

A Quantum recipe for Dynamics in Spin Ice

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Abstract

In Spin Ice the hopping of a magnetic monopole, from one site to the next, consists of the flip of a large spin from an easy axis configuration to the opposite one. At very low temperatures, this requires quantum-mechanical tunnelling through a large anisotropy barrier. Naively one would thus expect a single, very slow frequency in the low-temperature regime. On the other hand experiments reveal a broad range of temperature-independent (i.e. quantum) frequencies [1,2,3,4]. Here we study this question within a single ion (Ho^{3+}) picture. Starting from the CF Hamiltonian for $\text{Ho}_2\text{Ti}_2\text{O}_7$ we analyze how a transverse magnetic field B , induces a quantum spin flip from an anisotropic configuration to the opposite one. Interestingly, we find a broad range of frequencies that are characteristic of the system. In particular using the internal field distribution (from Monte Carlo simulations) due to the presence of a monopole we detect two different time scales for the hopping rate of a monopole. To model the hopping of a monopole, we use a classical Monte Carlo calculation to estimate the size and orientation of the magnetic field a spin is subjected to before (B_1) and after (B_2) a monopole arrives to an adjacent tetrahedron. The quantum quench from B_1 to B_2 induces a quantum mechanical flipping of the spin, and thus the hopping of the monopole. Our calculation, which doesn't depend on any adjustable parameter, suggests two main timescales for monopole hopping: $\tau_1 \approx 0.25\text{ms}$ and $\tau_2 \approx 2.5\mu\text{s}$.

Background

From Dipoles ...

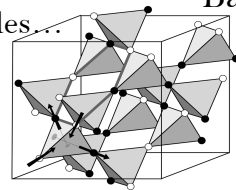
Spin Ice are *Geometrically Frustrated* ferromagnets (Pyrochlore Oxides).

In $\text{Dy}_2\text{Ti}_2\text{O}_7$ and $\text{Ho}_2\text{Ti}_2\text{O}_7$

The strong *crystal field* causes the spin to point along the $\langle 111 \rangle$ direction

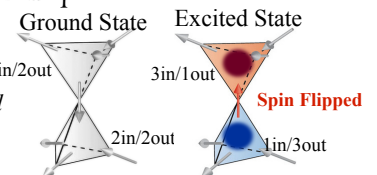
Ice Rule \rightarrow macroscopic degenerate GS \rightarrow non-zero entropy at $T=0$.

Large magnetic moments ($10 \mu_B$) \rightarrow Strong *Dipole-Dipole* interaction.



... to Monopoles

At low temperatures, the *local dipolar excitation* fractionalize into *deconfined* quasiparticles with magnetic charge.



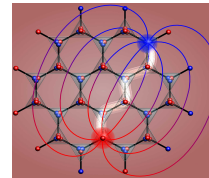
Magnetic monopoles interact via **Coulombic** potential

$$V(r_{\alpha\beta}) = \begin{cases} \frac{\mu_0}{4\pi} \frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} & \alpha \neq \beta \\ \frac{1}{2} v_0 Q_\alpha^2 & \alpha = \beta \end{cases}$$

Dipolar Hamiltonian

$$H = \frac{J}{3} \sum_{\langle ij \rangle} S_i S_j + Da^3 \sum_{\langle ij \rangle} \left[\frac{\hat{e}_i \cdot \hat{e}_j}{|\mathbf{r}_{ij}|^3} - \frac{3(\hat{e}_i \cdot \mathbf{r}_{ij})(\hat{e}_j \cdot \mathbf{r}_{ij})}{|\mathbf{r}_{ij}|^5} \right] S_i S_j$$

