Framing human inference by coherence based probability logic

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- Degrees of belief & subjective probabilities versus measure theory, limits of relative frequencies as $n \to \infty$

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- close to Bayesian statistics
- prob. semantics for non-classical logic systems

How is conditional probability introduced?

$P(E\|H)$ is basic	$P(E\|H)$ is defined	
$E\|H$	$E \wedge H, H$	$P(H), P(E \wedge H)$
1 conditional event	2 unconditional events	
$P(E\|H), \quad H \neq \emptyset$	$P(E\|H) = \frac{P(E \wedge H)}{P(H)}, \quad P(H) \neq 0$	
1 probability	2 probabilities	

Axioms (Popper, Rényi, ..., Coletti & Scozzafava)

Let $\mathcal{C} = \mathcal{G} \times \mathcal{B}^0$ be a set of conditional events $\{E|H\}$ such that $\mathcal{G}$ is a Boolean algebra and $\mathcal{B} \subseteq \mathcal{G}$ is closed with respect to (finite) logical sums, with $\mathcal{B}^0 = \mathcal{B} \setminus \{\emptyset\}$. A function $P : \mathcal{C} \mapsto [0,1]$ is a conditional probability iff the following three axioms are satisfied

A1 $P(H|H) = 1$, for every $H \in \mathcal{B}^0$,

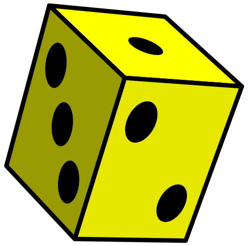
A2 $P(\cdot|H)$ is a (finitely additive) probability on $\mathcal{G}$ for any given $H \in \mathcal{B}^0$,

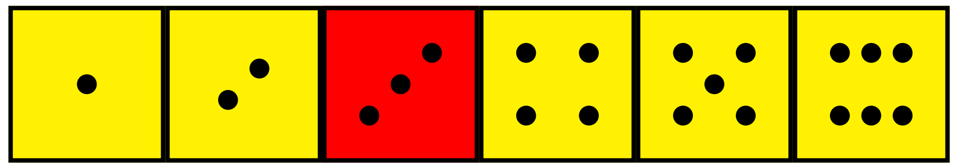
A3 $P(E \wedge A|H) = P(E|H)P(A|E \wedge H)$ for any $A, E \in \mathcal{G}, H, E \wedge H \in \mathcal{B}^0$.

Conditional: $\supset$ versus $|$

- "If H, then E" is interpreted as $E|H$ and not as $H \supset E$
- so that it is "weighted" by $P(E|H)$ and not as $P(H \supset E)$, $= 1 - P(H \wedge \neg E)$
- Does it make a difference?
  - Suppes: no, as P approaches 1
  - $P(E|H)$ does not lead to the paradoxes of material implication

Example





its a 3	0	0	1	0	0	0	
its even	0	1	0	1	0	1	
$\supset$	1	1	0	1	1	1	$\frac{5}{6}$
$\vert$	i	i	0	i	i	i	0

Historical notes

- Ramsey (1926) “... ’The degree of belief in p given q’. This does not mean the degree of belief in ’If p then q [material implication]’, or that ‘p entails q’ ... It roughly expresses the odds which he would now bet on p, the bet only be valid if q is true. Such conditional bets were often made in the eighteenth century.”
- de Finetti (1937 and before)
- Jeffreys, 1931 first use of vertical stroke $P(E|H)$ for conditional events
- Markov and Czuber (1902) used $P_H(E)$
- Carnap (1936) dispositional predicates

Indicators

$$T(E|H) = \begin{cases} 1 & \text{if} \quad E \wedge H & \text{win} \\ 0 & \text{if} \quad \neg E \wedge H & \text{loose} \\ p(E|H) & \text{if} \quad \neg H & \text{money back} \end{cases}$$

$$= 1 \cdot I_{E \wedge H} + 0 \cdot I_{\neg E \wedge H} + p(E|H) \cdot I_{\neg H}$$

$$X = \sum_{k=1}^{3} x_k I_{E^k}$$

Generalization (Coletti & Scozzafava): The third term may be considered as a function.

- allows the "derivation" of the axioms of conditional probabilities
- leads to possibility function

Coherence

- *Coherence* A precise probability assessment (L, A^p) on a set of conditional events $\mathcal{E}$ is coherent iff for every $\{\psi_1|\phi_1, \ldots, \psi_n|\phi_n\} \subseteq \mathcal{E}$ with $n \geq 1$ and for all real numbers $s_1, \ldots, s_n$

$$\max \sum_{i=1}^{n} s_i \cdot I(\phi_i) \cdot (I(\psi_i) - A(\psi_i|\phi_i)) \geq 0 \,. \qquad (1)$$

- *Total coherence* for interval probabilities ... iff all points are coherent (strong coherence, Walley (1991), Gilio)
- *g-coherence* An interval-probability assessment is g-coherent iff there exists at least one ... (weak coherence, Gilio in many papers, Walley (1991))

	Probability assessment	
	points	intervals
Unconditional events	coherence (linear)	total-coherence (linear) g-coherence (linear)
Conditional events	coherence (non-linear)	total coherence (non-linear) g-coherence (cubes)

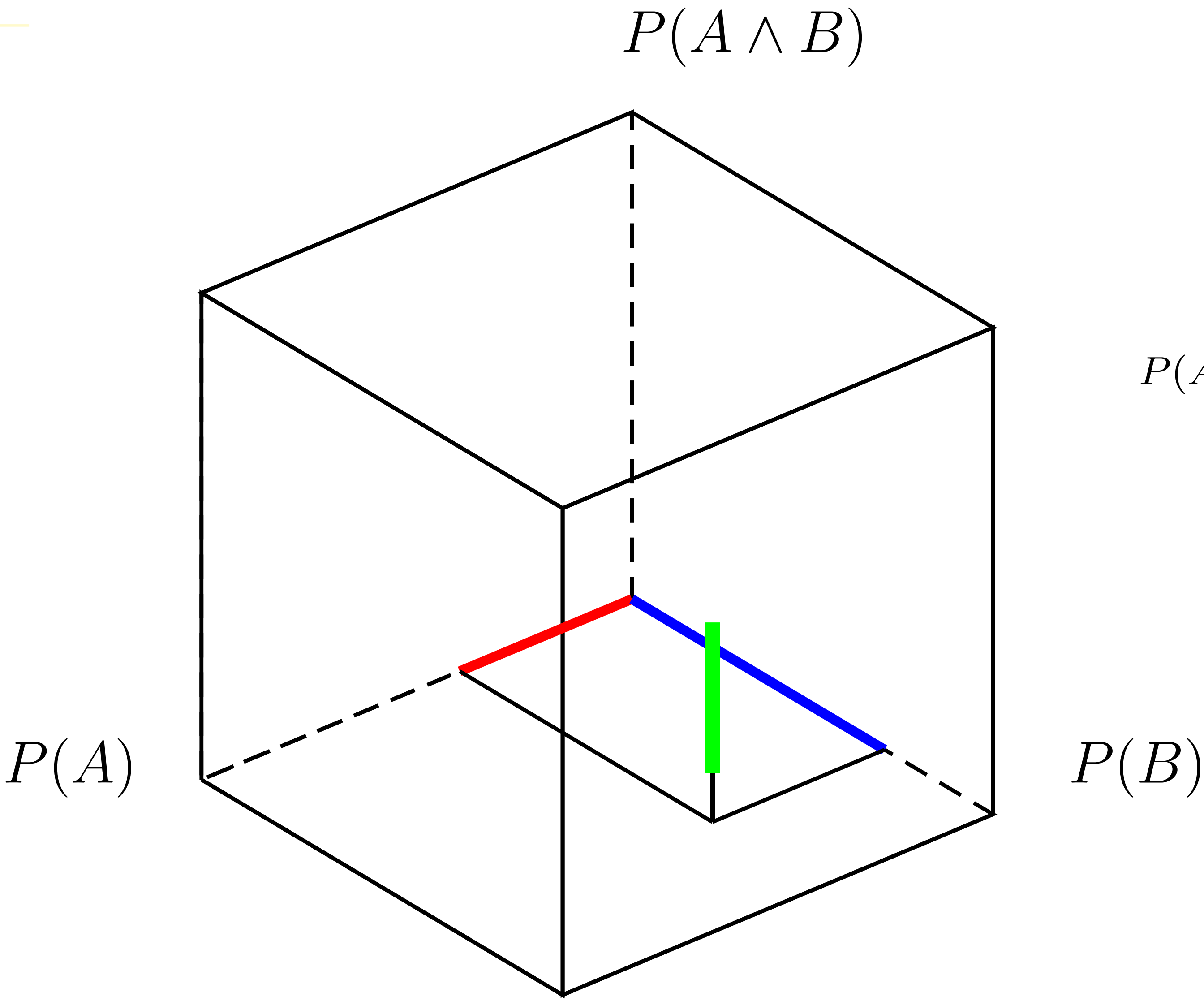
Fundamental Theorem (de Finetti, 1937)

Given the probabilities $P(E_1), \ldots, P(E_m)$ of a finite number of events, the probability of a further event E_{m+1},

$$P(E_{m+1}) \text{ is } \begin{cases} \text{precise} & \text{if } E_{m+1} \text{ is linearly dependent on } \{E_1, \ldots, E_m\}, \\ \in [0,1] & \text{if } E_{m+1} \text{ is logically independent on } \{E_1, \ldots, E_m\}, \\ \in [p', p''] & \text{if } E_{m+1} \text{ is logically dependent on } \{E_1, \ldots, E_m\}, \end{cases}$$

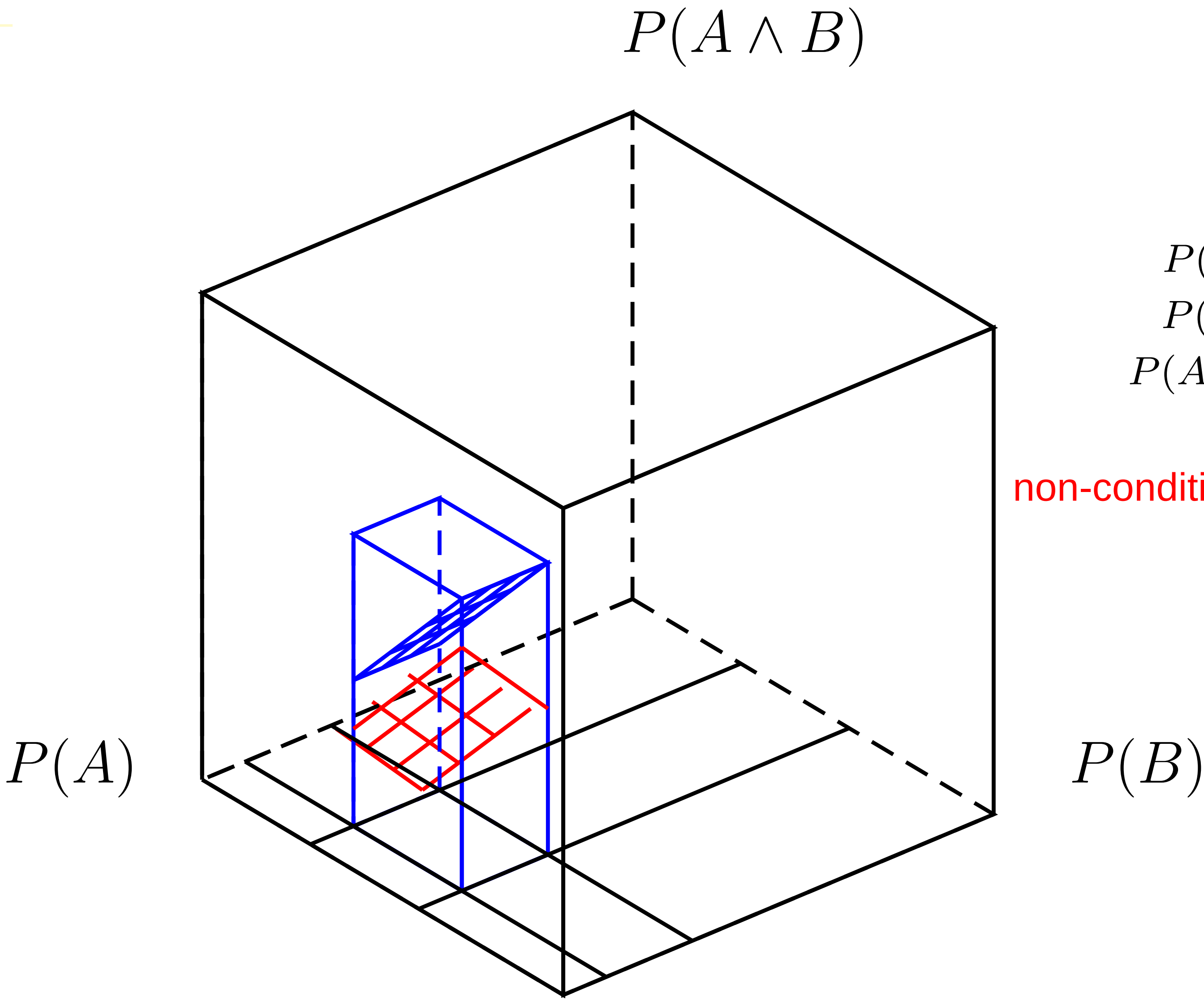
where p' and p'' are lower and upper probabilities.

