35TH INTERNATIONAL WORKSHOP ON OPERATOR THEORY AND ITS APPLICATIONS (IWOTA24)

AUGUST 12 TO AUGUST 16, 2024

University of Kent

UPDATES: Sunday Aug. 11

- S15, Natalia Bebiano cancelled talk. Tin-Yau Tam moved to Friday 15:00-15:25 from Friday 15:30-15:55.
- S22, Natalia Bebiano cancelled talk on Tuesday 18:00-18:25.
- S22, Elroy Zeekoei talk rescheduled from Monday 15:00-15:25 to Tuesday 18:00-18:25.
- S6, David Eelbode talk cancelled, Monday 14:30-14:55.
- S10, Claus Köstler talk cancelled, Thursday 15:00-15:25.

UPDATES: Monday Aug. 12

- S9, some changes to schedule, updated in programme online

UPDATES: Thursday Aug. 15

- S19 Thursday sessions moved from SIBSR5 to SIBSR4.

IWOTA 2024 is organized by:

- Ian Wood (chair), University of Kent, UK
- Marina Iliopoulou, University of Athens, Greece
- Bas Lemmens, University of Kent, UK
- Ana Loureiro, University of Kent, UK
- Marco Marletta, University of Cardif, UK

in collaboration with the IWOTA executive steering committee members:

- J. William Helton (chair), University of California, San Diego, USA
- Sanne ter Horst, North-West University, South Africa
- Igor Klep, University of Ljubljana, Slovenia
- Irene Sabadini, Politecnico Milano, Italy
- Hugo J. Woerdeman, Drexel University, USA

Conference email: `iwota2024@kent.ac.uk`

Supported by

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1. Scientific Program

1.1. Special Sessions.

S1 Harmonic Analysis and Related Areas
Jennifer Duncan (Edinburgh/ICMAT), Alan Chang (WUSTL)

S2 Orthogonal Polynomials and Special Functions
Jacob Christiansen (Lund) and Ana Loureiro (Kent)

S3 Jordan Algebra Structures and Operator Theory
Maria Cueto-Avellaneda (Lisbon), Bas Lemmens (Kent), Antonio Peralta (Granada)

S4 Differential Operators and Mathematical Physics
Sabine Boegli (Durham), Jean-Claude Cuenin (Loughborough), Petr Siegl (Graz)

S5 Multivariable Operator Theory
Sanne ter Horst (NWU), Tirtha Bhattacharyya (Indian Institute of Science), Nicholas Young (Newcastle & Leeds)

S6 Operator Theory and Hypercomplex Analysis
Daniel Alpay (Chapman), Uwe Kaehler (Aveiro), Irene Sabadini (Milan)

S7 Free Analysis and Convexity
Robert Martin (Manitoba), James Pascoe (Drexel, Philadelphia), Eli Shamovich (Ben Gurion)

S8 Quantum Information
Igor Klep (Ljubljana), David Gross (Cologne), William Slofstra (Waterloo)

S9 Noncommutative Geometry and Operator Spaces
Evgenios Kakariadis (Newcastle), Haluk Sengun (Sheffield)

S10 Random Matrices and Free Probability
Jani Virtanen (Reading), Haakan Hedenmalm (KTH Stockholm), Ian Charlesworth (Cardiff), Torben Krüger (FAU Erlangen-Nürnberg)

S11 Finite and Infinite Dimensional Moment Problems
Salma Kuhlmann (Konstanz), Maria Infusino (Cagliari)

S12 Herglotz Functions and Composite Materials
Victor Vinnikov (Ben Gurion), Joe Ball (Virginia Tech), Aaron Welters (Florida Tech)

S13 Linear Algebra and Control Theory
Andre Ran (Amsterdam), Volker Mehrmann (TU Berlin)

S14 Systems Theory
Felix Schwenninger (Twente)

S15 Numerical Ranges
Ilya Spitkovsky (NYU, Abu Dhabi), Tin-Yau Tam (Nevada)

S16 Spectral Problems and Computation
Christiane Tretter (Bern), Marco Marletta (Cardiff)

S17 Positive Operators and Their Dynamics
Jochen Glueck (Wuppertal), Anke Kalauch (Dresden)

S18 Operator Semigroups and Evolution Equations
Lyonell Boulton (Heriot-Watt), Christian Budde (Free State)

S19 Fractional Calculus Operators and Their Applications
Arran Fernandez (Eastern Mediterranean University), Milton Ferreira (Polytechnic of Leiria), Manuela Rodrigues (University of Aveiro), Nelson Vieira (University of Aveiro)

S20 Operators on Banach Spaces and Lattices
Niels Laustsen (Lancaster), Kevin Beanland (Washington & Lee University, VA, USA)

S21 Operator Theory on Analytic Function Spaces 1
Maribel Loaiza-Leyva (Centro de investigacion y de estudios avanzados del IPN, Mexico), Raul Quiroga Barranco (CIMAT, Guanajuato), Armando Sanchez-Nungaray (Universidad Veracruzana), Kehe Zhu (State University of New York at Albany)

S22 Special Matrices
Natalia Bebiano (University of Coimbra), Mikhail Tyaglov (Shanghai Jiao Tong University)

S23 Operator Theory and Applications
Marina Iliopoulou (Athens), Bas Lemmens, (Kent), Ana Loureiro (Kent), Marco Marletta (Cardiff), Ian Wood (Kent)

S24 Operator Theory on Analytic Function Spaces 2
Marina Iliopoulou (Athens), Bas Lemmens, (Kent), Ana Loureiro (Kent), Marco Marletta (Cardiff), Ian Wood (Kent)

1.2. **Schedule at a glance.**

	Monday	Tuesday	Wednesday	Thursday	Friday	
08:30	Registration (Woolf)					
09:00		**Erdős** (Woolf LT)	**Kurasov** (Woolf LT)	**Freitag** (Woolf LT)	**Kaltenbacher** (Woolf LT)	9:00
09:15	Opening (Woolf LT)					
10:00	**Kutyniok** (Woolf LT)	**Pramanik** (Woolf LT)	**Embree** (Woolf LT)	**Haragus** (Woolf LT)	**Gaubert** (Woolf LT)	10:00
11:00	Coffee	Coffee	Coffee	Coffee	Coffee	11:00
11:40	**Claeys** (SIBLT3) **Gross** (SIBLT1) **Lee** (CHLT)	**Bhat** (SIBLT3) **Laurent** (SIBLT1) **Martin** (CHLT)	S1, S2, S3, S5 S6, S8, S10, S11 S14, S15, S16, S21 S24	**Chandler-Wilde** (SIBLT3) **ter Horst** (SIBLT1) **Tylli** (CHLT)	**Boegli** (SIBLT3) **Dopico** (SIBLT1) **Mashreghi** (CHLT)	11:40
12:40	Lunch	Lunch	Lunch	Lunch	Lunch	12:40
14:00	S2, S4, S6 S8, S9, S11 S12, S17, S18 S20, S21, S23	S3, S4, S6 S7, S8, S9 S10, S12, S13 S17, S19, S21	S1, S2, S3, S5, S8, S10 S14, S15 S16, S24	S2, S3, S4 S5, S10, S13 S15, S16, S19 S20, S21,	S10, S15 S20, S24	14:00
15:00			Canterbury Walk			15:00
16:00	Coffee	Coffee		Coffee	Closing (SIBLT3)	16:00
16:30	S1, S2, S4 S6, S7, S8 S9, S12, S17 S21, S23	S1, S3, S5, S6 S8, S9, S11 S14, S15, S18 S20, S21, S22		S2, S3, S4 S5, S10, S13 S15, S16, S19 S23, S24	Coffee	16:30
						17:30
18:30						18:30
19:00				Drinks Reception		19:00
19:30				Conference Dinner		19:30

1.3. **List of Participants.** A list of participants can be found on:

https://blogs.kent.ac.uk/iwota2024/registered-participants/

2. Social Program and practicalities

2.1. **Canterbury City Tour.** The Canterbury Walking Tour will be on Wednesday afternoon. It will leave from **Westgate Towers (1 Pound Ln, Canterbury CT1 2BZ)**. At registration you will be given a ticket with the departure time of your tour. There are two departure times one at 15:30 and one at 16:00.

