Risk sharing for public pension schemes

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In context

Ageing population :

- increasing life expectancy
- decreasing birth rate
- *papy boom*

Ratio of pensioners to workers is expected to increase by 46% over the next two decades in Belgium (from 28% in 2017 to 41% in 2037). [1]

Impact of this ratio on the public pension scheme.

[1] The High Council of Finance, 2016

Outline

Pay As You Go pension scheme (PAYG)

Specific pension schemes

Stochastic optimal control

Benefits for specific career paths

Numerical application

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Specific pension schemes

Stochastic optimal control

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Numerical application

PAYG principle

Today workers pay for today pensioners.

PAYG scheme

Incomes from the contributors :

$$A_t \, \pi_t \, \overline{S}_t$$

with

- A : number of contributors
- π : contribution rate
- $\overline{S}$: mean salary

Outcomes for the pensioners :

$$B_t \, \overline{P}_t = B_t \, \delta_t \, \overline{S}_t$$

with

- B : number of pensioners
- δ : global replacement rate
- $\overline{P}$: mean pension

Equilibrium equation

The equilibrium equation of the PAYG scheme is

$$\begin{aligned} \text{Incomes} &= \text{Outcomes} \\ A_t \, \pi_t \, \overline{S}_t &= B_t \, \delta_t \, \overline{S}_t \\ \pi_t &= D_t \, \delta_t \end{aligned}$$

with the dependence ratio

$$D_t = \frac{B_t}{A_t}$$

Equilibrium equation

The equilibrium equation of the PAYG scheme is

$$\begin{aligned} \text{Incomes} &= \text{Outcomes} \\ A_t\,\pi_t\,\overline{S}_t &= B_t\,\delta_t\,\overline{S}_t \\ \pi_t &= D_t\,\delta_t \end{aligned}$$

with the dependence ratio

$$D_t = \frac{B_t}{A_t} \quad \text{risk factor}$$

Automatic balance mechanism

$$\pi_t = D_t \, \delta_t$$

In case of change of the risk factor D_t, how can π_t and δ_t be **automatically adjusted** to maintain the **equilibrium** ...

... while maintaining simultaneously financial sustainability and social adequacy ?

DB and DC schemes

Defined Benefit (DB)

$\overline{\delta}$ constant

$$\pi_t = D_t \, \overline{\delta}$$

Demographic risk borne by the **contributors**

Defined Contribution (DC)

$\overline{\pi}$ constant

$$\delta_t = \frac{\overline{\pi}}{D_t}$$

Demographic risk borne by the **pensioners**

An intermediate scheme : the Musgrave rule

Replacement rate **net of contribution** M constant

$$M = \frac{\overline{P}_t}{S_t\,(1-\pi_t)} = \frac{\delta_t}{1-\pi_t}$$

$$\begin{cases} \delta_t = \dfrac{M}{1+M\,D_t} \\ \pi_t = \dfrac{M\,D_t}{1+M\,D_t} \end{cases}$$

Risk shared by the contributors and the pensioners

Optimal criteria

Optimal risk sharing providing joint **stability** of π_t and δ_t around fixed targets $\overline{\pi}$ and $\overline{\delta}$.

Optimization and its loss function

$$\min_{\delta_s,\pi_s} \mathbb{E}\left[\int_0^T \left((1-\rho_s)\left(\frac{\delta_s}{\overline{\delta}}-1\right)^2+\rho_s\left(\frac{\pi_s}{\overline{\pi}}-1\right)^2\right) ds)\right]^2$$

with fix targets $\overline{\delta}$, $\overline{\pi}$ and a given weight process $\rho_s \in [0,1]$

The dependence ratio process follows a geometric Brownian motion

$$\frac{dD_t}{D_t} = \mu\, dt + \sigma\, dW_t$$

where W_t is a standard Brownian motion.