Coherence I



$$P(A) = .4$$
$$P(B) = .7$$
$$P(A \wedge B) \in [.1, .4]$$

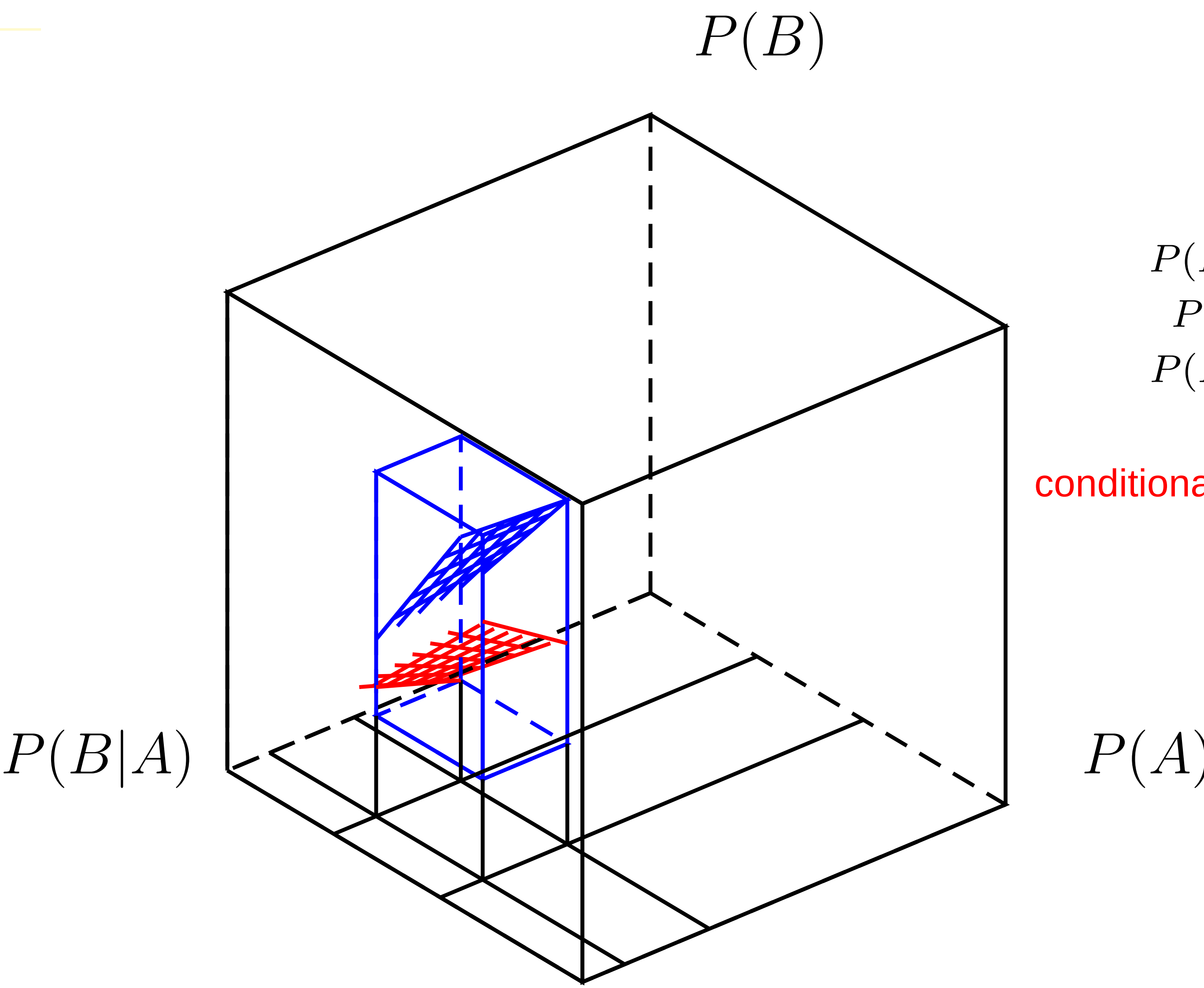
Coherence (imprecise conjunction)



$P(A) \in [.7, .9]$
$P(B) \in [.3, .6]$
$P(A \wedge B) \in [0, .6]$

non-conditional events $\longrightarrow$ linear

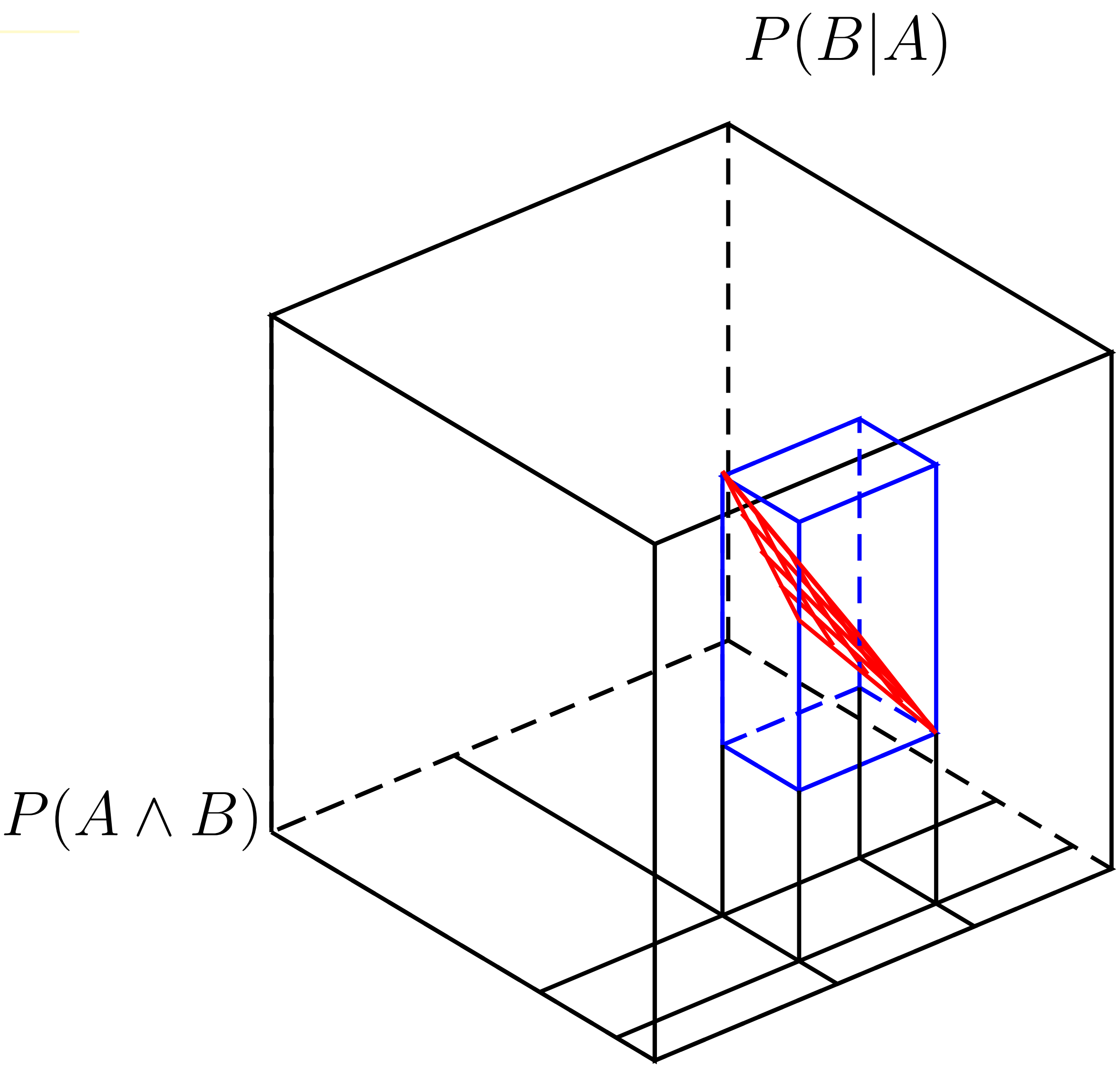
Coherence (imprecise MP)



$P(B|A) \in [.7, .9]$
$P(A) \in [.3, .6]$
$P(B) \in [.21, .72]$

conditional event $\rightarrow$ non-linear

Coherence (function)