Please make sure you are there 10 minutes in advance.

2.2. **Reception & Conference Dinner.** If you have signed up for the drinks reception and the conference dinner, please note that on Thursday the reception will be in Rutherford gardens outside of Rutherford College, where the dinner will be held. Reception starts 19:00 and the dinner starts at 19:30. Please remember if you requested the vegetarian or vegan option.

2.3. **Registration.** Registration opens at 8:30 in Woolf College in the foyer outside of Woolf Lecture Theatre (WoolfLT).

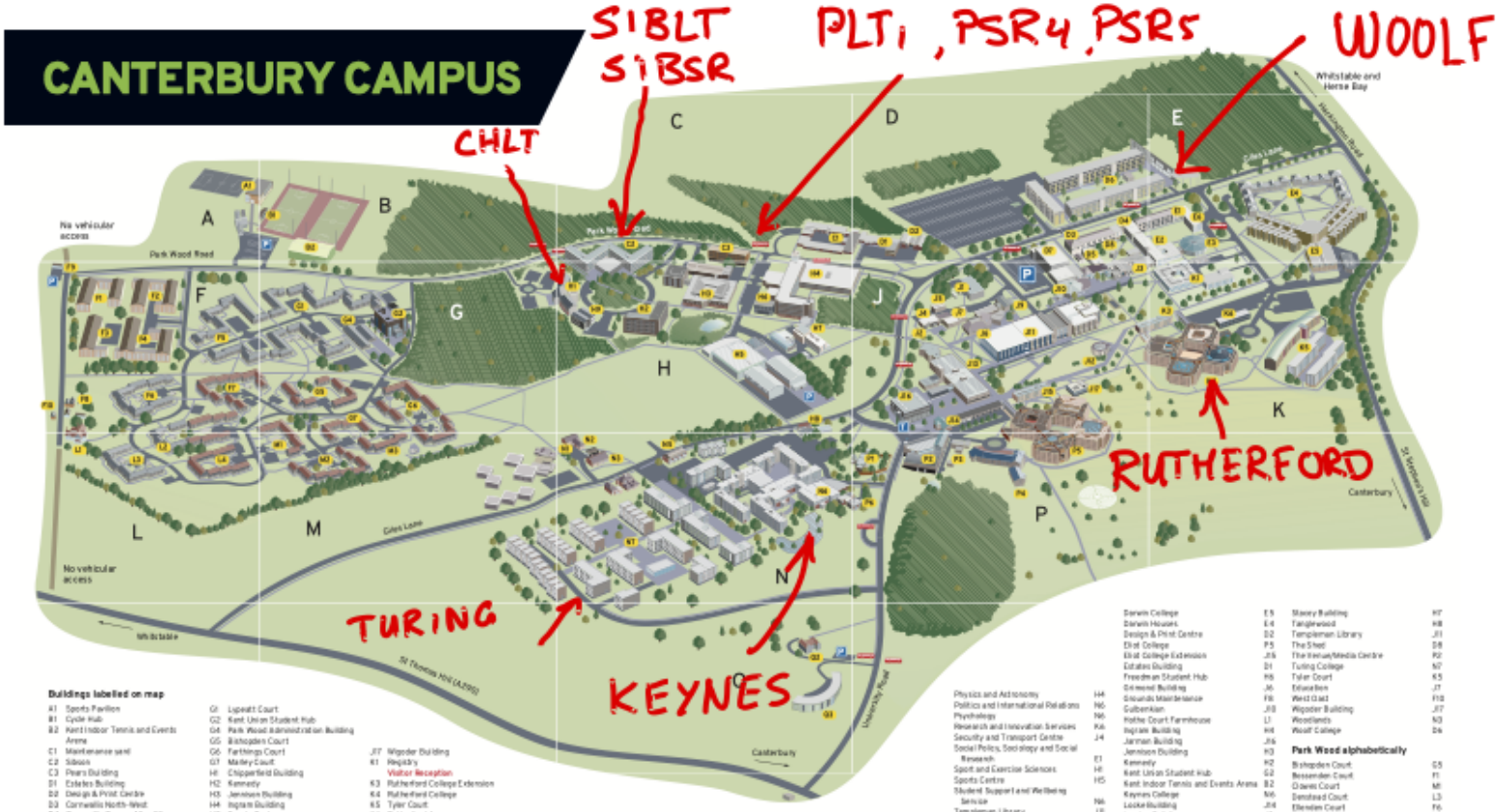
2.4. **Wi-Fi.**

Visitors with eduroam. Log in with your home institution details. If you don't connect straight away, check your connection settings have:
Security type set to WPA2-Enterprise
Encryption type set to AES

Other visitors. Find WiFi Guest in your list of available Wi-Fi networks and select it. Log in, or register if you're a first time user, to gain internet access.

2.5. **Maps.** Use the weblink below to search the online campus map for room location and facilities:

https://www.kent.ac.uk/maps/canterbury/canterbury-campus



Buildings labelled on map

A1 Sports Pavilion
B1 Cycle Hub
B2 Kent Indoor Tennis and Events Arena
C1 Maintenance yard
C2 Sibson
C3 Pears Building
D1 Estates Building
D2 Design & Print Centre
D3 Cornwallis North-West
D4 Cornwallis George Allen Wing
D5 Cornwallis West
D6 Woolf College
D7 Colyer-Fergusson Music Building
D8 The Shed
E1 Cornwallis North-East
E2 Cornwallis Central
E3 Cornwallis Octagon
E4 Darwin Houses
E5 Darwin College
E6 Cornwallis East
F1 Bessemlen Court
F2 Nickle Court
F3 Stock Court
F4 Kemsdale Court
F5 Purchas Court
F6 Ellenden Court
F7 Thornden Court
F8 Grounds Maintenance
F9 Oaks Nursery
F10 West Dell
G1 Lopeatt Court
G2 Kent Union Student Hub
G4 Park Wood Administration Building
G5 Bishopden Court
G6 Farthings Court
G7 Marley Court
H1 Chipperfield Building
H2 Kennedy
H3 Jennison Building
H4 Ingram Building
H5 Sports Centre
H6 Freedman Student Hub
H7 Stacey Building
H8 Tanglewood
H9 Chipperfield Building Extension
J1 Boiler House
J2 Nursing Services
J3 Cornwallis South & South West
J4 Security and Transport Centre
J5 Lupino Cinema
J6 Grimond Building
J7 Old Bank
J9 Lumley Building
J10 Gulbenkian
J11 Templeman Library
Nexus - Student Help Point
J12 Senate
J13 Mathew Building
J14 Locke Building
J15 Eliot College Extension
J16 Jarman Building
J17 Wigodar Building
K1 Registry
Visitor Reception
K3 Rutherford College Extension
K4 Rutherford College
K5 Tyler Court
K6 Rutherford Annexe
L1 Hothe Court Farmhouse
L2 Grimshill Court
L3 Dersingham Court
L4 Homestall Court
M1 Clowes Court
M2 Willows Court
M3 Tudor Court
N1 Medical Centre
N2 Pharmacy
N3 Woodlands
N5 Olive Cottages
N6 Keynes College
N7 Turing College
P1 Oaks Study Hub
P2 The Venue/Student Media Centre
P3 Mandela Building
P4 Becket Court
P5 Eliot College
P6 Careers and Employability Service
Q2 Beverley Farmhouse

