations to the logic, so this principle could be expanded in different ways.

for some paradoxes; given the boundary conditions, those paradoxes are erroneously given a score of zero *whatever* function is used for P.

The *long sorites* concerns the argument:

- 1 stone does not make a heap.
- If 1 stone does not make a heap, then 2 stones do not make a heap.
- If 2 stones do not make a heap, then 3 stones do not make a heap.

$\vdots$

- If $10^6 - 1$ stones don't make a heap, then 10^6 stones do not make a heap.
- Therefore 10^6 stones do not make a heap.

Paradox as before, except that the full set of principles includes all of the conditionals instead of the universally quantified one:

p_1: 1 stone does not make a heap.

p_2: If 1 stone does not make a heap, then 2 stones do not make a heap.

p_3: If 2 stones do not make a heap, then 3 stones do not make a heap.

$\vdots$

p_{10^6}: If $10^6 - 1$ stones do not make a heap, then 10^6 stones do not make a heap.

p_{10^6+1}: 10^6 stones *do* make a heap.

p_{10^6+2}: Classical logic.

Credence in every one of *these* principles will be very nearly 1, denoted as usual by π_{p_i}. The credence in the conjunction of the principles will be very nearly 0, so let $\pi_{\{p_1,\ldots,p_{10^6+2}\}}$ take some particular value. The credences are (at least very nearly) equal, which allows a high m_3. Now, as Paseau notes, the long and the short sorites ought to get at least roughly the same score, and that score ought to be high. So m_1, m_2 and m_3 *ought* to be all very nearly 1. But the $-\Sigma_{i=1}^{N}(1 - \pi_{p_i})$ term that occurs in both m_1 and m_2 gets this wrong: by sheer weight of numbers m_1 and m_2 for the long sorites will be low. That will make the product measure low. Moreover, since the $1 - \pi_{p_i}$ only need to be each greater than 10^{-6} to give *zero* for m_1, given the boundary conditions any $P(m_1, m_2, m_3)$ will give zero overall.

Some kind of fix is required, so that (a) the score for the long sorites comes out high, and (b) the score for the long sorites comes out at least *roughly* the same as the score for the short. One might try, perhaps, setting m_1 to the *minimum* credence π_N, or the mean of the credences (cf. Lycan's "average degree of plausibility all around"). Doing this and also simplifying m_2 to $1 - \pi_{\{p_1,\ldots,p_N\}}$ would stave off the immediate problems.

The measure m_3 ("The harder it is to choose which principle is at fault, the worse the paradox") again seems intuitively motivated, but goes wrong on examples. Suppose we have two sets of principles, where the credences are $\{0.79, 0.84, 0.89, 0.94, 0.99\}$ and $\{0.85, 0.85, 0.85, 0.95, 0.95\}$. Then Paseau's proposal says that the first is more paradoxical on this measure: m_3 is 0.95 for the first and 0.9 for the second. But, whether or not pinning the blame is *harder* for the second, it certainly does not seem *easier*. Here the problem seems to be that the focus should be wider, to take in all of the credences instead of just two. One could try using simply the spread $\pi_1 - \pi_N$, or the variance, or some suitable information theoretic measure; the notion of entropy springs to mind. We will not, however, pursue this here, in view of the following more general problem.

4 Against the class of credence-based measures

Paseau's measure depends only on the credences in individual principles and the credence in the conjunction of these principles. As we saw, the measure yields unsatisfactory results. Possibly, there is a different such measure, which yields better results. However, as we will now see, no measure $f(\pi_{p_1,\ldots,p_n}, \pi_{p_1}, \ldots, \pi_{p_N})$ can even approximately capture paradoxicality.

Consider the set of principles for some lottery for which there is exactly one winning ticket:

q_1: Ticket 1 will not win the lottery.

q_2: Ticket 2 will not win the lottery.

$\vdots$

q_{10^6}: Ticket 10^6 will not win the lottery.

q_{10^6+1}: Ticket $10^6 + 1$ will not win the lottery.

q_{10^6+2}: Ticket $10^6 + 2$ will not win the lottery.