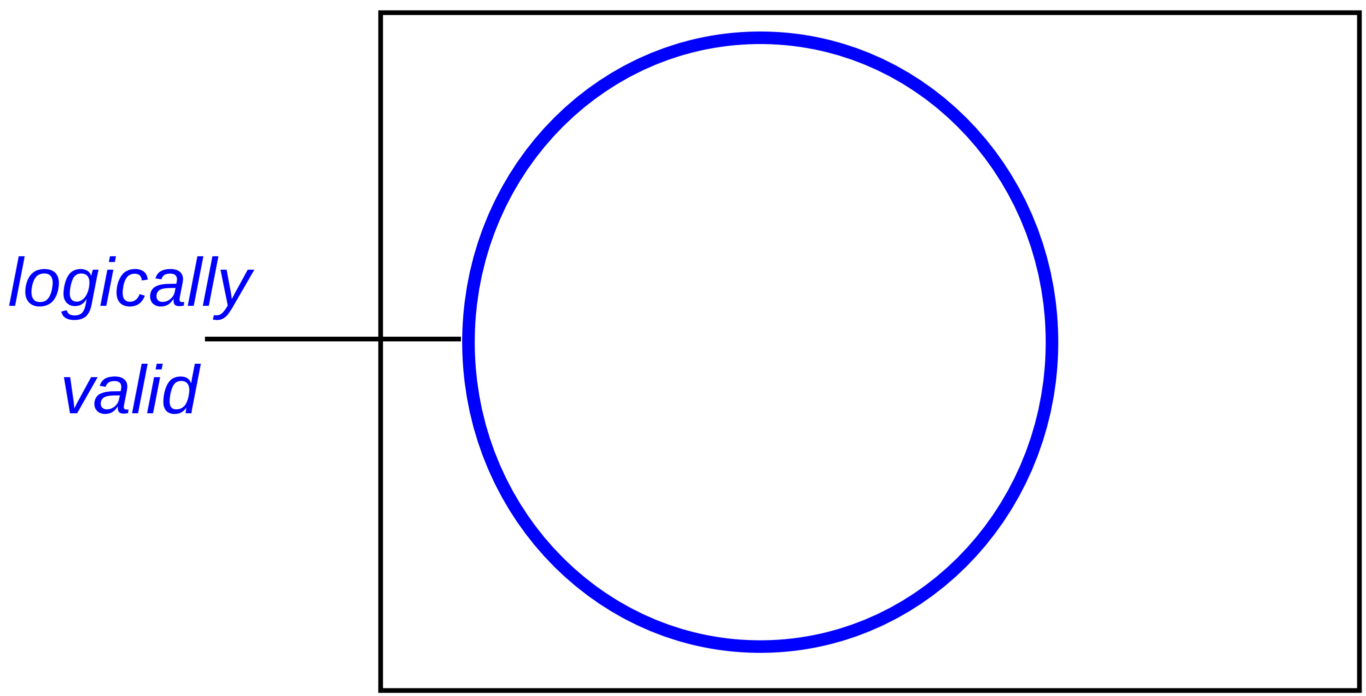
$$P(A \wedge B) = x \in [.3, .6]$$
$$P(A) = y \in [.7, .9]$$
$$P(B|A) = z = \frac{x}{y} = \in [.33, .86]$$

conditional event $\longrightarrow$ non-linear

Logical independence/dependence

- *Logical independence* Let $\{E_1, \ldots, E_m\}$ be a set of m unconditional events. If all 2^m atoms are possible conjunctions, then the set of events is logically independent. Otherwise they are dependent.
- *Linear dependence* If the rank $r(\mathbf{V}_m + 1) = k$ and the rank $r(\mathbf{V}_{m+2}) = k + 1$, then the premises and the conclusion are linearly independent. If $r(\mathbf{V}_m + 1) = r(\mathbf{V}_{m+2})$, then the conclusion is linearly dependent on the premises.

Logically valid–probabilistically informative



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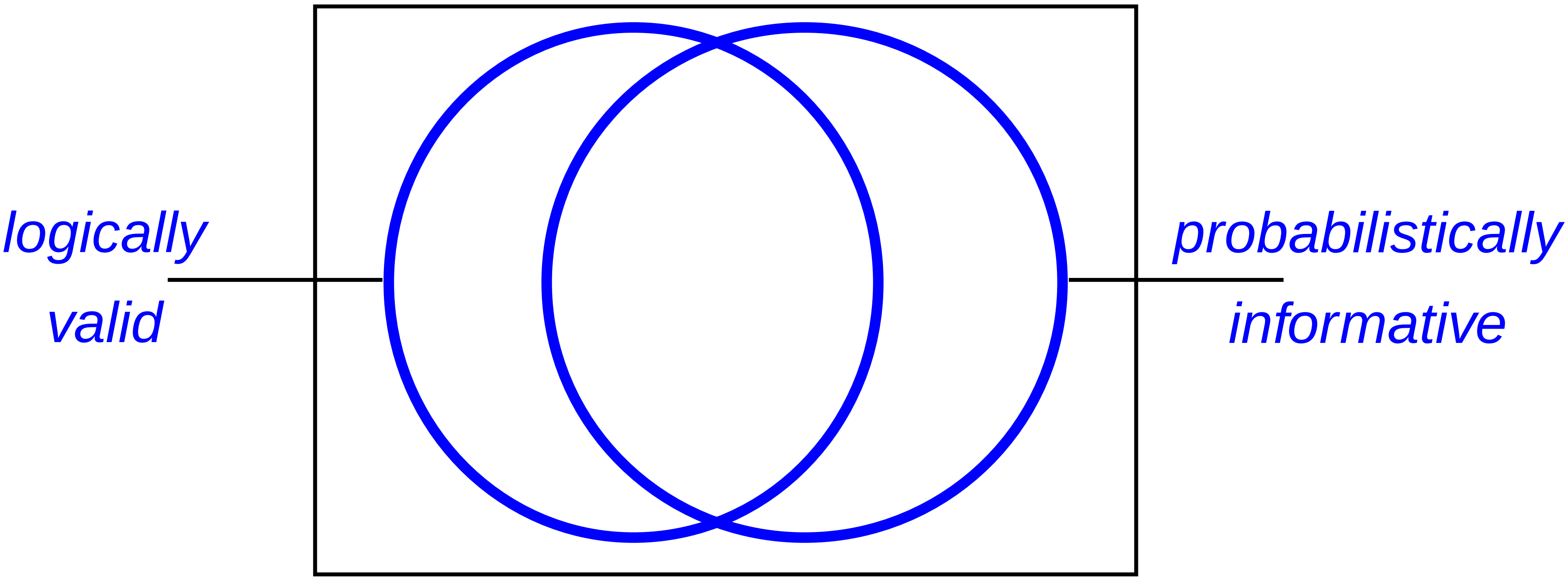

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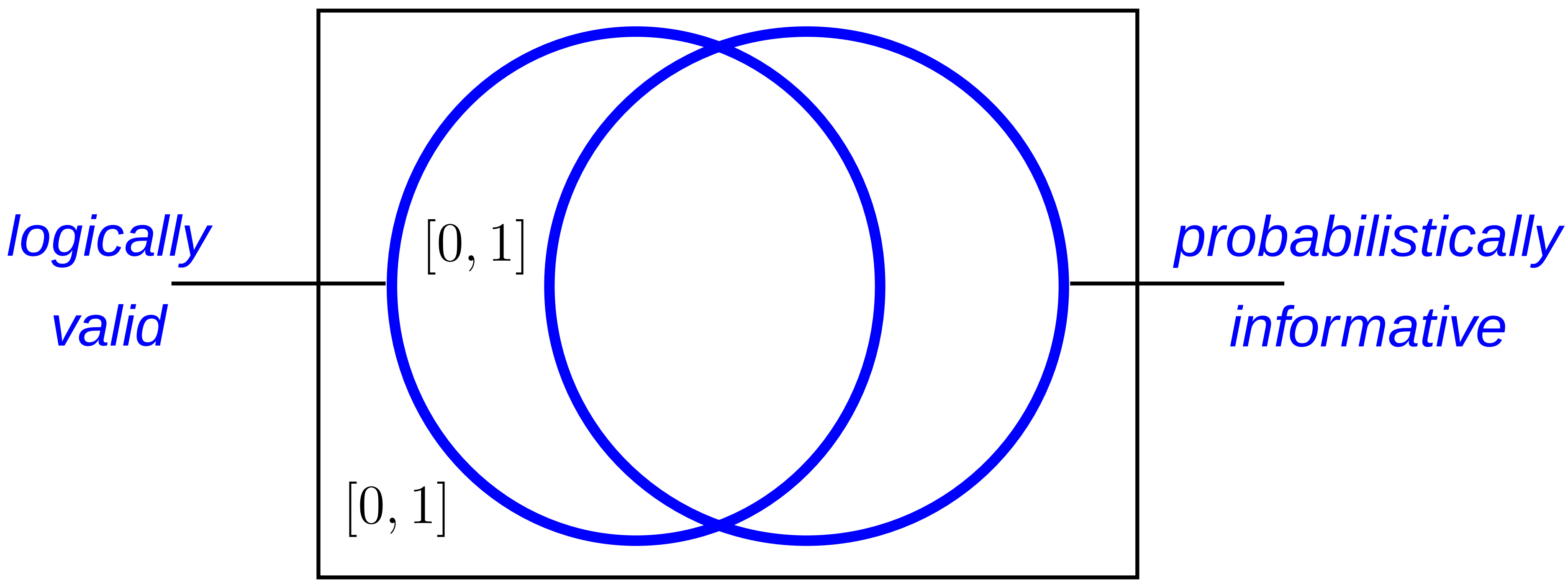

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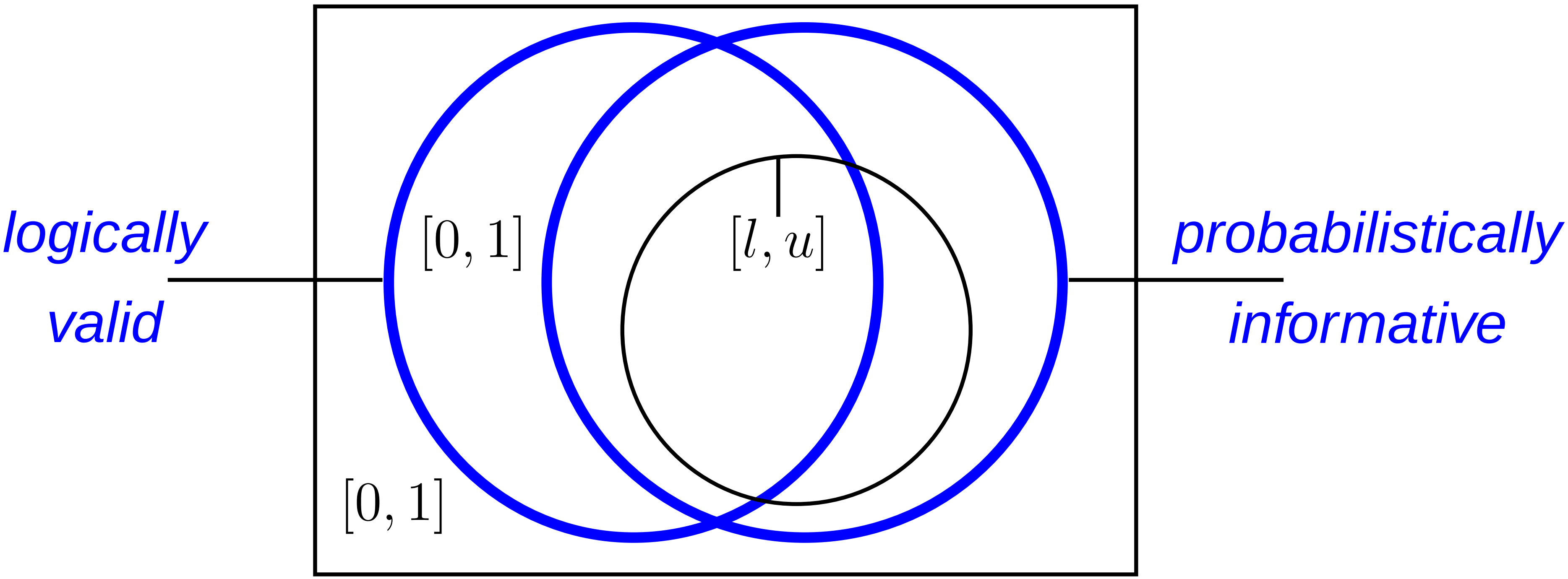