University departments/services

Accommodation Office H8
Anthropology and Conservation J13
Architecture and Planning J12
Arts J16
Biosciences H7
Careers and Employability Service P6
Chemistry and Forensic Science H4
Commercial Services & Estates D1 & H8
Computing H2
Conference Office H8
Economics C2
Education J7
Engineering H3
English K3
Engagement K6
Finance K1
Future Student and Brand K1
Gulbenkian J10
Graduate School E6
Health, Safety and Environmental Sustainability M6
History K4
Human Resources K1
Information Services J3 & J11
International Programmes M9
Kent and Medway Medical School C3
Kent Business School C2
Kent Law Clinic J17
Kent Law School J16
Mathematics, Statistics and Actuarial Science J5
Medical Centre N1
Modern Languages D3
Music at Kent J17
Nexus - Student Help Point J11
Nursing Services J2
Oaks Nursery F9
Physics and Astronomy H4
Politics and International Relations H6
Psychology H6
Research and Innovation Services K6
Security and Transport Centre J4
Social Policy, Sociology and Social Research E1
Sport and Exercise Sciences H1
Sports Centre H5
Student Support and Wellbeing Service H6
Templeman Library J11
Tizard Centre E1

Buildings alphabetically

Becket Court P4
Beverley Farmhouse Q2
Boiler House J1
Security and Transport Centre J4
Careers and Employability Service P6
Chipperfield Building H1
Chipperfield Building Extension H9
Colyer-Fergusson Music Building D7
Cornwallis Central E2
Cornwallis East E6
Cornwallis George Allen Wing D4
Cornwallis North-East E1
Cornwallis North-West D3
Cornwallis Octagon E3
Cornwallis South & South West J3
Cornwallis West D5
Cycle Hub B1
Darwin College E5
Darwin Houses E4
Design & Print Centre D2
Eliot College P5
The Shed D8
Eliot College Extension J15
Estates Building D1
Freedman Student Hub H6
Grimond Building J6
Grounds Maintenance F8
Gulbenkian J10
Hothe Court Farmhouse L1
Ingram Building H4
Jarman Building J16
Jennison Building H3
Kennedy H2
Kent Union Student Hub G2
Kent Indoor Tennis and Events Arena B2
Keynes College N6
Leake Building J14
Lumley Building J9
Lupino Cinema J5
Maintenance yard C1
Mandela Building P3
Mathew Building J13
Medical Centre N1
Nursing Services J2
Oaks Study Hub P1
Oaks Nursery F9
Olive Cottages N5
Pharmacy N2
Park Wood Administration Building G4
Pears Building C3
Pharmacy N2
Registry K1
Rutherford Annexe K6
Rutherford College K4
Rutherford College Extension K3
Senate J12
Sibson C2
Sports Centre H5
Sports Pavilion A1
Stacey Building H7
Tanglewood H8
Templeman Library J11
The Shed D8
The Venue/Media Centre P2
Tizard Centre E1
Turing College N7
Tyler Court K5
Ellenden J7
Wigodar Building J17
Woodlands N3
Woolf College D6

Park Wood alphabetically

Bishopden Court G5
Bessemlen Court F1
Clowes Court M1
Dersingham Court L3
Ellenden Court F6
Farthings Court G6
Grimshill Court L2
Homestall Court L4
Kemsdale Court F4
Lopeatt Court G1
Marley Court G7
Nickle Court F2
Purchas Court F5
Stock Court F3
Thornden Court F7
Tudor Court M3
Willows Court M2

Other organisations on campus

Blackwell's Bookshop/Caffè Nero J14
Co-op convenience stores G2 & J14
Canterbury Innovation Centre D3
Jobshop P3
Kent Union P3

3. Schedule and Abstracts of the Plenary and Semi-Plenary talks

Monday

9:15-9:55	Opening: Ian Wood (Main Organizer) J. William Helton (Chair of the IWOTA Steering Committee) Shane Weller (Deputy-Vice Chancellor for Research and Innovation)	
10:00-11:00	Gitta Kutyniok *An Operator Theoretical Perspective on Reliable AI*	(Woolf LT) chair: Felix Schwenninger
11:40-12:40	Tom Claeys *Fredholm determinants and determinantal point processes*	(SIBLT3) chair: Ana Loureiro
	David Gross *Quantum network correlations and polynomial optimization over states*	(SIBLT1) chair: Igor Klep
	Woo Young Lee *Backward-shift invariant subspaces*	(CHLT) chair: Raul Curto

3.0.1. *Abstracts.*

Gitta Kutyniok, LMU Munich

An Operator Theoretical Perspective on Reliable AI

Abstract. The new wave of artificial intelligence is impacting industry, public life, and the sciences in an unprecedented manner. In industrial and applied mathematics, it has by now already led to paradigm changes in several areas. A particular emphasis is on graph data due to the importance of application settings such as recommender systems, social media, or molecular dynamics. However, one current major drawback is the lack of reliability as well as the enormous energy problem.

The goal of this lecture is to first provide an introduction into this new vibrant research area. We will then survey recent advances from an operator theoretical perspective, in particular, concerning performance guarantees and explainability methods, which are key to ensure reliability. Finally, we will discuss fundamental limitations in terms of computability, which seriously affect diverse aspects of reliability, and reveal a surprising connection to novel computing approaches such as neuromorphic computing, which also bear the potential to ensure sustainability of future AI systems.

Tom Claeys, UCLouvain

Fredholm determinants and determinantal point processes

Abstract. Determinantal point processes are point processes with a specific determinantal structure arising for instance in random matrix theory, tiling models, and random growth models. The Laplace functionals of such point processes can be written as Fredholm series. I will explain how this can be used to study asymptotic behavior in determinantal point processes and in particular to obtain large deviation type estimates. I will illustrate these general considerations with concrete examples related to random matrices and polymer models.

David Gross, University of Cologne

Quantum network correlations and polynomial optimization over states

Abstract. The problem of deciding whether observed correlations are compatible with a quantum model has been extensively studied and is now well-understood. In particular, in the "commuting operator model" of locality, there is a family of semi-definite programs (the *NPA hierarchy*), whose feasible region converges to the set of compatible correlations from the outside. We are interested in analogous results for scenarios that have a non-trivial "network" or "causal" structure. The most basic example is the *bilocal scenario*, in which two independent bipartite quantum systems are distributed among three observers, one pair between Alice and Bob, and the other pair between Bob and Charlie. The independence condition between the two pairs gives rise to polynomial constraints in the global quantum state, which previous techniques could not handle. Addressing this issue, we have developed a convergent hierarchy of SDPs for the problem of optimizing over the state space of a universal C^*-algebra given by generators and relations, subject to polynomial constraints in the state. (The construction is based on the *quantum inflation technique* by Wolfe et al. and the convergence proof makes use of the *quantum de Finetti theorem* as stated by Raggio and Werner. Simultaneously, Klep, Magron, Volčič, and Wang laid out a different approach under the name of *state polynomial optimization*). The method achieves our goal for the quantum bilocal scenario in the sense that it gives rise to a family of SDPs whose feasible region converges to the set of compatible correlations from the outside. The situation is more complex for general causal scenarios, for which I will state partial results and open questions.

References

[1] L.T. Ligthart, M. Gachechiladze, D. Gross, A convergent inflation hierarchy for quantum causal structures, *Comm. Math. Phys.* **401**, (2003), 2673-2714.
[2] L.T. Ligthart, D. Gross, The inflation hierarchy and the polarization hierarchy are complete for the quantum bilocal scenario, *J. Math. Phys.* **64**, (2023), 072201.