For our subject, the credences in each of these principles are high. In fact, let them have credence $\pi_{q_i} = \pi_{p_i}$, the same as for the principles p_i above. This may be arranged by assuming that our subject has biases, if the π_{p_i} are not all equal. The credence in the conjunction, $\pi_{\{q_1,\ldots,q_{10^6+2}\}}$, is low; in fact, let it be the same as $\pi_{\{p_1,\ldots,p_{10^6+2}\}}$ above. So, for this subject, this set of principles gets the *same* score $f(\pi_{\{p_1,\ldots,p_{10^6+2}\}}, \pi_{p_1}, \ldots, \pi_{p_{10^6+2}})$ as the long sorites. But, unlike the long sorites, this is not a paradox at all. It is simply an illustration of a familiar fact about probability: the probability of a conjunction may be lower, often much lower, than probability of any of the conjuncts. In general, the only constraint for the probability P of the conjunction of propositions p_i is that $max\{0, \Sigma_i 1 - P(p_i)\} \leq P(\bigwedge_i p_i) \leq min\{P(p_i)\}$.

Any adequate measure of paradox should score the long sorites high and the lottery set of principles $\{q_1, ..., q_{10^6+2}\}$ low. But no function just of the various credences can do that for this subject. These sets of principles are *credence isomorphic*, so to speak, for this subject, in the sense that the numerical value for the credence in each item in the first matches the numerical value for the credence in the corresponding item in the second. Therefore any function of credences must score them the same. Moreover, this subject's credences are not unusual. This shows that paradoxicality cannot be simply a function of the various credences.

Our point may be obscured by the fact that there *is* a genuine paradox that is closely related, a version of the *Lottery Paradox* (Kyburg, 1961). *If* the set of principles involved claims such as "*I know that* Ticket 1 will not win the lottery", "*I know that* Ticket 2 will not win the lottery", as well as "*I know that* someone will win the lottery" and as well as statements of plausible intuitions about knowledge, such as fallibilism and closure under conjunction introduction, *then* we would have a genuine paradox. That is an interesting paradox, since these claims appear to be licensed by our concept of knowledge, so a solution to the paradox will tell us something about that concept (or about knowledge itself, depending on metaphilosophical outlook). That is what makes the lottery paradox a genuine paradox and the focus of a substantial literature. But this is not the present example: those plausible intuitions are not among the q_i, which don't even mention knowledge (or rational acceptability, which concerns Kyburg).

5 Objection: Realistic subjects?

Since the measures at issue here are supposed to be generally applicable for normal subjects, they would seem to be refuted by one counterexample involving a normal subject; we have used those above, in the weight-of-numbers and lottery-matching arguments. The subjects' credences involved there seem to us to be entirely realistic. But it might be objected that we have cooked up the numbers to make the point, or that the measure need not apply to all subjects. We find that loophole unlikely, but for the sake of completeness show in this section how to close it.

For example, perhaps for some subjects all the π_{p_i} may be very close to 1, i.e. our weight of numbers argument falls short since $\sum_{i=1}^{10^6+2} 1 - \pi_{p_i} < 1$. To make our argument in such a case we replace the principles $p_1, \ldots, p_{10^6+2}$ by $b_1 = p_1 \wedge \theta, \ldots, b_{10^6+2} = p_{10^6+2} \wedge \theta$ where θ is some completely unrelated proposition such that $\sum_{i=1}^{10^6+2} 1 - \pi_{b_i} = 1$; where π_{b_i} shall denote the subject's credence in b_i. Surely, for a typical subject some such proposition θ may be found; θ may for instance express that the sun rises tomorrow. Now note that the set $\{b_1, \ldots, b_{10^6+2}\}$ is in essence the long sorites in bad disguise and is clearly paradoxical. However, Paseau's boundary conditions enforce that any Paseau measure assigns $\{b_1, \ldots, b_{10^6+2}\}$ paradoxicality degree zero.