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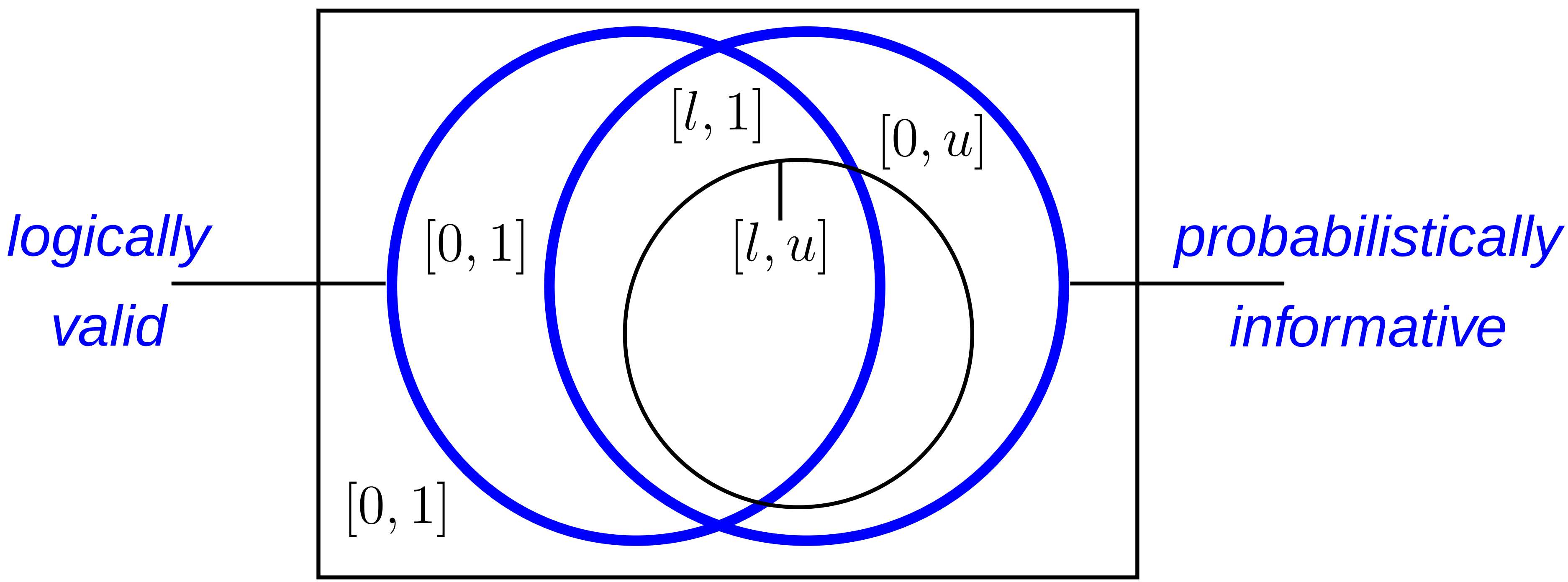

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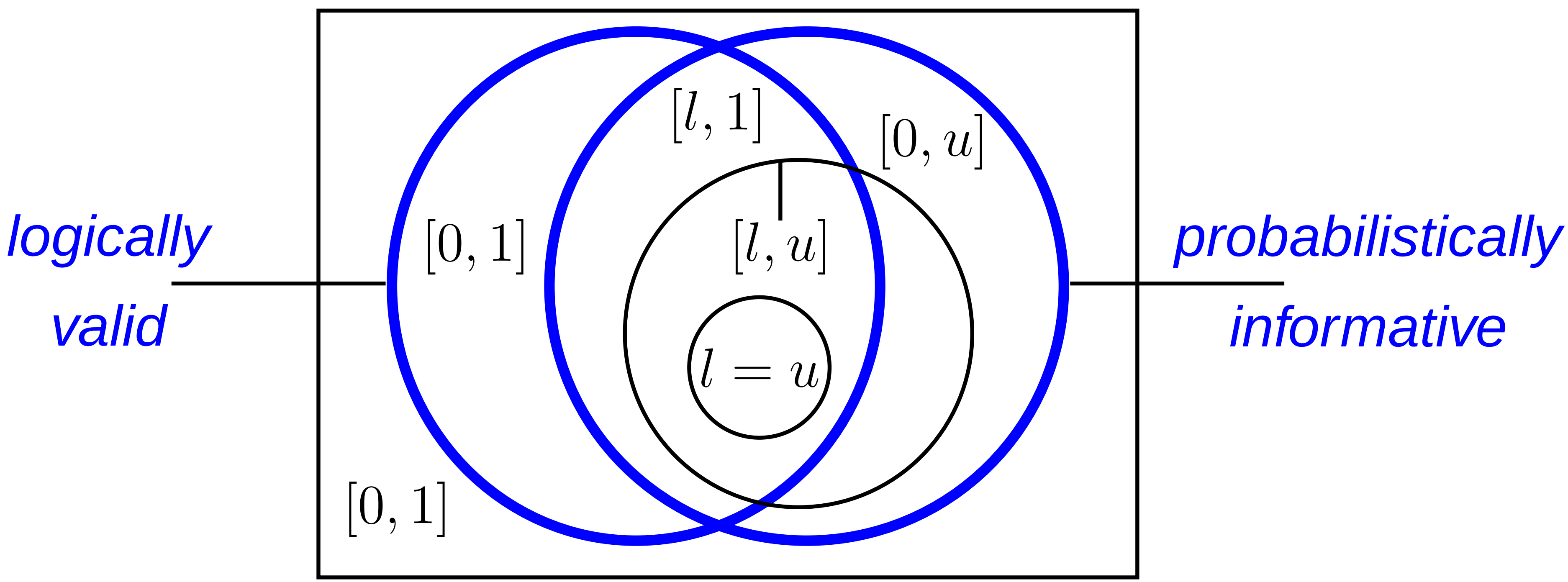

Combining logic and probability in psychology

Probabilistic approaches to human deductive reasoning

Postulated interpretation of the "IF A, THEN B"

$$P(A \supset B)$$

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Probabilistic extension of the *mental model* theory

Johnson-Laird et al.

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Theoretical problems:

Paradoxes of the material implication:
e.g., from IF A, THEN B infer IF A AND C, THEN B

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The material implication is not a genuine conditional

$$(A \supset B) \quad \Leftrightarrow \quad (\neg A \vee B)$$

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No paradoxes of the material implication:
If $P(B|A) = x$, then $P(B|A \wedge C) \in [0, 1]$,

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The conditional event $B|A$ is a genuine conditional

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Deductive relation between premise(s) and conclusion
Mental probability logic
Pfeifer & Kleiter

Mental probability logic

- investigates IF A, THEN B as nonmontonic conditionals in a probability logic framework
  - A, normally B iff $P(B|A) = high$

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- coherence

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- In logic
  from $\boxed{A \text{ and } A \supset B}$ infer $\boxed{B}$

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  infer $\boxed{P(B) \in [xy, xy + (1 - x)]}$

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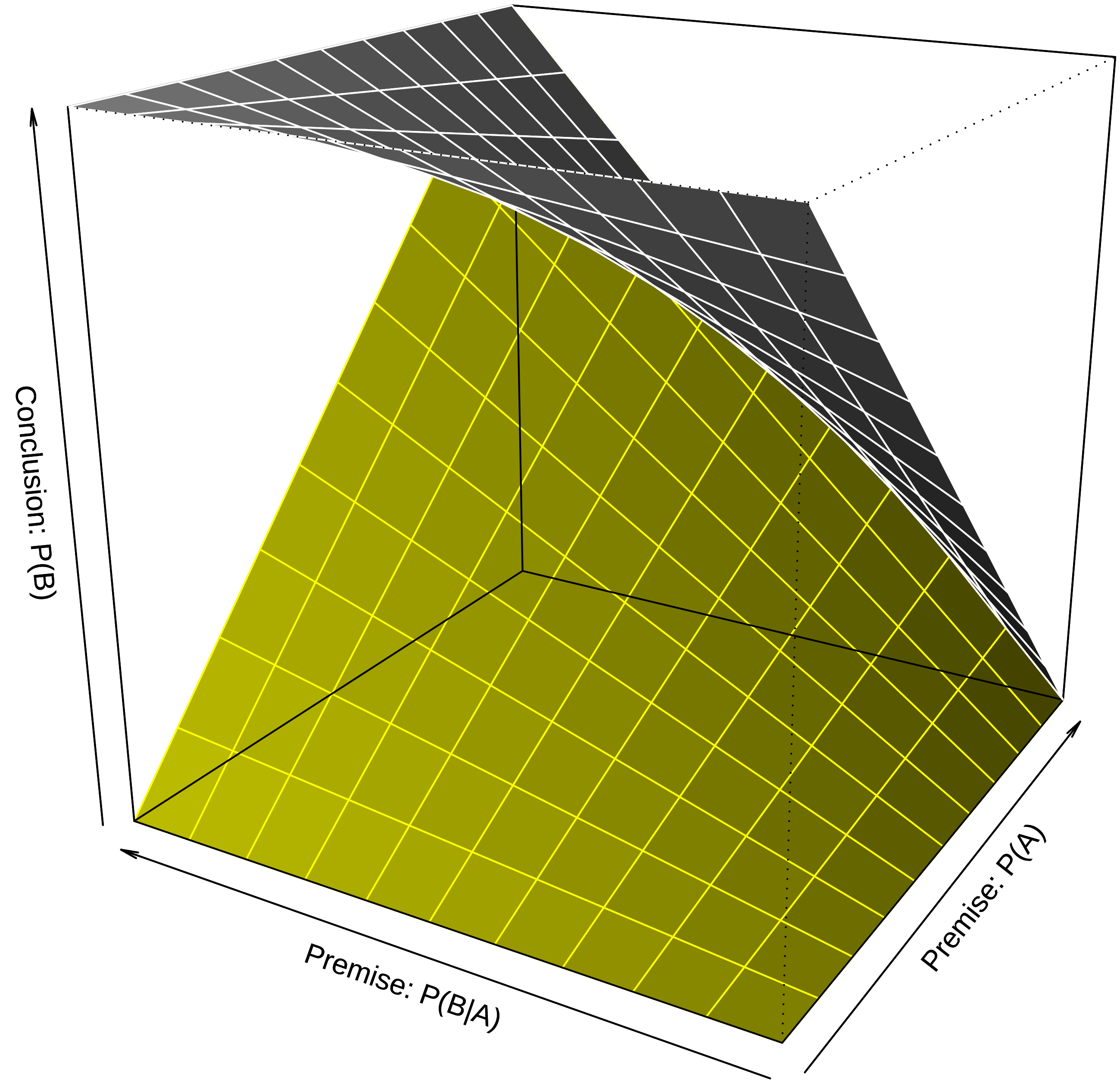
- In probability logic

  from $\boxed{P(A) = x \text{ and } P(B|A) = y}$

  infer $\boxed{P(B) \in [\underbrace{xy}_{at\ least}, \underbrace{xy + (1 - x)}_{at\ most}]}$

Probabilistic MODUS PONENS



Example task: MODUS PONENS

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Claudia is 100% certain:

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How certain should Claudia be that a recent donated blood is Rhesus-positive?

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The solution is either a point percentage or a percentage between two boundaries (from at least ... to at most ...):

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The solution is either a point percentage or a percentage between two boundaries (from at least . . . to at most . . .):

Claudia is at least . . . .% and at most . . . .% certain, that the donated blood is Rhesus-positive.

Within the bounds of:

0 25 50 75 100 %

Results

Premise 1	2	*coherent* LB.	UB.	*response* LB.	UB.	*coherent* LB.	UB.	*response* LB.	UB.
		MODUS PONENS				NEGATED MODUS PONENS			
1	1	1	1	1	1	.00	.00	.00	.00
.7	.9	.63	.73	.62	.69	.27	.37	.35	.42
.7	.5	.35	.85	.43	.55	.15	.65	.41	.54
		DENYING THE ANTECEDENT				NEGATED DENYING THE ANTECEDENT			
1	1	.00	1	.37	.85	.00	1	.01	.53
.7	.2	.20	.44	.19	.42	.56	.80	.52	.76
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"certain" MODUS PONENS tasks: all participants inferred correctly "1" or "0"

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"certain" DENYING THE ANTECEDENT tasks: most participants inferred intervals close to $[0, 1]$

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overall good agreement between the normative bounds and the mean responses

Conjugacy

All participants inferred a probability (interval) of a conclusion $P(\mathfrak{C}) \in [z', z'']$ and the probability of the associated negated conclusion, $P(\neg\mathfrak{C})$.