Woo Young Lee, Korea Institute for Advanced Study

Backward-shift invariant subspaces

Abstract. Let $S_{\mathbb{C}^n}$ be the shift operator on $\mathbb{C}^n$-valued Hardy space $H^2_{\mathbb{C}^n}$, i.e., $(S_{\mathbb{C}^n} f)(z) = zf(z)$ for each $f \in H^2_{\mathbb{C}^n}$. The Beurling-Lax Theorem states that every invariant subspace of $S_{\mathbb{C}^n}$ is of the form $\Delta H^2_{\mathbb{C}^r}$ for some $n \times r$ inner matrix function Δ. Equivalently, every invariant subspace of the backward shift operator $S^*_{\mathbb{C}^n}$ is of the form $\mathcal{H}(\Delta) := H^2_{\mathbb{C}^n} \ominus \Delta H^2_{\mathbb{C}^r}$, which is often called a model space. Thus, for a subset F of $H^2_{\mathbb{C}^n}$, if E^*_F denotes the smallest $S^*_{\mathbb{C}^n}$-invariant subspace containing F, i.e., $E^*_F := \bigvee\{(S^*_{\mathbb{C}^n})^i F : i \geq 0\}$, then $E^*_F = \mathcal{H}(\Delta)$ for some inner matrix function Δ. In this context, for a given inner matrix function Δ, we may ask: What is the smallest number of vectors in F satisfying $E^*_F = \mathcal{H}(\Delta)$? This question is closely related to a decomposition problem of matrix-valued L^2-functions. In this talk, we give a canonical decomposition of matrix-valued L^2-functions, which reduces to the Douglas-Shapiro-Shields factorization for a special case of bounded type functions. This idea invites a new notion of the "Beurling degree" of inner matrix functions. Eventually, we answer the above question in terms of the Beurling degree. The talk is based on the recent research with Raúl E. Curto and In Sung Hwang. Korea

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Tuesday

9:00-10:00	Laszlo Erdős *Universality phenomena for random matrices*	(Woolf LT) chair: Bill Helton
10:00-11:00	Malabika Pramanik *Circles, projections and operators*	(Woolf LT) chair: Alan Chang
11:40-12:40	V Rajarama Bhat *Dilation theory of quantum dynamical maps*	(SIBLT3) chair: Baruch Solel
	Monique Laurent *Performance Analysis of Sums of Squares Approximations in Polynomial Optimization*	(SIBLT1) chair: Hugo Woerdeman
	Rob Martin *Operator realizations of non-commutative functions*	(CHLT) chair: Nicholas Young

3.0.2. *Abstracts.*

Laszlo Erdős, IST Austria

Universality phenomena for random matrices

Abstract. Large random matrices tend to exhibit universal fluctuations. Beyond the well-known Wigner-Dyson and Tracy-Widom eigenvalue distributions, we overview other universality results for Hermitian and non-Hermitian matrices. We discuss the emergence of normal distribution involving eigenvectors, especially the random matrix version of quantum unique ergodicity. We also explain why results on non-Hermitian random matrices are much harder than their Hermitian counterparts and highlight our new methods to tackle them.

Supported by ERC Advanced Grants *RANMAT* and *RMTBeyond*.

Malabika Pramanik, University of British Columbia

Circles, projections and operators

Abstract. How large can a planar set be if it contains a circle of every radius? This is the quintessential example of a curvilinear Kakeya problem, central to many areas of harmonic analysis and incidence geometry.

Large sets in Euclidean space should have large projections in most directions. Projection theorems in geometric measure theory make this intuition precise, by quantifying the words "large" and "most". What do circles and projections have in common?

The talk will survey a few landmark results in these areas and point to a newly discovered connection between the two, in the form of a geometric maximal operator.

B V Rajarama Bhat, Indian Statistical Institute, Bangalore

Dilation theory of quantum dynamical maps

Abstract. It is a recurring theme in mathematics to view complicated structures as parts or components of larger but simpler looking objects. Sz. Nagy dilation of contractions on Hilbert spaces to unitaries is a great example of the same. Here we dilate unital completely positive maps on C^*-algebras to unital endomorphisms of larger algebras. Not only such dilations exist, but also there exist a *minimal dilation* which is unique up to unitary isomorphism ([2-3]).

The theory can also be developed in a von Neumann algebra setting and it can be extended to one parameter semigroups of unital completely positive maps. Then the dilations are one parameter semigroups of endomorphisms or E-semigroups in the sense of Arveson [1] and Powers. It is

a useful tool for constructing E_0-semigroups of type I factors [8]. It has applications in the classification theory of type II factors [7].

In the context of quantum theory of open systems the dilation presented above amounts to a construction of a quantum Markov process of a given quantum dynamical semigroup [4]. Recently it has been demonstrated that non-commutative peripheral Poisson boundaries admit very simple descriptions through dilation theory [6].

References

[1] W. Arveson, *Noncommutative dynamics and E-semigroups.* Springer, (2003). [2] B. V. Rajarama Bhat, An index theory for quantum dynamical semigroups, *Trans. Amer. Math. Soc.* **3**48(2), (1996), 561–583.
[3] –, Minimal dilations of quantum dynamical semigroups to semigroups of endomorphisms of C^*-algebras, *J. Ramanujan Math. Soc.* **1**4(2), (1999), 109-124.
[4] B. V. Rajarama Bhat and K. R. Parthasarathy, Markov dilations of nonconservative dynamical semigroups and a quantum boundary theory, *Ann. Inst. H. Poincaré Probab. Statist.* , **3**1(4), (1995) 601–651.
[5] B. V. Rajarama Bhat and M. Skeide, Tensor product systems of Hilbert modules and dilations of completely positive semigroups, *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* **3**(4), (2000), 519–575.
[6] B. V. Rajarama Bhat, S. Kar and B. Talwar, Peripheral Poisson boundary, https://arxiv.org/abs/2209.07731 (To appear in the Israel J. of Math.)
[7] S. Das and J. Peterson, Poisson boundaries of II_1 factors, Compositio Mathematica, 158(8), (2022),1746–1776.
[8] R.T. Powers, Construction of E_0-semigroups of $\mathscr{B}(\mathcal{H})$ from CP-flows, in: Advances in Quantum Dynamics, in: Contemp. Math., vol. 335, AMS, Providence, RI, (2003), 57–97.

I would like to thank SERB(India) for their generous support through J C Bose Fellowship.

Monique Laurent, Centrum Wiskunde & Informatica (CWI) Amsterdam / Tilburg University, The Netherlands

Performance Analysis of Sums of Squares Approximations in Polynomial Optimization

Abstract. We discuss hierarchies of approximations for polynomial optimization, based on using sums of squares of polynomials as a tractable proxy to certify polynomial positivity. Our focus will be on analyzing the quality of these approximations as a function of the degrees of the involved sums of squares. We will present recent state-of-the-art results for polynomial optimization over various classes of semi-algebraic sets and some of the main techniques used to obtain these results, which include Fourier analysis, reproducing kernels, and extremal roots of orthogonal polynomials.

Rob Martin, University of Manitoba

Operator realizations of non-commutative functions

Abstract. Realization theory is powerful tool in non-commutative (NC) function theory. Here, a *realization* is a triple, (A, b, c), consisting of a $d-$tuple, $A = (A_1, \cdots, A_d)$, of bounded linear operators on a separable, complex Hilbert space, $\mathcal{H}$, and vectors $b, c \in \mathcal{H}$. Any such realization defines a (uniformly analytic) NC function in an open neighbourhood of the origin, $0 = (0, \cdots, 0)$, of the *NC universe* of $d-$tuples of square matrices of any fixed size, via the formula $h(X) = I_n \otimes b^*(I_n \otimes I_{\mathcal{H}} - \sum X_j \otimes A_j)^{-1} I_n \otimes c$.

It is well-known that an NC function has a finite–dimensional realization if and only if it is a non-commutative rational function that is defined at 0. Such finite realizations contain valuable information about the NC rational functions they generate. By considering more general, infinite–dimensional realizations we study, construct and characterize more general classes of NC functions. In particular, we show that an NC function is (uniformly) entire, if and only if it has a jointly compact and quasinilpotent realization. Restricting our results to one-variable shows that an analytic Taylor series extends to an entire or meromorphic function on the complex plane if and only if it has a realization whose component operator is compact and quasinilpotent, or compact, respectively. This then motivates our definition of the set of global uniformly meromorphic NC functions as the (universal) skew field (of fractions) generated by NC rational expressions in the (semi-free ideal) ring of NC functions with jointly compact realizations.

This is joint work with Méric L. Augat (James Madison) and Eli Shamovich (Ben-Gurion).

Wednesday

9:00-10:00 Pavel Kurasov (Woolf LT) chair: Ian Wood
From metric graphs to Fourier quasicrystals

10:00-11:00 Mark Embree (Woolf LT) chair: Sanne Ter Horst
Contour Integral Eigensolvers through the Lens of System Identification

3.0.3. *Abstracts.*

Pavel Kurasov, Stockholm University

From metric graphs to Fourier quasicrystals

Abstract. Spectra of Laplacians on metric graphs have been intensively studies in recent years due to possible applications in nano-physics. Consider finite metric graphs formed from compact intervals $e_n,\ n=1,2,\dots,N$, of lengths ℓ_n and Laplace operators $-\frac{d^2}{dx^2}$ with standard vertex conditions:

- the function is continuous at the vertex (continuity condition);
- the sum of outgoing first derivates at the vertex is equal to zero (Kirchhoff condition).