In reply to the lottery-matching argument, it might be objected that we have underestimated the credence that a typical subject has in each of the principles that go into the sorites. In order to construct our lottery parallel, we need the uncertainties in those principles to sum to 1; that will not be the case if the credences in the sorites principles are really very high. That loophole can be closed in the following way. (This also strengthens our argument: While it is already a serious blow to Paseau's measure that there is *some* reasonable subject for which that measure fails to approximately capture paradoxicality, we now show that for (almost) *any* subject for which the long sorites constitutes a non-trivial paradox the measure falls short.)

Begin by considering the long sorites and our subject's credences $\pi_{p_1}, \ldots, \pi_{p_n}, \pi_{p_1,\ldots,p_n}$. There are three mutually exclusive cases: A) $\sum_{i=1}^{k} 1 - \pi_{p_i} = 1$, B) $\sum_{i=1}^{10^6+2} 1 - \pi_{p_i} < 1$ and C) $\sum_{i=1}^{10^6+2} 1 - \pi_{p_i} > 1$.

In case of A), simply proceed as above and arrange that $\pi_{q_i} = \pi_{p_i}$ holds for all i. These equalities may be arranged for instance, by publicly truthfully announcing that ticket i has a winning chance of π_{p_i}.

In case of B) we shall do as above and consider $b_i = p_i \wedge \theta$ with credences $\sum_{i=1}^{10^6+2} \pi_{b_i} = 1$. We may thus arrange that the credences satisfy $\pi_{q_i} = \pi_{b_i}$ by considering a suitable (unfair) lottery.

In case of C) we consider the long sorites paradox for k stones. We shall choose k such that with c_1 = "Classical logic" and c_k = "k stones do make a heap" it holds that $1 - \pi_{c_1} + 1 - \pi_{c_k} + \sum_{i=1}^{k} 1 - \pi_{p_i} = 1$ and proceed as in case A. [2]

All that we need to assume now is that $\pi_{\{p_1,\ldots,p_k\}}$ is very nearly $\pi_{\{q_1,\ldots,q_k\}}$ (respectively $\pi_{\{p_1\wedge\theta,\ldots,p_k\wedge\theta,c_1,c_k,\}}$ is very nearly $\pi_{\{q_1,\ldots,q_k,q_{k+1},q_{k+2}\}}$), which seems to be a rather benign assumption since all credences will be very nearly zero.

Thus, for (almost) *any* subject for which the long sorites constitutes a non-trivial paradox there is a long sorites paradox on $k \leq 10^6$ stones (possibly inconsequently modified by adding θ) and a lottery-non paradox with $k + 2$ tickets which are (almost) credence-isomorphic.

6 Objection: Measuring vs Demarcation

Of course, our argument is only an objection, if the aim is to measure the extent to which some set of principles – any set of principles – is paradoxical. If one had already, by some other means, separated the genuine paradoxes from the non-paradoxes, one could then take some such measure as scoring the paradoxes for seriousness. Then the above lottery example would be no problem, since it would already have been declared a non-paradox by those other means, and thus outside the scope of the measure. (The problem of the long versus the short sorites would remain, however.)

A more interesting project does both at once: if some set of principles is an utter non-paradox, then the measure will score it zero; and if some set of principles is not scored zero, then it is not utterly non-paradoxical. We take Paseau to be engaged in this more interesting project, as is evidenced by his statements that a zero score means "no paradox at all" (p.23, example 1) and that a middling score gives "half a paradox" (p.23, example 4), rather than "a paradox, which is half way up the scale" or similar.

Lycan seems to have this project in mind too: his discussion of *degrees of plausibility* of the ingredients of a putative paradox comes at the end of a paper that aims to answer the question "What, exactly, is a paradox?". In fact, our present objection appears to work not just against Paseau's precise measure, but also against Lycan's characterisation of what a paradox *is*: "a Paradox is an inconsistent set of propositions, each of which is very plausible". The propositions "This ticket will not win the raffle", "That ticket will not win the raffle", ... , "one of these tickets will win the raffle" *are* all very plausible, and the set as a whole *is* inconsistent, so by Lycan's account this is a paradox; but it is not a paradox, for the reasons given above.