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(*Premise 1, Premise 2*)	$(1, 1)$	$(.7, .9)$	$(.7, .5)$	$(.7, .2)$
MODUS PONENS	100%	53%	50%	
DENYING THE ANTECEDENT	67%		30%	0%

...percentages of participants satisfying both $z'_{\mathfrak{C}} + z''_{\neg\mathfrak{C}} = 1$ and $z'_{\neg\mathfrak{C}} + z''_{\mathfrak{C}} = 1$

Concluding remarks

- Framing human inference by coherence based probability logic
  - investigating nonmonotonic conditionals in agrument forms
  - interpreting the if–then as high conditional probability
  - coherence based
  - competence theory ("Mental probability logic")
  - MODUS PONENS, conjugacy, forward & affirmative

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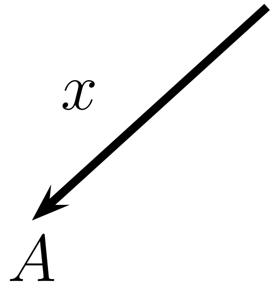
Towards a process model of human conditional inference

Propositional graph: Notation



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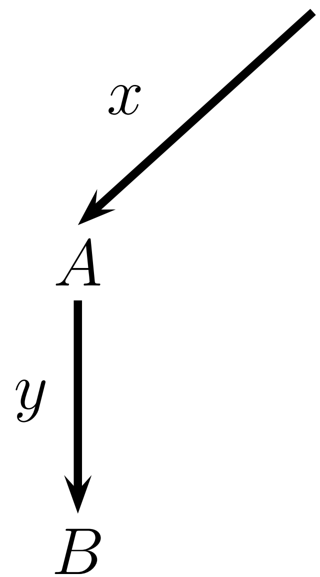

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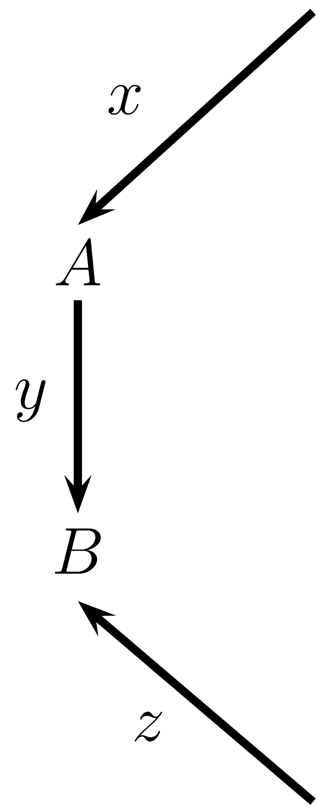
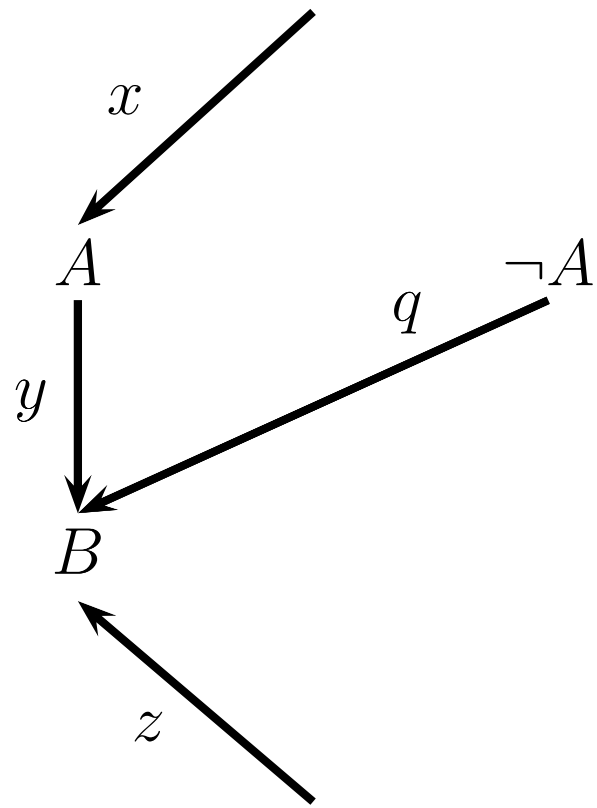


Propositional graph: Notation



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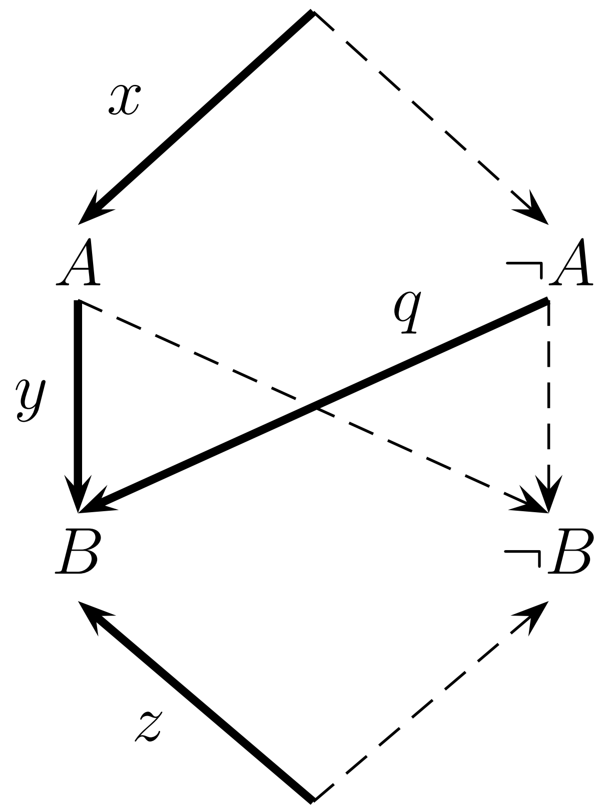
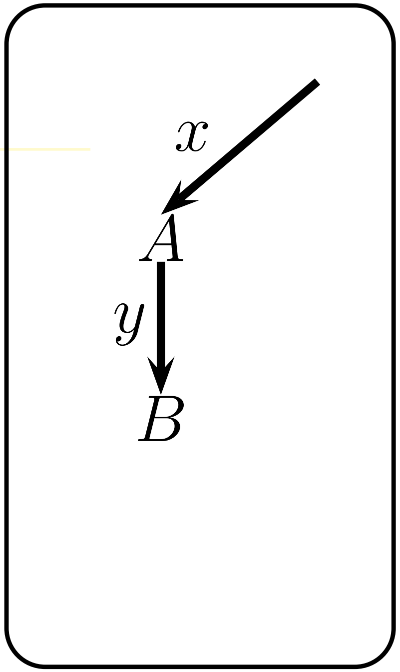
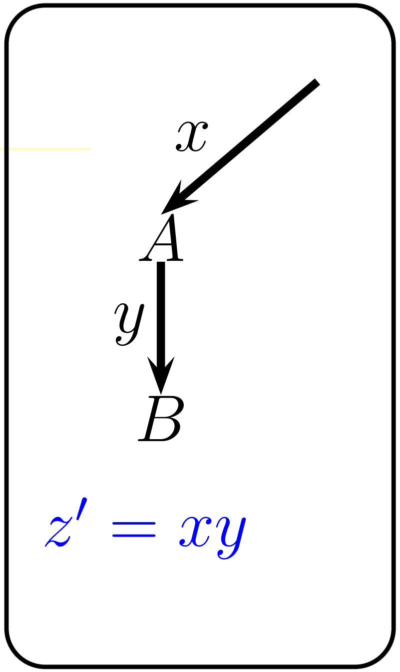




MODUS PONENS

$$P(B) = ?$$



MODUS PONENS

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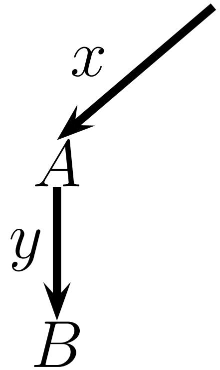