One of the most interesting results in the area is the trace formula connecting the spectrum $\lambda_j = k_j^2$ to geometric and topologic properties of the metric graph [10,3,5,4]:

$$\underbrace{2\delta(k)+\sum_{k_n\neq 0}\left(\delta(k-k_n)+\delta(k+k_n)\right)}_{\text{spectral information}} = \underbrace{\underbrace{(1-\beta_1)}_{=\chi}\,\delta(k)+\frac{\mathcal{L}}{\pi}+\frac{1}{\pi}\sum_{p\in\mathcal{P}} l(\mathrm{prim}\,(p))S_{\mathrm{v}}(p)\cos k\ell(p)}_{\text{geometric/topologic information}}$$

where

- $\mathcal{L}=\sum_{n=1}^{N}\ell_n$ - the total length of the graph;
- χ – Euler characteristic of Γ;
- β_1 – number of independent cycles in Γ;
- $\mathcal{P}$ – the set of closed oriented paths p on Γ;
- $\ell(p)$ – length of the closed path p;
- $S_{\mathrm{v}}(p)$ – product of all vertex scattering coefficients along the path p.

This formula can be seen as a direct generalisation of the classical Poisson summation formula and coincides with it if the graph is just one interval with Neumann conditions at the end points. In contrast to Chazarian-Duistermaat-Guilemin-Melrose trace formulas the obtained formula is exact.

It appears that this formula is extremely interesting for Fourier analysis since it provides explicit examples of crystalline measure, which can be defined following Y. Meyer as [8]:

A discrete measure μ is **crystalline** *if it is a tempered distribution and if the measure itself and its Fourier transform $\hat{\mu}$ are sums of delta functions with discrete supports:*

$$\mu=\sum_{\lambda\in\Lambda} a_\lambda\delta_\lambda \qquad \hat{\mu}=\sum_{s\in S} b_s\delta_s.$$

Collecting all delta functions to the left hand side the trace formula can be written as:

$$(2-\chi)\delta(k)+\sum_{k_n\neq 0}\left(\delta(k-k_n)+\delta(k+k_n)\right)=\frac{\mathcal{L}}{\pi}+\frac{1}{\pi}\sum_{p\in\mathcal{P}} l(\mathrm{prim}\,(p))S_{\mathrm{v}}(p)\cos k\ell(p).$$

One may obtain similar summation formulas starting from multivariate stable polynomials [6]. The supports of the corresponding measures are described as zeroes of trigonometric polynomials. If the multivariate polynomials in addition are symmetric, *i.e.* invariant under involution $z_j \mapsto 1/z_j$, then the trigonometric polynomials have only real zeroes and the corresponding measures are crystalline measures. It appears that all one-dimensional crystalline measures are given by real rooted trigonometric polynomials [9]. It was proven recently that all such polynomials can be obtained using our construction via multivariate stable polynomials [1]. Spectral properties of Laplacians on metric graphs are further described in [7,4].

This is a joint work with Peter Sarnak.

Described approach is generalised in collaboration with L. Alon, M. Kummer, and C. Vinzant to obtaine crystalline measures in several dimensions.

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Supported by the Swedish Research Council.

Mark Embree, Virginia Tech

Contour Integral Eigensolvers through the Lens of System Identification

Abstract. In the past two decades contour integral methods have emerged as an intriguing technology for numerically computing eigenvalues within some target region of the complex plane. These algorithms are remarkably flexible, applying to self-adjoint and non-self-adjoint operators, pencils, and (thanks to a classical theorem of Keldysh) even to nonlinear eigenvalue problems. These methods, such as the Sakurai–Sugiura algorithm, typically reduce the problem to an equivalent low-dimensional Hankel matrix pencil, whose elements are matrix moments that are computed approximately with quadrature. We will show how such methods can be seen as Ho-Kalman system identification problems with noisy data. This connection to systems theory suggests new algorithms that leverage modern system identification tools based on the Loewner framework for rational interpolation [1]. In theory, these approaches yield equivalent matrix pencils with the exact desired eigenvalues – *provided the data is exact.* Yet these

spectrally equivalent pencils can have starkly different perturbation properties, giving different degrees of sensitivity when operating on the inexact data that is inevitable in practical problems. We will frame this discussion in terms of pseudospectra of matrix pencils [2] and the location of Loewner interpolation points [3], and demonstrate use of these contour methods to solve nonlinear eigenvalue problems arising from networks of vibrating strings [4].

References

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I would like to thank the US National Science Foundation for their support (DMS-1720257).

Thursday

9:00-10:00 Melina Freitag (Woolf LT) chair: Marco Marletta
Solving parameter dependent eigenvalue problems using Taylor series and Chebyshev expansion

10:00-11:00 Mariana Haragus (Woolf LT) chair: Marco Marletta
Spectral and semigroup methods for hydrodynamic and optical waves

11:40-12:40 Simon Chandler-Wilde (SIBLT3) chair: Lyonell Boulton
On Spectral Inclusion Sets and Computing the Spectra and Pseudospectra of Bounded Linear Operators

Sanne ter Horst (SIBLT1) chair: Andre Ran
Unbounded Toeplitz operators with rational symbols

Hans-Olav Tylli (CHLT) chair: Niels Laustsen
Closed ideals and subideals associated to the algebra of compact-by-approximable operators

3.0.4. *Abstracts.*

Melina Freitag, University of Potsdam

Solving parameter dependent eigenvalue problems using Taylor series and Chebyshev expansions

Abstract. We discuss two approaches to solving the parametric (or stochastic) eigenvalue problem. One of them uses a Taylor expansion and the other a Chebyshev expansion. The parametric eigenvalue problem assumes that the matrix A depends on a parameter μ, where μ might be a random variable. Consequently, the eigenvalues and eigenvectors are also functions of μ. We compute a Taylor approximation of these functions about μ_0 by iteratively computing the Taylor coefficients. The complexity of this approach is $\mathcal{O}(n^3)$ for all eigenpairs, if the derivatives of $A(\mu)$ at μ_0 are given. The Chebyshev expansion works similarly. We first find an initial approximation iteratively which we then refine with Newton's method. This second method is more expensive but provides a good approximation over the whole interval of the expansion instead around a single point. We present numerical experiments confirming the complexity and demonstrating that the approaches are capable of tracking eigenvalues at intersection points. Further experiments shed light on the limitations of the Taylor expansion approach with respect to the distance from the expansion point μ_0.

References

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Mariana Haragus, University of Franche-Comté, France

Spectral and semigroup methods for hydrodynamic and optical waves

Abstract. Motivated by two concrete examples from optics and hydrodynamics we present some general results for linear differential operators with periodic coefficients. These examples share the property that the underlying mathematical models possess stationary periodic solutions. We are interested in the stability of these periodic solutions under spatially localized perturbations.

The first example is a nonlinear Schrödinger equation with damping and forcing that arises in nonlinear optics. The linear operator obtained by linearizing the equation at a stationary periodic solution is a matrix differential operator with periodic coefficients. We discuss its spectral

properties and analyze the decay of the evolution semigroup. The result for the semigroup holds for differential operators with periodic coefficients under some rather general hypotheses.

The second example is the classical water-wave problem in hydrodynamics. Linear operators arising in the study of water waves typically have a product structure JL in which J and L are respectively skew- and self-adjoint operators. Classical counting results show that, under suitable conditions, the number of unstable eigenvalues of the operator JL is bounded by the number of nonpositive eigenvalues of the self-adjoint operator L. We extend these results by showing that the operator L can be replaced by another self-adjoint operator K, provided the operators JL and JK commute.

This talk is based on joint works with Mat Johnson, Wesley Perkins, Björn de Rijk, Jin Li and Dmitry Pelinovsky.