It might be, of course, that "plausibility" can be construed so that not all of the principles in our lottery set

[2] There is one final technical detail we need to take care of. Possibly, there is no such k : It could be the case that there is a γ such that $1 - \pi_{c_1} + 1 - \pi_{c_{\gamma-1}} + \sum_{i=1}^{\gamma-1} 1 - \pi_{p_i} < 1 < 1 - \pi_{c_1} + 1 - \pi_{c_\gamma} + \sum_{i=1}^{\gamma} 1 - \pi_{p_i}$. In this case let $k = \gamma$ and proceed as in case B).

are plausible. In that case our objection would not after all apply to Lycan's definition as it does to Paseau's exact measure. Unfortunately we can't see how to thus understand plausibility, and think that the burden of proof is on Lycan to further explicate what sets degree of plausibility significantly apart from credence.

Thus by reflecting on this attempt to *measure* paradox, we get insight into what a paradox *is*. Paradoxicality is not a function just of plausibility, any more than it is a function only of credence in the p_i and in $\bigwedge_i p_i$. In standard statements such as those from Quine and Sainsbury above, one finds references to apparently acceptable premises, apparently acceptable reasoning, and apparently unacceptable conclusions. We suggest that more work is being done here by "apparent acceptability", and by the reference to reasoning, than can be captured by simplifications such as Lycan's and Paseau's.

7 Syntax, credence, rules of inference and paradoxicality

Our basic objection to Paseau's measure and to Lycan's definition is that there are two sets of principles that are credence isomorphic, so any function of credences must score them the same, even though one is paradoxical and the other is not. Thus any sensible measure of paradox must depend on more than credences. Plausibly, such a measure tracking paradoxicality may not only depend on credence but also on the *syntactical form* of the paradox and the *rules of inference used* to arrive at the conclusion of the paradox. For one final time, will we tweak our pet examples to show that such a move cannot be successful.

We will construct two sets of principles, classically inconsistent, such that in both cases each principle gets high credence, while the conjunction gets low credence. One set of principles is a paradox and the other is not. First, note that the conclusion of the sorites is usually of the form "k stones do not make a heap", but the same reasoning also establishes the conclusion "One stone does not make a heap, and two stones don't make a heap, and ... and a k stones don't make a heap". That's just as bad a conclusion, just as paradoxical. To be sure, some of that conclusion is *redundant*, but it is essentially the same paradox. We can use this to push the sorites towards lottery form, whilst making it no less paradoxical. Second, note that it is a fact about probability that if $P(A) \gg P(B)$ then $P(A) \approx P(A \vee B)$ and that if $P(A) \approx 1$ then $P(A \vee B) \approx 1$. We can use this to push the lottery towards sorites form, whilst making it no more paradoxical. Then our sets of principles are:

Long Sorites Paradox:	Lottery Non-paradox:
$\sim H(1)$	$\sim W(1)$
$\sim H(1) \supset \sim H(2)$	$\sim W(1) \supset \sim W(2)$
$\ldots$	$\ldots$
$\sim H(k-1) \supset \sim H(k)$	$\sim W(k) \supset \sim W(k)$
$H(1) \vee \ldots \vee H(k)$	$W(1) \vee \ldots \vee W(k)$

We may again assume that these sets of principles are credence isomorphic by the same arguments we gave above. They also have the exact same syntactical form, so any paradoxicality measure only depending on these credences and the syntactical form wrongly assigns both sets of principles the same score.

8 Conclusion

What makes a paradox paradoxical is the reasoning from apparently true principles via prima facie correct inferences to a conclusion that seems unacceptable or even contradictory. Our analysis shows that this tension cannot be measured by mere numbers, at least if these are taken simply to be credences in the principles and their conjunction. Rather, a deeper treatment of the underlying principles, the inferences and the conclusion is required. This is in line with our intuition that paradoxes are phenomena of *reasoning* and not merely of credence, and these ingredients will all have to be quantified in order to construct better measures of paradoxicality. Whether or not such a measure proves ultimately to be successful, simply undertaking that construction promises further insights into the nature of paradoxes and the sense in which there is more to a paradox than credence.

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