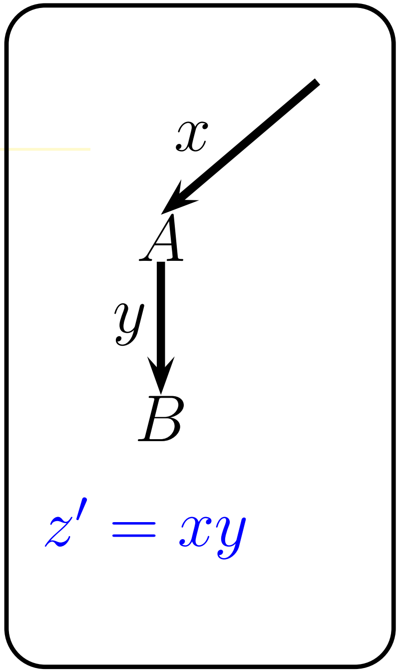
$z' = xy$

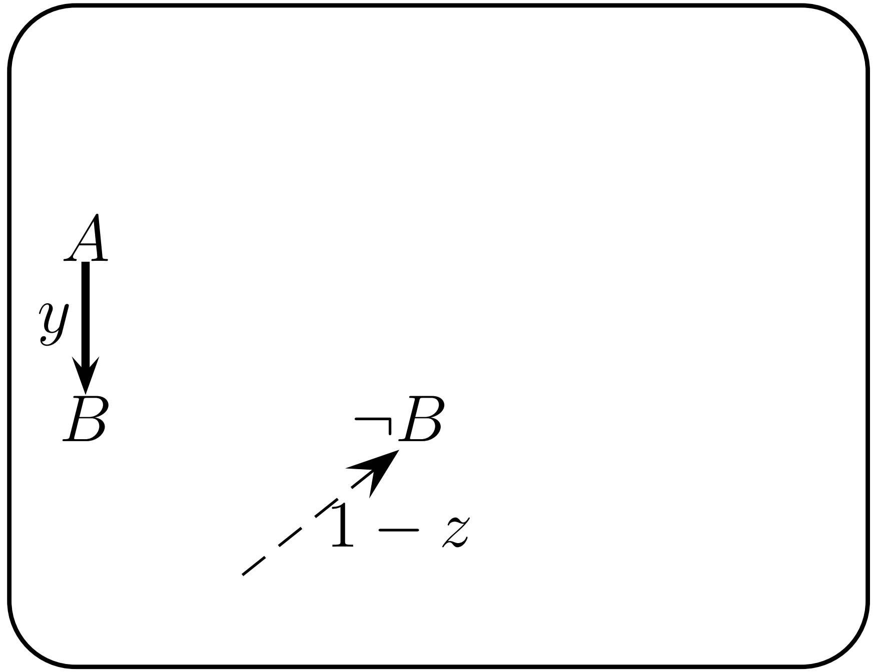
MODUS PONENS

$$P(B) = ?$$

forward

affirmative





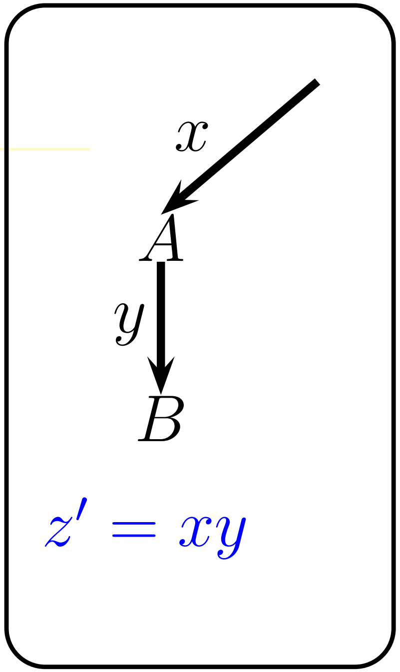
MODUS PONENS

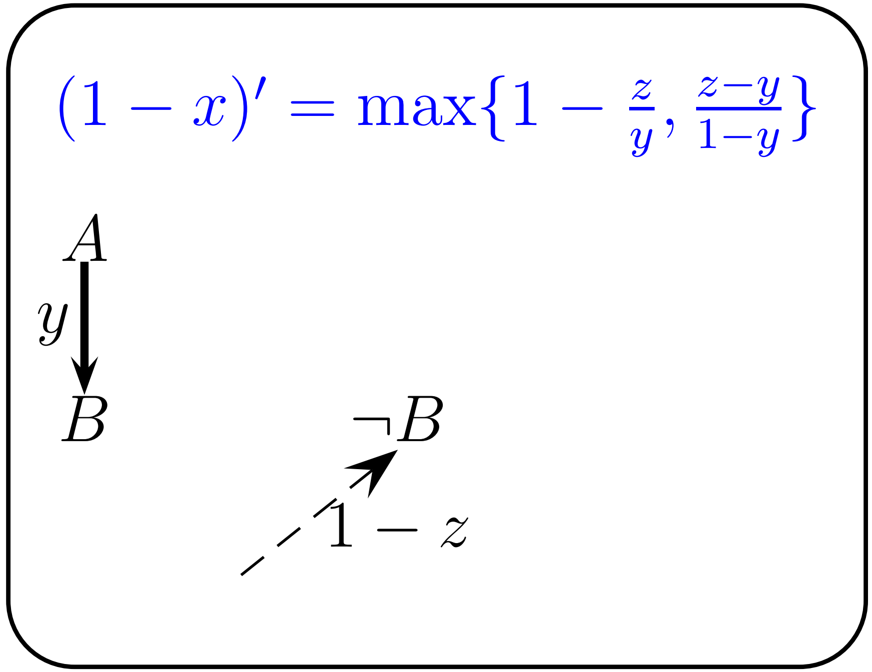
MODUS TOLLENS

$$P(B) = ?$$

forward
affirmative

$$P(\neg A) = ?$$





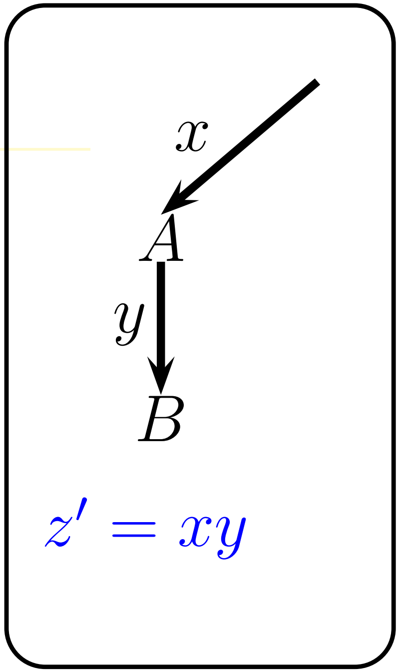
MODUS PONENS

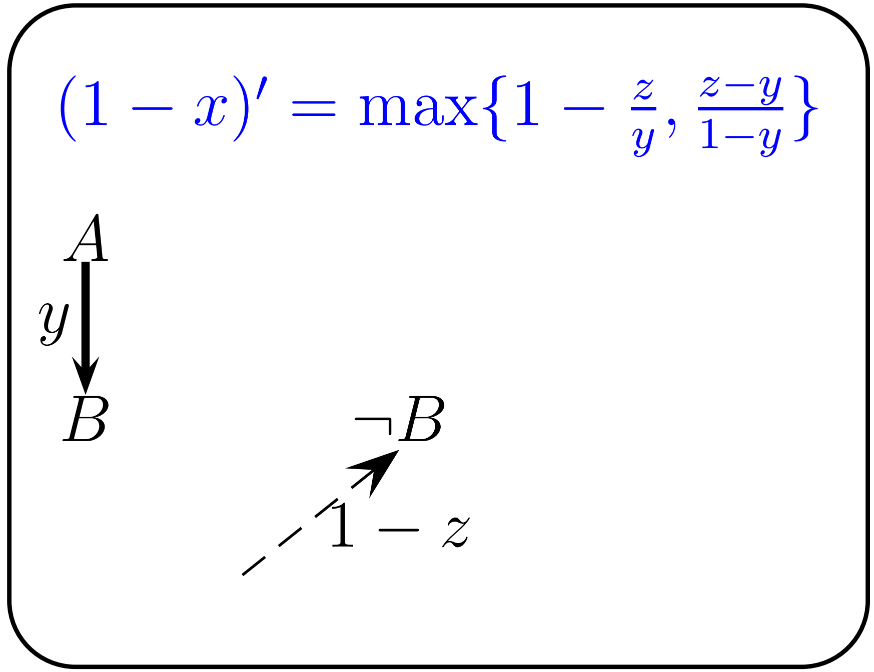
MODUS TOLLENS

$$P(B) = ?$$

forward
affirmative

$$P(\neg A) = ?$$





MODUS PONENS

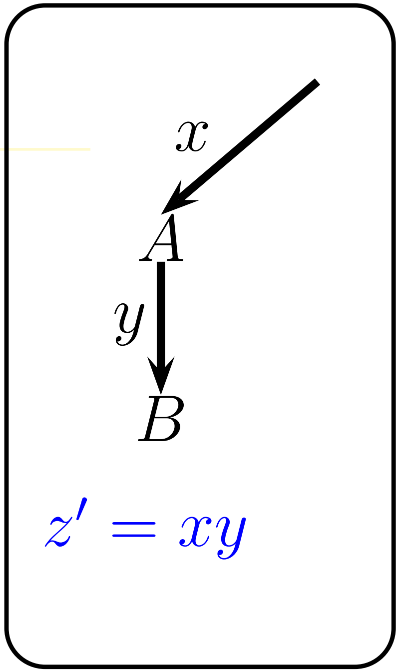
$P(B) = ?$

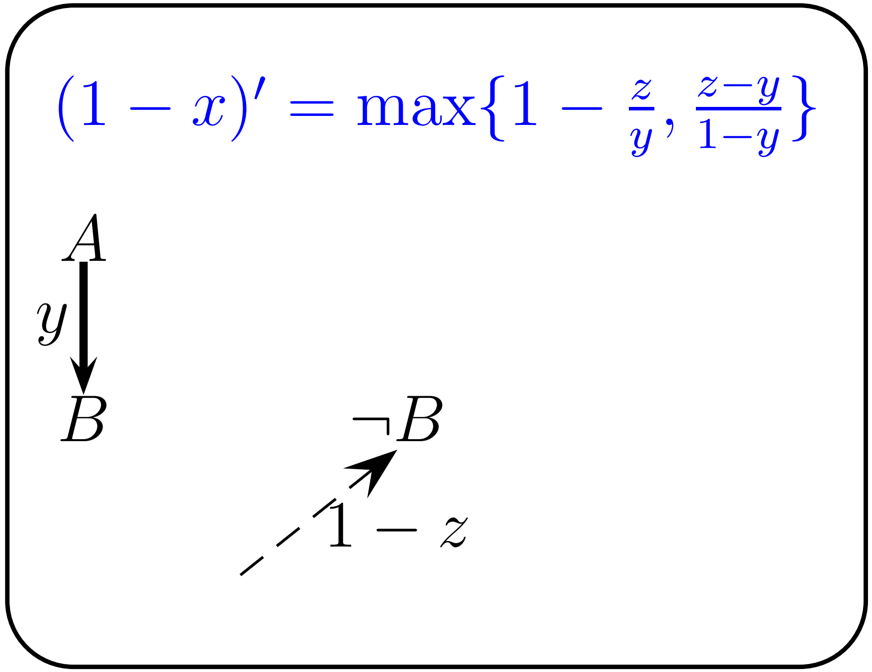
forward
affirmative

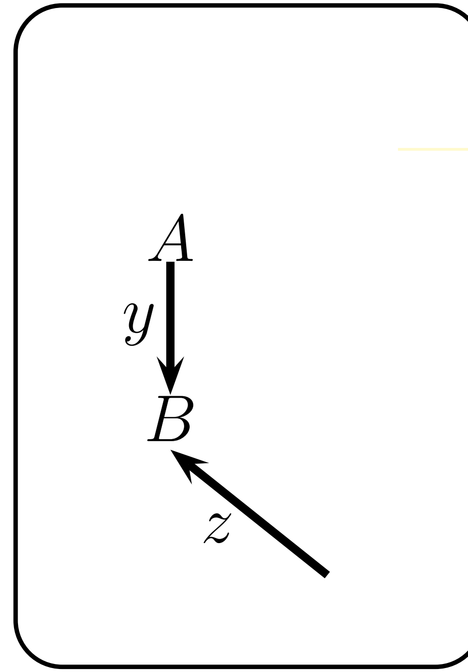
MODUS TOLLENS

$P(\neg A) = ?$

backward
negated







MODUS PONENS

$$P(B) = ?$$

forward
affirmative

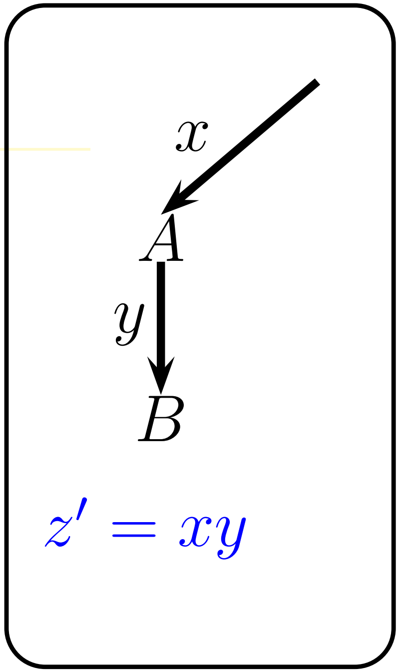
MODUS TOLLENS

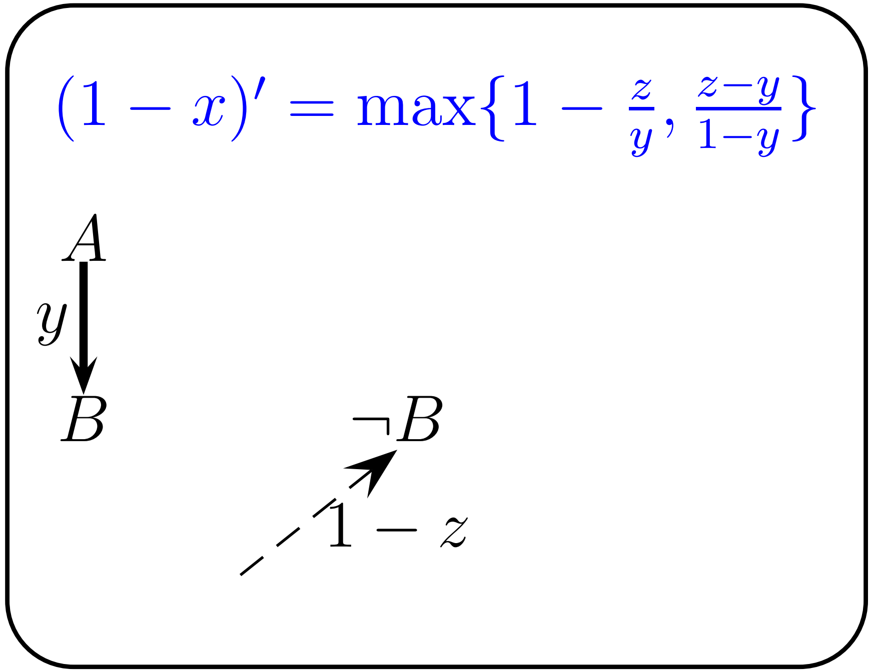
$$P(\neg A) = ?$$

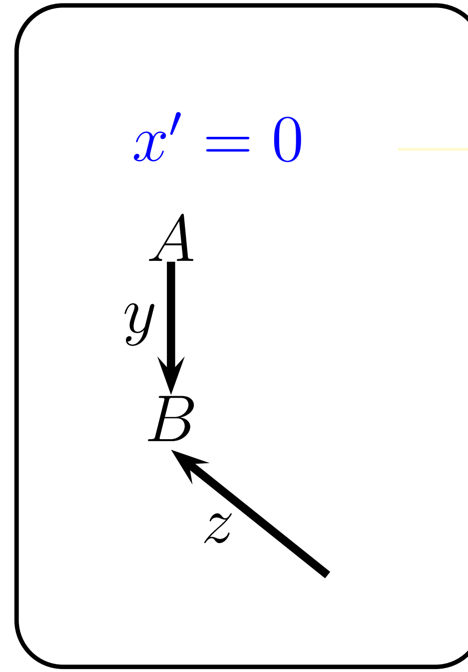
backward
negated

AFFIRMING THE CONSEQUENT

$$P(A) = ?$$







MODUS PONENS

$$P(B) =?$$

forward
affirmative

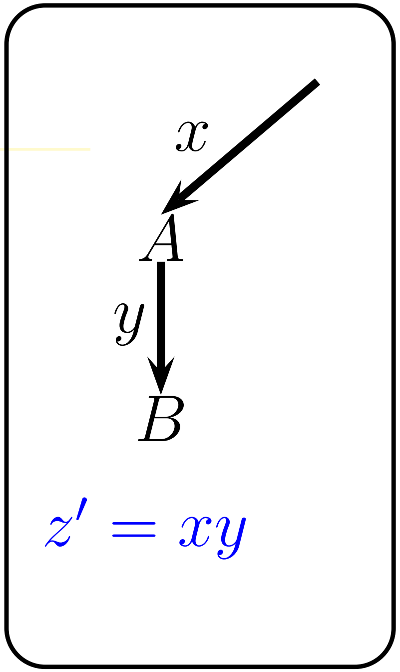
MODUS TOLLENS

$$P(\neg A) =?$$

backward
negated

AFFIRMING THE CONSEQUENT

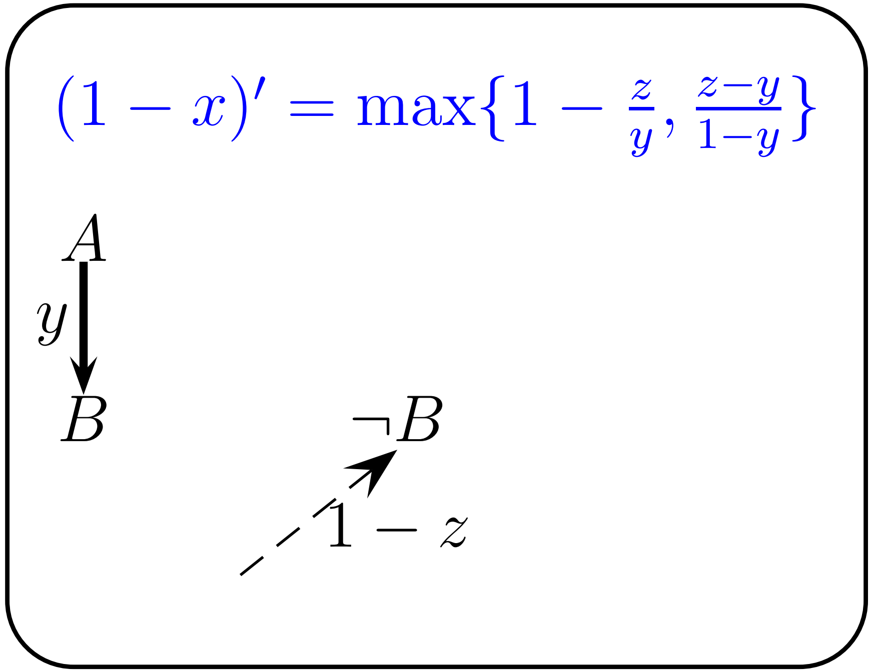
$$P(A) =?$$



MODUS PONENS

$$P(B) = ?$$

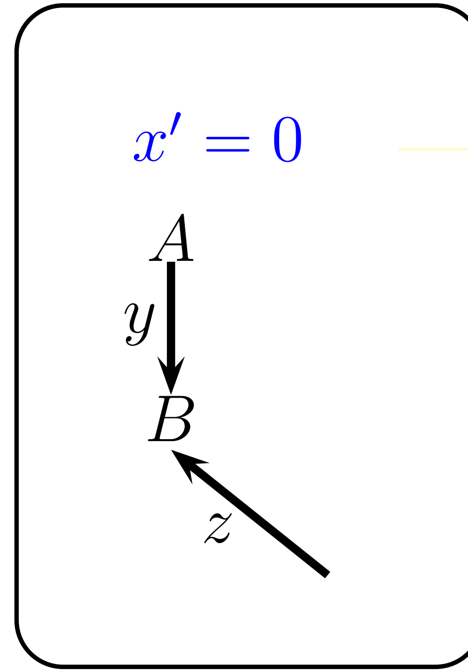
forward
affirmative



MODUS TOLLENS

$$P(\neg A) = ?$$

backward
negated



AFFIRMING THE CONSEQUENT

$$P(A) = ?$$

backward
affirmative

Logical validity vs. soundness

	MP
P_1:	$A \supset B$
P_2:	A
$\mathfrak{C}$:	B

Logical validity vs. soundness

	MP	NMP
P_1:	$A \supset B$	$A \supset B$
P_2:	A	A
$\mathfrak{C}$:	B	$\neg B$

Logical validity vs. soundness

	MP	NMP	DA	NDA
P_1:	$A \supset B$	$A \supset B$	$A \supset B$	$A \supset B$
P_2:	A	A	$\neg A$	$\neg A$
$\mathfrak{C}$:	B	$\neg B$	$\neg B$	B

Logical validity vs. soundness

	MP	NMP	DA	NDA
P_1:	$A \supset B$	$A \supset B$	$A \supset B$	$A \supset B$
P_2:	A	A	$\neg A$	$\neg A$
$\mathfrak{C}$:	B	$\neg B$	$\neg B$	B
L-valid:	yes	no	no	no

Logical validity vs. soundness

	MP	NMP	DA	NDA
P_1:	$A \supset B$	$A \supset B$	$A \supset B$	$A \supset B$
P_2:	A	A	$\neg A$	$\neg A$
$\mathfrak{C}$:	B	$\neg B$	$\neg B$	B
L-valid:	yes	no	no	no
$V(\mathfrak{C})$	t	f	?	?

$V(\mathfrak{C})$ denotes the truth value of the conclusion $\mathfrak{C}$ under the assumption that the valuation-function V assigns t to each premise.

Probabilistic argument forms

	Probabilistic versions of the			
	MP	NMP	DA	NDA
P_1:	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$