Simon Chandler-Wilde, University of Reading

On Spectral Inclusion Sets and Computing the Spectra and Pseudospectra of Bounded Linear Operators

Abstract. In this talk, based substantially on our recent paper [2], we derive novel families of inclusion sets for the spectrum and pseudospectrum of large classes of bounded linear operators, and establish convergence of particular sequences of these inclusion sets to the spectrum or pseudospectrum, as appropriate. Our results apply, in particular, to bounded linear operators on a separable Hilbert space that, with respect to some orthonormal basis, have a representation as a bi-infinite matrix that is banded or band-dominated. More generally, our results apply in cases where the matrix entries themselves are bounded linear operators on some Banach space. In the scalar matrix entry case we show that our methods, given the input information we assume, lead to a sequence of approximations to the spectrum, each element of which can be computed in finitely many arithmetic operations, so that, with our assumed inputs, the problem of determining the spectrum of a band-dominated operator has solvability complexity index one, in the sense of Ben-Artzi et al. [1]. As a concrete application, we apply our methods to the determination of the spectra of non-self-adjoint bi-infinite tridiagonal matrices that are pseudoergodic in the sense of Davies [3].

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Sanne ter Horst, North-West University

Unbounded Toeplitz operators with rational symbols

Abstract. Although there are some earlier sources on Unbounded Toeplitz operators, starting in the 1950s and 1960s, it was not until the 2008 paper by Sarason [3], in which he studies a class of unbounded Toeplitz operators in the context of truncated Toeplitz operators, that the topics gained in popularity. We begin with a brief overview of some types of unbounded Toeplitz operators that have appeared in the recent literature, after which we shall focus on a class of unbounded Toeplitz operators with rational symbols that have poles on the unit circle [1]. In that case, more concrete results can be obtained regarding the spectrum and Fredholm theory of such operators. If time permits, the case of rational matrix symbols will be discussed, in which further challenges arise [2].

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Hans-Olav Tylli, University of Helsinki

Closed ideals and subideals associated to the algebra of compact-by-approximable operators

Abstract. I will describe recent work [1] with Henrik Wirzenius (Tampere), where we construct explicit non-trivial closed ideals $\mathcal{A}(X) \subset \mathcal{J} \subset \mathcal{K}(X)$ for various Banach spaces X that fail to have the approximation property. Here $\mathcal{K}(X)$ is the Banach algebra of compact operators on X, and the ideal of the approximable operators $\mathcal{A}(X) := \overline{\mathcal{F}(X)}$ is the uniform closure of the bounded finite rank operators $\mathcal{F}(X)$. Our work was motivated by longstanding problems about the structure of the elusive compact-by-approximable algebra $\mathcal{K}(X)/\mathcal{A}(X)$, and it complements current work on closed ideals of the bounded operators $\mathcal{L}(Z)$ for various classical Banach spaces Z.

Results include a Banach space Z together with an uncountable lattice $\{\mathcal{J}_\alpha : \alpha \in \Lambda\}$ of closed ideals of $\mathcal{K}(Z)$, which are not ideals of the algebra $\mathcal{L}(Z)$ of bounded operators. This family has the unexpected property that $\mathcal{J}_\alpha$ and $\mathcal{J}_\beta$ are isomorphic as Banach algebras whenever $\alpha \neq \beta$, which is not possible for closed ideals of $\mathcal{L}(Z)$. Time permitting I will describe further examples from [2] of non-trivial closed subideals of $\mathcal{L}(X)$. (Here $\mathcal{J}$ is a non-trivial closed subideal of $\mathcal{L}(X)$ if $\mathcal{J} \subset \mathcal{I} \subset \mathcal{L}(X)$, where $\mathcal{J}$ is a closed ideal of $\mathcal{I}$ and $\mathcal{I}$ is a closed ideal of $\mathcal{L}(X)$, but J is not an ideal of $\mathcal{L}(X)$.)

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Friday

9:00-10:00 Barbara Kaltenbacher (Woolf LT) chair: Irene Sabadini
Inverse problems for fractional PDEs

10:00-11:00 Stéphane Gaubert (Woolf LT) chair: Bas Lemmens
Tropical geometry, operators and games

11:40-12:40 Sabine Bögli (SIBLT3) chair: Christiane Tretter
On the discrete eigenvalues of Schrödinger operators with complex potentials

Froilán Dopico (SIBLT1) chair: Christian Mehl
Polynomial and rational matrices with prescribed data

Javad Mashreghi (CHLT) chair: Marek Ptak
Linear polynomial approximation schemes

3.0.5. *Abstracts.*

Barbara Kaltenbacher, University of Klagenfurt

Inverse problems for fractional PDEs

Abstract. This talk is based on recent joint work with William Rundell, Texas A&M University, USA, cf. [1,2,3]. In particular, we plan to focus on two prototypical problems in which the order of the fractional time derivative in the governing PDE has a crucial influence on the degree of ill-posedness of the inverse problem under consideration. In the first one, this effect will be exploited for designing a regularization method with the time fractional differentiation order as a regularization parameter; in the second one, the order of the fractional derivative is not at our disposal but imposed by the physical model. Modelling with fractional derivatives, in particular in ultrasonics, will be therefore be topic of this talk as well. If time permits, some selected further inverse problems for fractional PDEs will be highlighted.

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Stéphane Gaubert, Inria and CMAP, École polytechnique

Tropical geometry, operators and games

Abstract. Tropical convex sets arise as log-limits of parametric families of convex sets, or as images by a non-archimedean valuation of convex sets over order non-archimedean fields. This sets up a correspondence between several classes of zero-sum repeated games that have an unsettled complexity, and several classes of convex semialgebraic programs. This has unexpected consequences, leading to a proof that interior point methods in linear programming are not strongly polynomial. An operator approach, studying zero-sum games by means of nonexpansive and order preserving mappings (dynamic programming operators), plays an essential role here. We will provide an introductive survey of these developments which combine methods from tropical geometry, non-linear Perron-Frobenius theory and game theory.

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Sabine Bögli, Durham University

On the discrete eigenvalues of Schrödinger operators with complex potentials

Abstract. In this talk I shall present constructions of Schrödinger operators with complex-valued potentials whose spectra exhibit interesting properties. One example shows that for sufficiently large p, the discrete eigenvalues need not be bounded in modulus by the L^p norm of the potential. This is a counterexample to the Laptev–Safronov conjecture (Comm. Math. Phys. 2009). Another construction proves optimality (in some sense) of generalisations of Lieb–Thirring inequalities to the non-selfadjoint case - thus giving us information about the accumulation rate of the discrete eigenvalues to the essential spectrum. This talk is based on joint works with Jean-Claude Cuenin (Loughborough) and František Štampach (Prague).

Froilán Dopico, Universidad Carlos III de Madrid, Spain

Polynomial and rational matrices with prescribed data

Abstract. We study necessary and sufficient conditions for the existence of polynomial and rational matrices with different prescribed data. First, we consider the problem for polynomial matrices when the size, degree, rank, invariant factors, infinite elementary divisors, and the minimal indices of their left and right null spaces are prescribed and prove that a polynomial matrix with these data exists if and only if these data satisfy a surprisingly simple unique condition related to a fundamental constraint known as the "index sum theorem". In the second place, we extend this result to rational matrices when the size, rank, invariant rational functions, invariant orders at infinity, and minimal indices of their left and right null spaces are prescribed. The data prescribed so far are called in the literature the *complete structural data* or the *complete eigenstructure* of the polynomial or rational matrix. Finally, in addition to the complete eigenstructure, we also prescribe the minimal indices of the row and column spaces and show that the simple condition found in the previous problems must be completed with a majorization relation among the involved indices. The results presented in this talk are based on the references [1], [3] and the work still in preparation [2].

References

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This work has been partially supported by the Agencia Estatal de Investigación of Spain MCIN/AEI/10.13039/501100011033/ through grants PID2019-106362GB-I00 and RED2022-134176-T.

This talk is based on joint work with L. M. Anguas, I. Baragaña, F. De Terán, R. Hollister, D. S. Mackey, S. Marcaida, A. Roca, and P. Van Dooren.