[2] A. Cairns, 2000

Stochastic optimal control

We use the PAYG equilibrium equation

$$\pi_t = D_t \, \delta_t \, .$$

- The loss function is

$$(1-\rho_t)\left(\frac{\delta_t}{\overline{\delta}} - 1\right)^2 + \rho_t \left(\frac{D_t \, \delta_t}{\overline{\pi}} - 1\right)^2 \, .$$

- The state variable is

$$D \, .$$

- The control variable is

$$\delta \, .$$

Optimal solutions

By applying the stochastic optimal control theory (HJB equation), we obtain

$$\begin{aligned}
\delta_t^\star &= \frac{(1-\rho_t)\,\frac{1}{\overline{\delta}} + \rho_t\,\frac{D_t}{\overline{\pi}}}{(1-\rho_t)\,\frac{1}{\overline{\delta}^2} + \rho_t\,\frac{D_t^2}{\overline{\pi}^2}} \\
\pi_t^\star &= D_t\,\delta^\star(t)
\end{aligned}$$

The obtained result does not depend on the type of the process D_t.

This result can be directly obtained by optimizing the loss function.

Optimal solutions

Extreme DB and DC schemes

$$\delta_t^\star = \frac{(1-\rho_t)\,\frac{1}{\overline{\delta}} + \rho_t\,\frac{D_t}{\overline{\pi}}}{(1-\rho_t)\,\frac{1}{\overline{\delta}^2} + \rho_t\,\frac{D_t^2}{\overline{\pi}^2}}$$

$$\pi_t^\star = D_t\,\delta^\star(t)$$

$$\text{DB} : \rho_t = 0 \quad \begin{cases} \delta_t^\star = \overline{\delta} \\ \pi_t^\star = D_t\,\overline{\delta} \end{cases} \qquad \text{DC} : \rho_t = 1 \quad \begin{cases} \delta_t^\star = \dfrac{\overline{\pi}}{D_t} \\ \pi_t^\star = \overline{\pi} \end{cases}$$

Calibration of the targets

The targets $\overline{\pi}$ and $\overline{\delta}$ are determined according following constraints

$$\begin{aligned} \delta^{\star}_{t_0} &= \delta_0 && \text{(initialization)} \\ \overline{\pi} &= \overline{\delta}\, D_\infty && \text{(PAYGO equation).} \end{aligned}$$

For a constant ρ, we obtain

$$\begin{aligned} \overline{\pi} &= \delta_0 \, \frac{(1-\rho)\, D_\infty^2 + \rho\, D_0^2}{(1-\rho)\, D_\infty + \rho\, D_0} \\ \overline{\delta} &= \delta_0 \, \frac{(1-\rho) + \rho\, \dfrac{D_0^2}{D_\infty^2}}{(1-\rho) + \rho\, \dfrac{D_0}{D_\infty}}\,. \end{aligned}$$

Benefits for specific career paths

Individual δ^i depends on the specific career profile $\{S_x^i\}$.

The **points system** is used to determine the pension P^i according to the career of each affiliate.

Points system

Benefits :

$$P_t^i = \Pi^i \, v_t$$

Number of points :

$$\Pi^i = \sum_{x=x_0}^{x_r-1} \frac{S_x^i}{\overline{S}}$$

constant mean salary over time $\overline{S}$

Value of the point :

$$\begin{aligned} v_t &= \delta_t \, \overline{S} \, \frac{1}{\Pi_{ref}} \\ &= \delta_t \, \overline{S} \, \frac{1}{x_r - x_0} \end{aligned}$$

Points system

Benefits :

$$\begin{aligned} P_t^i &= \Pi^i \, v_t \\ &= \frac{1}{x_r - x_0} \, \delta_t \sum S_x^i \end{aligned}$$

Individual replacement rate :

$$\begin{aligned} \delta_t^i &= \frac{P_t^i}{S_{x_{r-1}}^i} \\ &= \frac{1}{x_r - x_0} \, \frac{1}{S_{x_{r-1}}^i} \, \delta_t \sum S_x^i \end{aligned}$$

Another risk sharing

With the proposed risk sharing and points system, we can not define a **DB system on final salary** with the same replacement rate for everyone independently of the career profile. In order to obtain a DB system on final salary, we propose a new risk sharing :

$$\gamma \, DB + (1-\gamma) \, DC \qquad \text{with} \, \gamma \in [0,1] \, .$$

Another risk sharing

Extreme DB and DC schemes

$$\gamma\, DB + (1-\gamma)\, DC \qquad \text{with}\, \gamma \in [0,1]$$

DB ($\gamma = 1$)

$$\begin{cases} \pi_t = D_t\, \overline{\delta} \\ \delta_t^i = \delta_t = \overline{\delta} \end{cases}$$