P_2:	$P(A) = y$	$P(A) = y$	$P(\neg A) = y$	$P(\neg A) = y$
$\mathfrak{C}$:	$P(B) = z$	$P(\neg B) = z$	$P(\neg B) = z$	$P(B) = z$

The "IF A, THEN B" is interpreted as a conditional probability, $P(B|A)$.

Probabilistic argument forms

	Probabilistic versions of the			
	MP	NMP	DA	NDA
P_1:	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$
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$\mathfrak{C}$:	$P(B) = z$	$P(\neg B) = z$	$P(\neg B) = z$	$P(B) = z$
z'	xy		$(1-x)(1-y)$	
z''	$1-(y-xy)$		$1-x(1-y)$	

$z = f(x, y)$ and $z \in [z', z'']$

Probabilistic argument forms

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	MP	NMP	DA	NDA
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$\mathfrak{C}$:	$P(B) = z$	$P(\neg B) = z$	$P(\neg B) = z$	$P(B) = z$
z'	xy	$y - xy$	$(1-x)(1-y)$	$x(1-y)$
z''	$1-(y-xy)$	$1-xy$	$1-x(1-y)$	$1-(1-x)(1-y)$

…by conjugacy: $P(\neg\mathfrak{C}) = 1 - P(\mathfrak{C})$

Probabilistic argument forms

	Probabilistic versions of the			
	MP	NMP	DA	NDA
P_1:	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$
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Chater, Oaksford, et. al: Subjects' endorsement rate depends only on the conditional probability of the conclusion given the categorical premise, $P(\mathfrak{C}|P_2)$

- the conditional premise is ignored
- the relation between the premise(s) and the conclusion is uncertain

Probabilistic argument forms

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	MP	NMP	DA	NDA
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$\mathfrak{C}$:	$P(B) = z$	$P(\neg B) = z$	$P(\neg B) = z$	$P(B) = z$

Mental probability logic: most subjects infer coherent probabilities from the premises

- the conditional premise is not ignored
- the relation between the premise(s) and the conclusion is deductive

Results—Certain Premises (Pfeifer & Kleiter, 2003*, 2005a**, 2006)

Condition	lower bound		upper bound		
(Task B7)	M	SD	M	SD	n_i
CUT1	95.05	22.14	100	0.00	20
CUT2	93.75	25.00	93.75	25.00	16
RW	95.00	22.36	100	0.00	20
OR	99.63	1.83	99.97	0.18	30
CM*	100	0.00	100	0.00	19
AND**	75.30	43.35	90.25	29.66	40
M*	41.25	46.63	92.10	19.31	20
TRANS1	95.00	22.36	100	0.00	20
TRANS2	95.00	22.36	100	0.00	20
TRANS3	77.95	37.98	94.74	15.77	19

Inference from imprecise premises – “Silent bounds”

“Silent” bounds

A probability bound b of a premise is silent iff b is irrelevant for the probability propagation from the premise(s) to the conclusion.