Javad Mashreghi, Laval University

Linear polynomial approximation schemes

Abstract. In this presentation, we explore polynomial approximation schemes within function spaces. While Taylor polynomials are fundamental in polynomial approximation theory, there are instances where they may not be the most suitable candidates. Without entering into technical details, we will discuss some summation methods, with a particular emphasis on the well-known Cesaro means. Our focus remains primarily on Hardy and Dirichlet spaces, although other function spaces also make appearances in the discussion. Moreover, within the broader context of super-harmonically weighted Dirichlet spaces, we establish that Fejer polynomials and de la Vallee Poussin polynomials serve as appropriate approximation schemes.

This work has evolved over an extended period and is the result of collaborative efforts with O. El-Fallah, E. Fricain, K. Kellay, H. Klaja, M. Nasri, P. Parisé, M. Shirazi, V. Verreault, T. Ransford, and M. Withanachchi in various combinations.

4. Schedules and abstracts of the special sessions

4.1. **Harmonic Analysis and Related Areas.**

Organizers: Alan Chang (WUSTL) and Jennifer Duncan (ICMAT)

Monday 16:30-19:00 (SIBSR4) chair: Jennifer Duncan

16:30-16:55 K. N. Sridharan
Orlicz space on Groupoids

17:00-17:25 Deborpita Biswas
Near-Riesz bases in Hilbert Spaces

17:30-17:55 Jane McDougall
The Rosette minimal surface, its deformation to the classical Enneper surface, and a non-Jenkins Serrin surface

18:00-18:25 Ahmad Al-Salman
Singular Integral Operators Along twisted Surfaces

18:30-19:00 Lyazzat Sarybekova
Some results on Fourier multipliers

Tuesday 16:30-19:00 (SIBSR4) chair: Alan Chang

16:30-16:55 Ritika Singhal
Paley inequality for the Weyl transform and its applications

17:00-17:25 Ferenc Weisz
Hardy-Littlewood-type theorems for multi-dimensional Fourier transforms

17:30-17:55 Himani Sharma
Vertical maximal functions on manifolds with ends

18:00-18:25 Ole Christensen
Frames and Redundancy

Wednesday 11:40-12:40 (SIBSR4) chair: Jennifer Duncan

11:40-12:05 Itamar Oliviera
A phase-space approach to weighted Fourier extension inequalities

12:10-12:35 Hrit Roy
Estimates for cone multipliers in $\mathbb{R}^3$ associated with rough convex domains

Wednesday 14:00–15:00 (SIBSR4) chair: Alan Chang

14:00-14:25 Lars Becker
Carleson Operators on Doubling Metric Measure Spaces

14:30-14:55 Carmelo Puliatti
On Fourier transforms of fractal measures on the parabola

4.1.1. *Abstracts.*

Ahmad Al-Salman, Sultan Qaboos University

Singular Integral Operators Along twisted Surfaces

Abstract. Singular integral operators on product domains have been introduced by R. Fefferman and E. M. Stein as a natural generalization of the double Hilbert transform on $\mathbb{R}^2$. Here, $\mathbb{R}^d$ $(d = n, m \geq 2)$ is the d-dimensional Euclidean space. Also, we let $\mathbb{S}^{d-1}$ be the unit sphere in $\mathbb{R}^d$ equipped with the normalized Lebesgue measure $d\sigma_d$. In this talk, we are interested in studying the L^p boundedness of a class of singular integrals on product domains along special surfaces. For an $\Omega \in L^1(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ satisfying

$$\Omega(tx, sy) = \Omega(x, y) \text{ for any } t, s > 0; \tag{1}$$

$$\int_{\mathbb{S}^{n-1}} \Omega(u, \cdot)\, d\sigma_n(u) = \int_{\mathbb{S}^{m-1}} \Omega(\cdot, v)\, d\sigma_m(v) = 0, \tag{2}$$

we let

$$K_\Omega(x, y) = \frac{\Omega(x', y')}{|x|^n |y|^m}$$

where $x' = x/|x|$ for $x \neq 0$ and $y' = y/|y|$ for $y \neq 0$. For mappings $\phi_1, \phi_2 : (0, \infty) \to \mathbb{R}$, we introduce the class of operators

$$T_\Omega^{(\phi_1 \leftrightarrow \rho \phi_2)}(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - \phi_1(|v|)u, y - \phi_2(|u|)v) \frac{\Omega(u, v)}{|u|^n |v|^m} du dv. \tag{3}$$

When ϕ_1 and ϕ_2 are non zero constants, then the operator $T_\Omega^{(\phi_1 \leftrightarrow \rho \phi_2)}$ reduces to the classical operator T_Ω given by

$$T_\Omega(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - u, y - v) \frac{\Omega(u, v)}{|u|^n |v|^m} du dv. \tag{4}$$

If Ω satisfies some regularity conditions, R. Fefferman and E. M. Stein showed that the special operator T_Ω is bounded on $L^p(\mathbb{R}^n \times \mathbb{R}^m)$ for all $1 < p < \infty$. Subsequently, J. Duoandikoetxea proved that T_Ω is bounded on L^p $(1 < p < \infty)$ provided that Ω satisfies the weaker condition $\Omega \in L^q(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$, $q > 1$. In 2006, A. Al-Salman, H. Al-Qassem, and Y. Pan showed that T_Ω is bounded on $L^p(\mathbb{R}^n \times \mathbb{R}^m)$ for all $1 < p < \infty$ provided that $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$, i.e.,

$$\int_{\mathbb{S}^{n-1} \times \mathbb{S}^{m-1}} |\Omega(u, v)| \log^2(2 + |\Omega(u, v)|)\, d\sigma_n(u)\, d\sigma_m(v) < \infty. \tag{5}$$

Here, we remark that $\Omega \in L^q(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1}) \subset L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ for all $q > 1$. Furthermore, the same authors proved that the condition $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ is nearly optimal in the sense that the exponent 2 in $L(\log L)^2$ can not be replaced by any smaller numbers.

We remark here that the general operator $T_\Omega^{(\phi_1 \leftrightarrow \rho \phi_2)}$ arises naturally when considering singular integrals in the form

$$S_{\Omega, b}(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - u, y - v) \frac{b(|u|\,|v|)\Omega(u, v)}{|u|^n |v|^m} du dv \tag{6}$$

where b is bounded function. In particular, if $b(r) = r^{-m} \chi_{(1,\infty)}$, then by change of variables, it is radially seen that the operator $S_{\Omega, b}$ reduces to the operator

$$S_{\Omega,, b}(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - |v|^{-1} u, y - v) \frac{b(|u|)\Omega(u, v)}{|u|^n |v|^m} du dv. \tag{7}$$

with $b(t) = \chi_{(1,\infty)}$. It is shown by A. Al-Salman that if $\varphi(t) = t$ or $\phi(t) = t$, then the corresponding operator may fail to be bounded on L^p for any $1 < p < \infty$. Furthermore,

it is shown that the operator $T_{\Omega}^{(\phi_1 \leftarrow \rho \phi_2)}$ is bounded on L^p for $1 < p < \infty$ provided that $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ and that the functions φ and ϕ belong to the class $\mathcal{F}$ of smooth functions $\Phi : (0, \infty) \to \mathbb{R}$ which satisfy $\Phi(0) = 0$ and the following growth conditions:

$$\sup_{0<t<\infty} t^{-d_\Phi} |\Phi(t)| \leq C_1 \text{ and } \inf_{0<t<\infty} t^{2-d_\Phi} \left|\Phi''(t)\right| \geq C_2 \tag{8}$$

for some $d_\Phi \neq 0$ where C_1 and C_2 are positive constants.