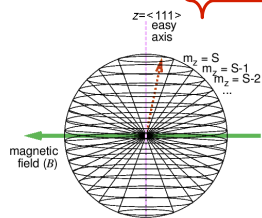
Long Range Coupling



RESULTS

Quantum mechanics of an Ho^{3+} ion in a transverse magnetic field

$$\hat{H} = -g_J \mu_B \hat{\mathbf{J}} \cdot \mathbf{B} + \hat{H}_{CF}$$



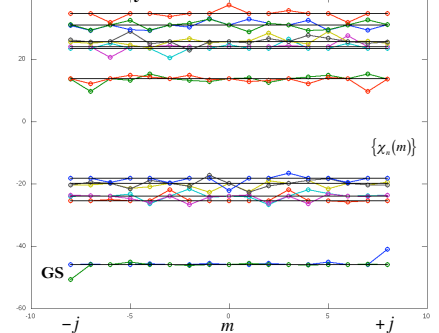
Example of a large spin (blue dotted) under magnetic transverse field B (red solid).

$$\hat{H}_{CF} = \Gamma_0^2 \hat{O}_0^2 + \Gamma_0^4 \hat{O}_0^4 + \Gamma_3^4 \hat{O}_3^4 + \Gamma_0^6 \hat{O}_0^6 + \Gamma_3^6 \hat{O}_3^6 + \Gamma_6^6 \hat{O}_6^6$$

Crystal Field Hamiltonian in terms of Stevens operators $\hat{O}_q^k(\hat{J}_x, \hat{J}_y, \hat{J}_z)$ (functions of angular momentum operators)

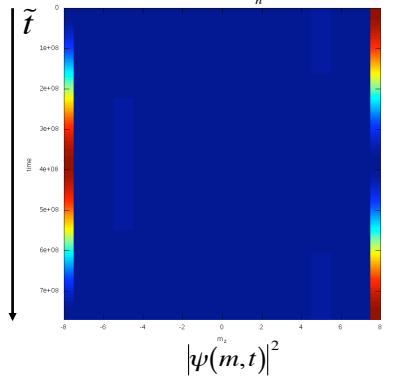
$\text{Ho}_2\text{Ti}_2\text{O}_7 \rightarrow |J, J_z\rangle = |8, m_z\rangle$

Stationary States for the CF Hamiltonian

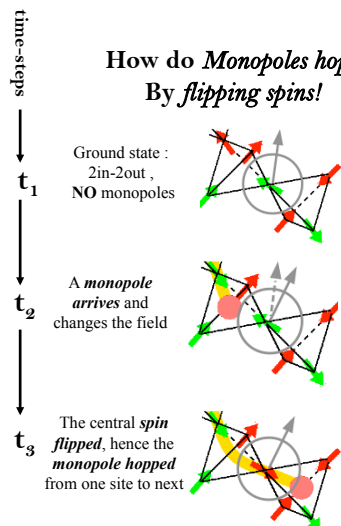


QUANTUM DYNAMICS

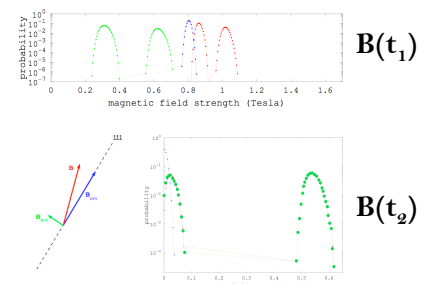
$$|\psi_t\rangle = e^{-\frac{i}{\hbar} \hat{H} t} |\psi_0\rangle = \sum_n c_n^0 e^{-\frac{i}{\hbar} E_n t} |\chi_n\rangle$$



How do Monopoles hop? By flipping spins!



Histograms of internal fields from classical Monte Carlo



Quantum Quench

$B(t_2)$ $\left\{ \begin{array}{l} \tau_1 \approx 0.25 \text{ ms} \\ \tau_2 \approx 2.5 \mu\text{s} \end{array} \right.$ **TWO** different time scales for the monopole hopping

[1] Bramwell et al. *Nature* **461** (2009); [2] Dunsiger et al *PRL* **107** (2011); [3] S.Giblin et al. *Nature-Phys* **7** (2011); [4] K.Matsushira et al. *JPSJ* **80** (2011)

ACKNOWLEDGMENTS

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