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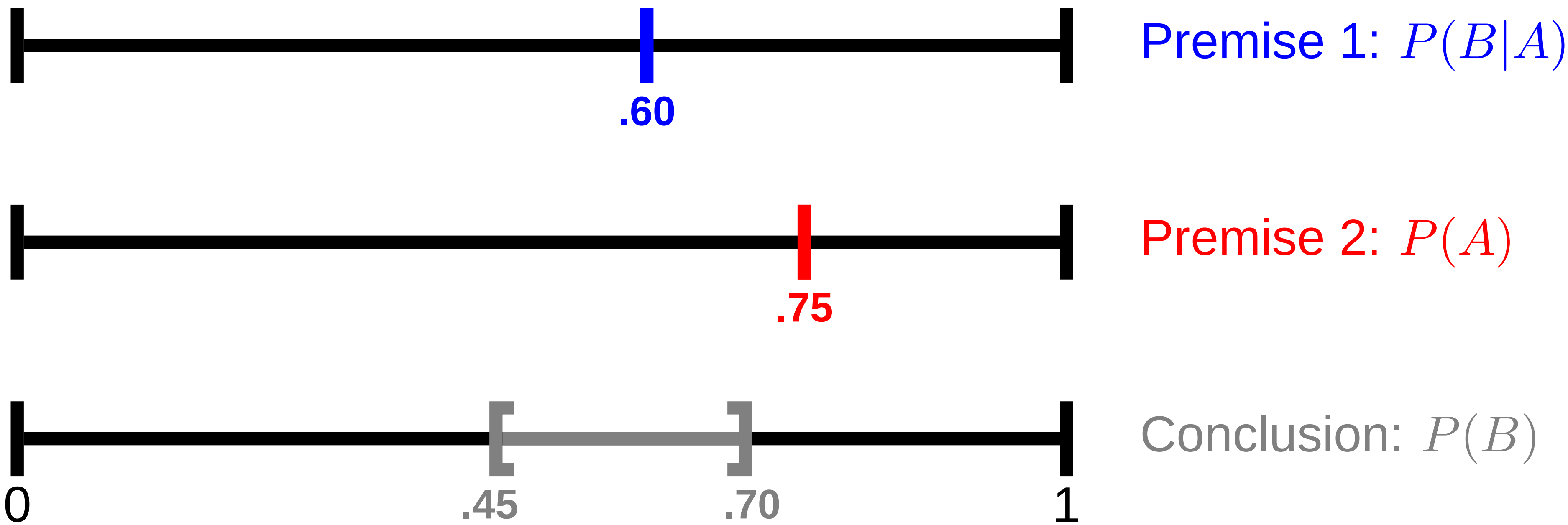

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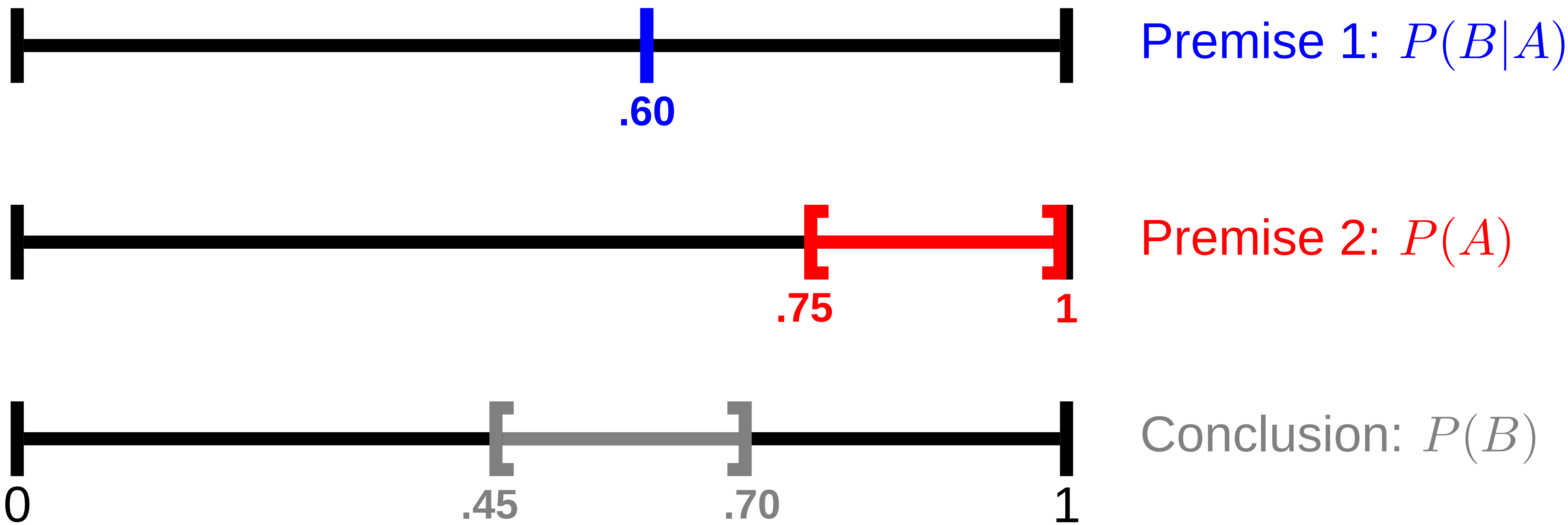

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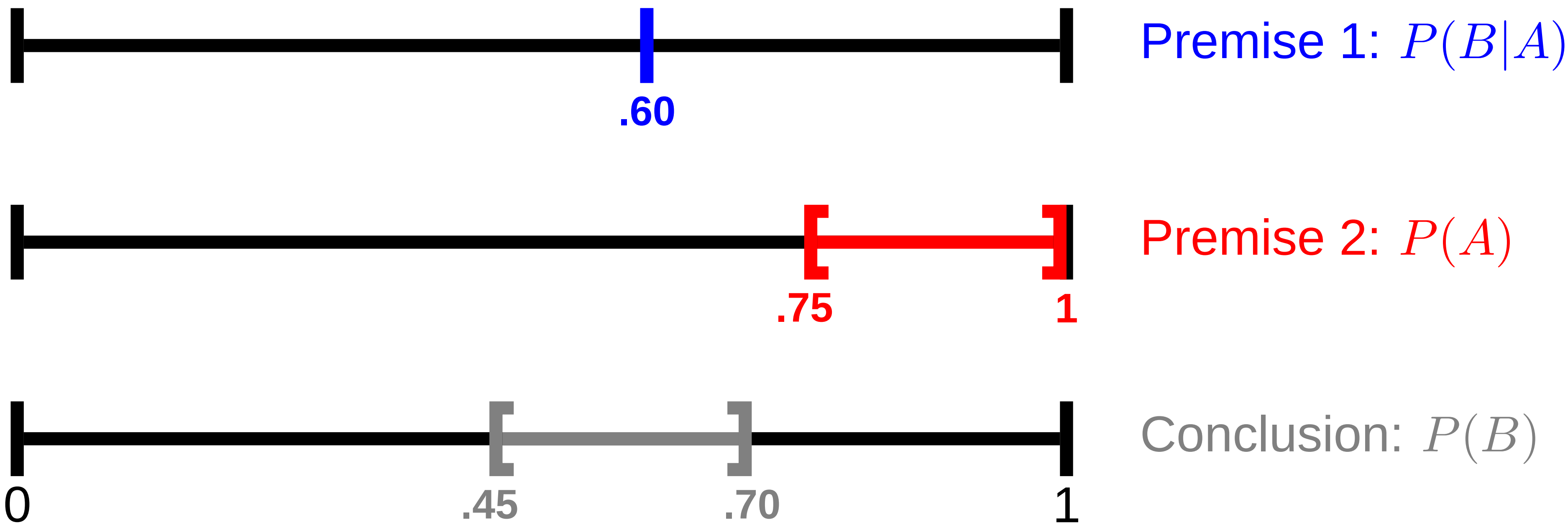

$$P(B|A) \in [x', x''], \; P(A) \in [y', \; y''] \;\therefore\; P(B) \in [x'y', 1 - y' + x''y']$$

“Silent” bounds

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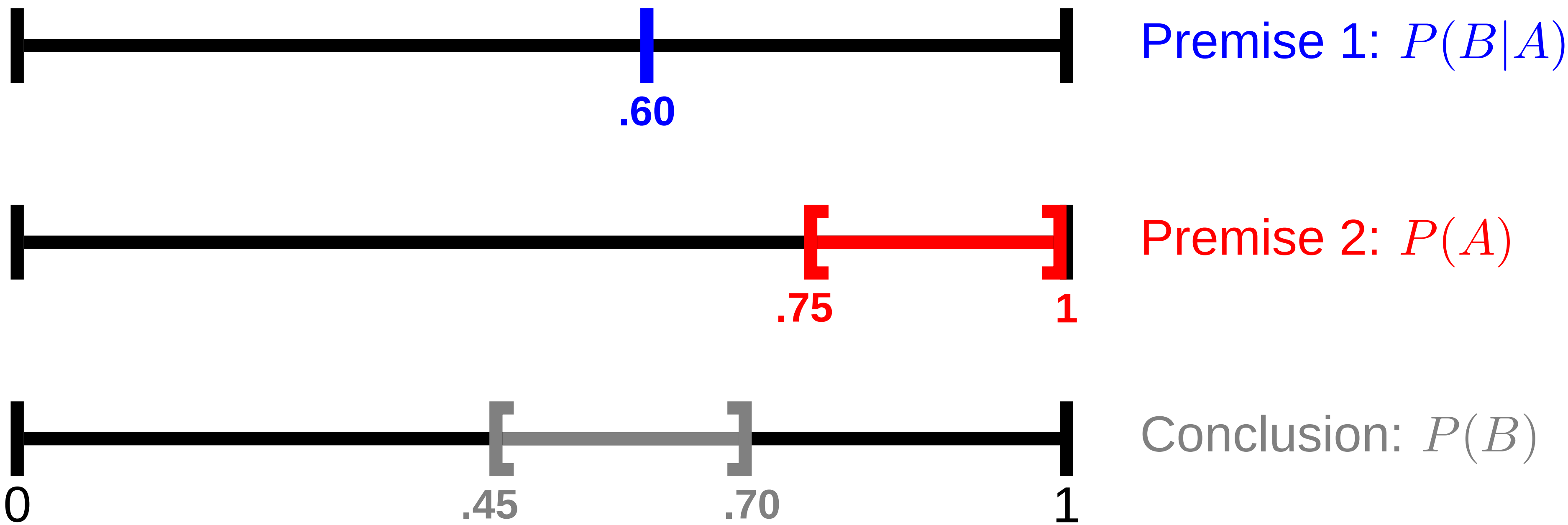

$$P(B|A) \in [x', x''],\ P(A) \in [y', \overbrace{\ y''\ }^{silent}] \therefore P(B) \in [x'y', 1 - y' + x''y']$$

MODUS PONENS **task with silent bound** (Bauerecker, 2006)

Claudia works at blood donation services. She investigates to which blood group the donated blood belongs and whether the donated blood is Rhesus-positive.

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Claudia is 60% certain: If the donated blood belongs to the blood group 0, then the donated blood is Rhesus-positive.

Claudia knows that donated blood belongs with more than 75% certainty to the blood group 0.

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How certain should Claudia be that a recent donated blood is Rhesus-positive?

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Claudia is 60% certain: If the donated blood belongs to the blood group 0, then the donated blood is Rhesus-positive.

Claudia knows that donated blood belongs with exactly 75% certainty to the blood group 0.

How certain should Claudia be that a recent donated blood is Rhesus-positive?

Results: Mean Responses (Bauerecker, 2006)

Task	*Premise*		*Coherent*		*Response*	
	1	*2*	*LB*	*UB*	*LB*	*UB*
MP	.60	.75-1^*	.45	.70	.45	.72
	.60	.75	.45	.70	.47	.60
NMP	.60	.75-1^*	.30	.55	.17	.46
	.60	.75	.30	.55	.23	.42

- Participants inferred higher intervals in the MP tasks: participants are sensitive to the complement

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MP	.60	.75-1^*	.45	.70	.45	.72
	.60	.75	.45	.70	.47	.60
NMP	.60	.75-1^*	.30	.55	.17	.46
	.60	.75	.30	.55	.23	.42

- Participants inferred higher intervals in the MP tasks: participants are sensitive to the complement
- Participants inferred wider intervals in the tasks with the silent bound, 1^*: they are sensitive to silent bounds (i.e., they neglect the irrelevance of 1^*)
- More than half of the participants inferred coherent intervals