DC ($\gamma = 0$)

$$\begin{cases} \pi_t = \overline{\pi} \\ \delta_t = \dfrac{\overline{\pi}}{D_t} \\ \delta_t^i = \frac{1}{x_r - x_0}\, \frac{1}{S_{x_r}^i}\, \frac{\overline{\pi}}{D_t}\, \sum S_x^i \end{cases}$$

Another risk sharing

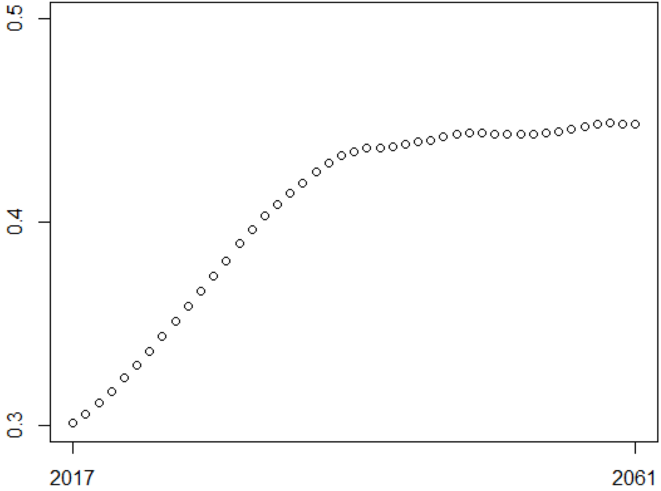
$$\gamma\, DB + (1-\gamma)\, DC \qquad \text{with}\, \gamma \in [0,1]$$

Risk sharing

$$\begin{cases} \pi_t = \gamma\, D_t\, \overline{\delta} + (1-\gamma)\, \overline{\pi} \\ \delta_t = \gamma\, \overline{\delta} + (1-\gamma)\, \frac{\overline{\pi}}{D_t} \\ \delta_t^i = \gamma\, \overline{\delta} + (1-\gamma) \frac{1}{x_r - x_0}\, \frac{1}{S^i_{x_r}}\, \frac{\overline{\pi}}{D_t}\, \sum S^i_x \end{cases}$$

Dependence ratio[3]

Complete career from $x_0 = 20$ years to $x_r = 65$ years



[3] Federal Planning Bureau, 2014

Dependence ratio process

The dependence ratio is a mean reverting process and follows a lognormal distribution : the **Black-Karasinski model**

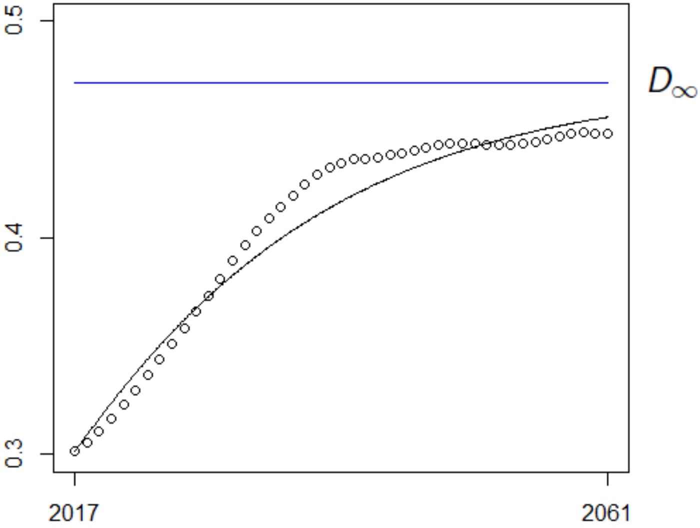
$$d \ln D(t) = \alpha \left(\ln D_\infty - \ln D(t) \right) dt + \sigma \, dW(t)$$

where $\alpha > 0$, $\sigma > 0$ and W_t is a standard Brownian motion.

Calibration using least square regression provides the parameters

$$\alpha = 0.059 \,, \quad D_\infty = 0.47 \quad \text{and} \quad \sigma = 0.0046 \,.$$

Dependence ratio process



Initial conditions

The initialisation of our model in $t_0 = 2017$:

- Dependence ratio : $D_0 = 30\%$
- Contribution rate : $\pi_0 = 15\%$
- Replacement rate : $\delta_0 = 50\%$
- Net replacement rate : $M = 59\%$

Risk sharing under stochastic optimal control

Remember, the loss function to minimize is

$$(1-\rho_t)\left(\frac{\delta_t}{\overline{\delta}}-1\right)^2+\rho_t\left(\frac{D_t\,\delta_t}{\overline{\pi}}-1\right)^2 \ .$$

We simulate scenarios with

$\rho = \{0, 0.25, 0.5, 0.75, 1\}$ and the Musgrave rule.

Risk sharing under stochastic optimal control

Six scenarios

$$(1-\rho_t)\left(\frac{\delta_t}{\overline{\delta}}-1\right)^2+\rho_t\left(\frac{D_t\,\delta_t}{\overline{\pi}}-1\right)^2$$

Simulated scenarios

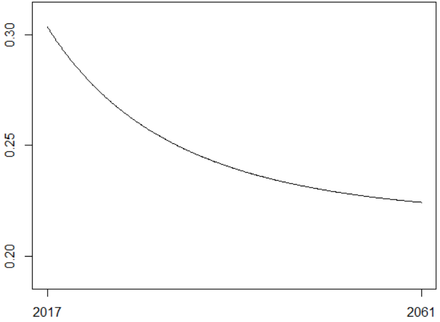
scheme	ρ_t	$\overline{\pi}$	$\overline{\delta}$
DB	0	24%	50%
risk sharing	0.25	22%	47%
Musgrave	$\tilde{\rho}_t$	22%	46%
risk sharing	0.5	20%	43%
risk sharing	0.75	18%	38%
DC	1	15%	32%

Risk sharing under stochastic optimal control

Musgrave rule

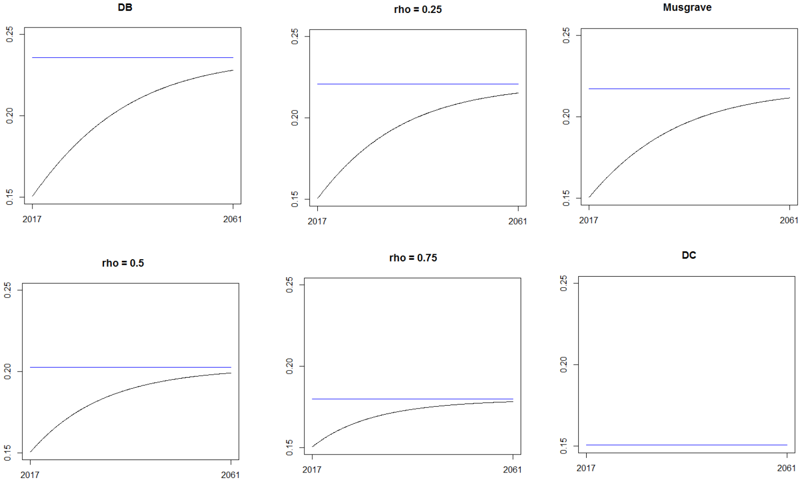
The weight process is

$$\tilde{\rho}_t = \frac{\frac{1}{\overline{\delta}}\left(1 - \frac{M}{\overline{\delta}} + M\ D_t\right)}{\frac{1}{\overline{\delta}}\left(1 - \frac{M}{\overline{\delta}} + M\, D_t\right) - \frac{D_t}{\overline{\pi}}\left(1 - \frac{M\, D_t}{\overline{\pi}} + M\, D_t\right)}\,.$$



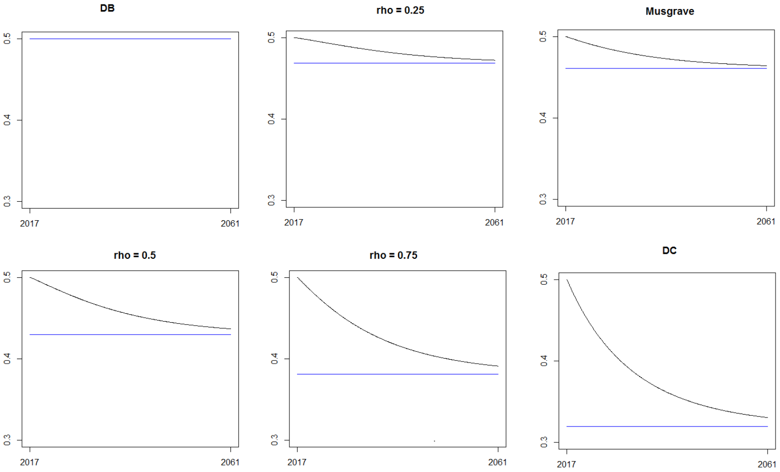
Risk sharing under stochastic optimal control

Contribution rate



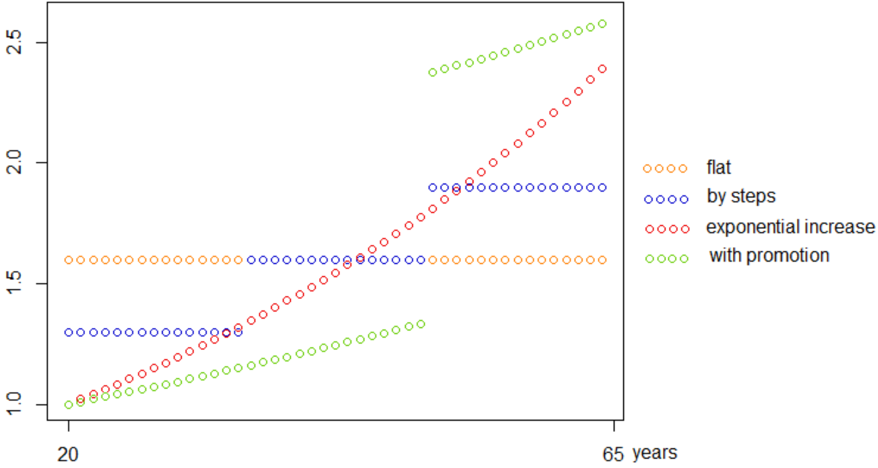
Risk sharing under stochastic optimal control

Replacement rate



Salaries

4 career profiles



Another risk sharing

Remember, the second risk sharing model is

$$\gamma \, DB + (1-\gamma) \, DC \; .$$

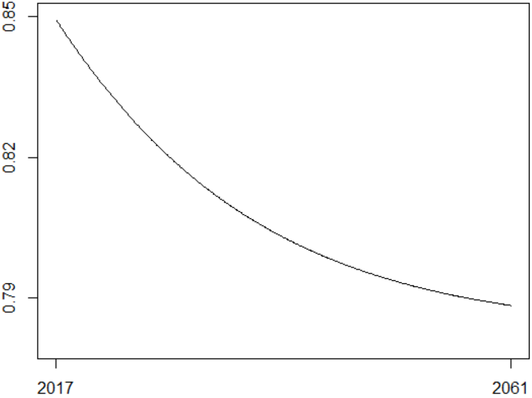
We simulate scenarios with

$\gamma = \{0, 0.25, 0.5, 0.75, 1\}$ and the Musgrave rule with $\tilde{\gamma}_t$.

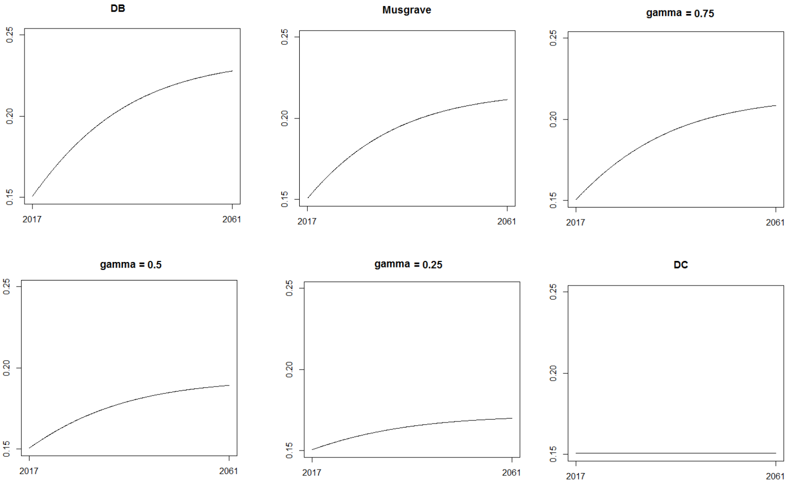
Another risk sharing

Musgrave rule

$$\tilde{\gamma}_t = \frac{M\,D_t - \pi_0 - \pi_0\,M\,D_t}{(1 + M\,D_t)\,(\delta_0\,D_t - \pi_0)}$$

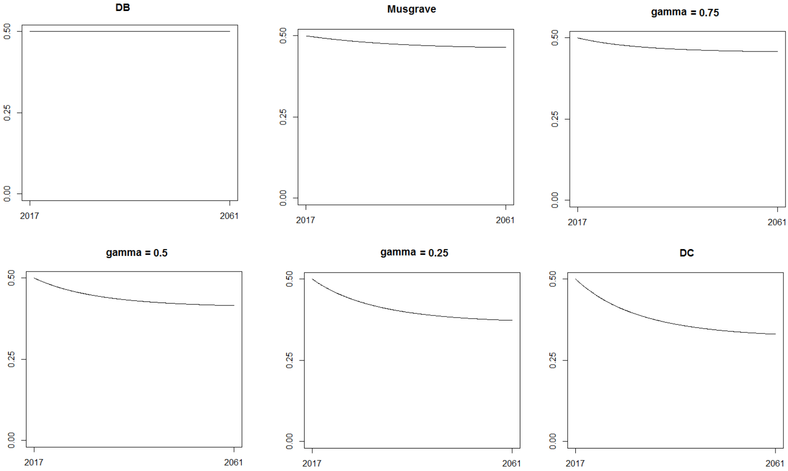


Another risk sharing
Contribution rate



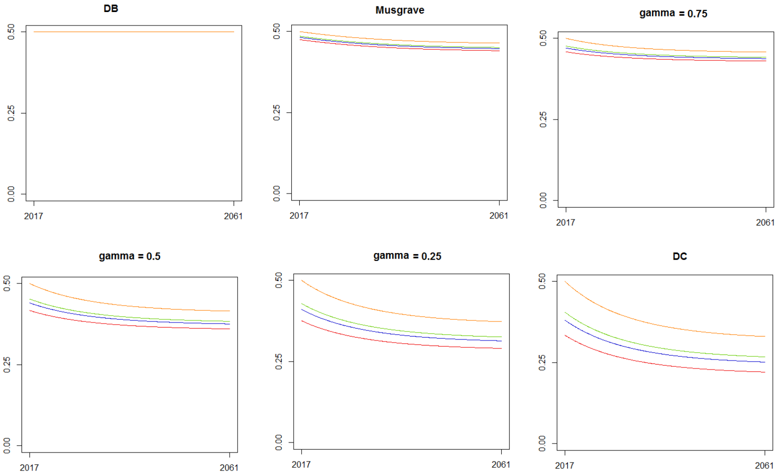
Another risk sharing

Mean replacement rate



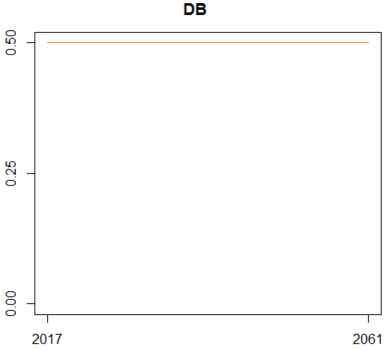
Another risk sharing

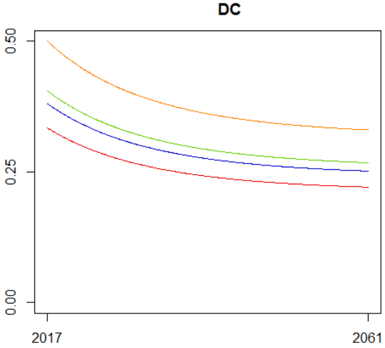
Individual replacement rate



Another risk sharing

Individual replacement rate





The replacement rate is

- constant,
- the same for everyone.

Two effects :

- the demographic risk,
- the salary risk.

Conclusions

- Ageing induces an ineluctable and significant increase of the dependence ratio in the coming decades.

- In the aim to maintain a balanced PAYG system, we propose two different **risk sharing** :

  mix between the extreme DB and DC schemes.

Future research

- **Optimal** risk sharing through the processes ρ_t and γ_t .

- Integration of the proposed risk sharing models within the **NDC system** (with a variable contribution rate).

Risk sharing for public pension schemes

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