In this talk, we are interested in investigating the boundedness of the operator $T_{\Omega}^{(\phi_1 \leftarrow \rho \phi_2)}$ under weak conditions on the kernel Ω. In fact, we shall consider the following question:

Question. *Suppose that $\phi_1, \phi_2 \in \mathcal{F}$. Under what conditions on the kernel function Ω, the corresponding operator $T_{\Omega}^{(\phi_1 \leftarrow \rho \phi_2)}$ is bounded on $L^p(\mathbb{R}^n \times \mathbb{R}^m)$ for some $1 < p < \infty$?*

Lars Becker, University of Bonn

Carleson Operators on Doubling Metric Measure Spaces

Abstract. A classical theorem of Carleson states that the maximally modulated Hilbert transform is bounded on L^2. More famous might be its corollary, the pointwise convergence of Fourier series of functions in L^2. We give a new generalization of Carleson's theorem, in which the Hilbert transform is replaced by a singular integral operator on a doubling metric measure space. Our theorem unifies various generalizations of Carleson's theorem that are already in the literature. One application is pointwise convergence of expansions in certain orthogonal polynomials.

This is joint work with Floris van Doorn, Asgar Jamneshan, Rajula Srivastava and Christoph Thiele.

Deborpita Biswas, Clemson University

Near-Riesz bases in Hilbert Spaces

Abstract. James R. Holub, in one of his papers in 1994, introduced the influential concept of near-Riesz bases as frames which become Riesz bases after removal of finitely many terms. We recently extended his definition of near- Riesz basis to sequences which are not frames. In this talk I will present a characterization of our extended near-Riesz bases in terms of the Fredholmness of their associated synthesis operator. I will also present some perturbation results for our near-Riesz bases.

Ole Christensen, Technical University of Denmark

Frames and Redundancy

Abstract. A frame in a Hilbert space can be considered as an "redundant basis:" every element in the Hilbert space has an expansion in terms of the frame element, but the corresponding coefficients might not be unique. We first give a short introduction to general frame theory, and then discuss redundancy properties for the so-called Carleson frames constructed by Aldroubi et al. in 2016; they have the form $\{T^k\varphi\}_{k=0}^{\infty}$ for a bounded linear operator on the underlying Hilbert space. We show that such frames have a number of remarkable features that have not been identified for any other frames in the literature. Most importantly, the subfamily obtained by selecting each Nth element from the frame is itself a frame, regardless of the choice of $N \in \mathbf{N}$. Furthermore, the frame property is kept upon removal of an arbitrarily finite number of elements. The new results are joint work with M. Hasannasab, F. M. Philipp, and D. Stoeva.

Jane McDougall, Colorado College

The Rosette Minimal Surface, its deformation to the classical Enneper surface, and a non-Jenkins Serrin Surface

Abstract. A harmonic mapping f is a complex valued univalent harmonic function defined on a region in the complex plane. Rosette harmonic mappings are generalizations of the polynomial harmonic mappings $z + \overline{z}^{n-1}/(n-1)$ through modifying the canonical decomposition with hypergeometric ${}_2F_1$ factors. These mappings were discovered through a fortuitous application of Clunie and Sheil-Small's famous shear construction. For appropriate parameters, the harmonic mapping 'lifts', via the Weierstrass Enneper equations, to a Triply Periodic Minimal Surface (TPMS) known as the Rosette Minimal Surface. We describe the continuous deformation of this TPMS into the classical Enneper surface, using a further generalization of the rosette harmonic mappings. In contrast, the Poisson integral formula, applied to piecewise constant functions on the boundary of the unit disk, frequently lifts to a Jenkins Serrin minimal surface (for which the classical doubly-periodic Schwarz surface is a prototypical example). Poisson extensions of piecewise constant functions have in common with the rosette harmonic mappings that there are arcs of constancy. However we explore examples where the dilatation is not a power function, and the corresponding minimal surface is not a Jenkins Serrin surface.

I would like to thank the Science Division Research and Development fund at Colorado College for their generous support

Itamar Oliveira, University of Birmingham

A phase-space approach to weighted Fourier extension inequalities

Abstract. The goal of the talk is to present a certain *ray bundle representation* of the Fourier extension operator in terms of the Wigner transform to investigate two longstanding conjectures in the restriction theory of the Fourier transform, namely *Stein's* and the *Mizohata-Takeuchi conjecture*. In joint work with Bennett, Gutierrez and Nakamura, we show how Sobolev estimates for the Wigner transform can be converted into certain tomographic bounds for the Fourier extension operator to the paraboloid, which imply weaker variants of these conjectures. We are also able to extend this analysis to a wide class of hypersurfaces, a step that requires finding and understanding a good "geometric" replacement for the classical Wigner transform. Our results do not depend on lower bounds for the Gaussian curvature of these manifolds, which contrasts the intuition behind the classical Fourier restriction conjecture. If time allows, we will make a connection between our results and Flandrin's conjecture in the plane.

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The speaker is supported by EPSRC Grant EP/W032880/1.

Carmelo Puliatti, Universitat Autònoma de Barcelona

On Fourier transforms of fractal measures on the parabola

Abstract. I will discuss L^p bounds for the Fourier transform of Borel measures supported on the parabola $\mathbb{P} = \{(x, x^2) : x \in [-1, 1]\}$ satisfying an s-dimensional Frostman condition, $s \in [0, 1]$. These estimates are closely connected to incidence problems of points and lines and, in particular, rely on K. Ren and H. Wang's recent solutions to the Furstenberg set conjecture

in the plane. Additionally, I will talk about related lower bounds for the Hausdorff dimension of the n-fold sum-set $nK = K + \cdots + K$ of a Borel subset K of the parabola.

This is a joint work with T. Orponen and A. Pyörälä.

Hrit Roy, University of Edinburgh

Cone multipliers associated to general convex domains

Abstract. In this talk, we shall discuss variants of Mockenhaupt's cone multiplier in $\mathbb{R}^3$ associated to cones generated by arbitrary planar convex domains Ω. We obtain estimates for general cones described in terms of a fractal dimension κ_Ω introduced by Seeger–Ziesler, which measures the flatness of the boundary $\partial\Omega$. Our argument relies on an L^4 square function estimate for general cones, which is a generalization of the Guth–Wang–Zhang square function estimate for the light cone.

Lyazzat Sarybekova, King's College London

Some results on Fourier multipliers

Abstract. This talk is devoted to the Fourier series and Fourier transform multipliers.

A generalization and sharpening of the Lizorkin theorem concerning Fourier transform multipliers between L_p and L_q is proved [1]. Moreover, the work deals with the Fourier series multipliers in the more general case with strong regular system. This system is rather general. For example, all trigonometrical systems, the Walsh system and all multiplicative systems with bounded elements are strong regular. A generalization and sharpening of the Lizorkin type theorem concerning Fourier series multipliers between the spaces L_p and L_q is obtained [2]. In addition, in this work the Fourier series multipliers in the generalized Haar system in Lebesgue and Lorentz spaces are studied [3].

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Himani Sharma

Vertical maximal functions on manifolds with ends

Abstract. We consider a class of Riemannian manifolds with ends, denoted by $\mathcal{M}$, which is obtained by taking the connected sum of a finite number of N-dimensional Riemannian manifolds of the form $\mathbb{R}^{n_i} \times \mathcal{M}_i$, where $\mathcal{M}_i$ is a compact manifold, with product metric. An interesting case of these manifolds occurs when the Euclidean dimensions n_i are not equal, making $\mathcal{M}$ a non-doubling space. This talk is based on the $L^p(\mathcal{M})$ boundedness of the vertical maximal function operator $M^{res,\nabla} f(x) := \sup_{t>0} |\sqrt{t}\nabla(1+t\Delta)^{-m} f(x)|$, where $m \geq 1$ and Δ is the Laplace-Beltrami operator. We show that $M^{res,\nabla}$ is weak type (1,1) and bounded on $L^p(\mathcal{M})$ for $1 < p \leq n^*$ where $n^* = \min_i\{n_i\}$. The techniques we use here come from the papers of Hassell-Sikora and Bailey-Sikora on non-doubling manifolds with ends. We also show Fefferman-Stein inequality for the vector-valued version of the vertical maximal function on $L^p(\mathcal{M})$ for $1 < p < n^*$.

This talk is based on my joint work with Adam Sikora.

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Ritika Singhal, Indian Institute of Technology Delhi, India

Paley inequality for the Weyl transform and its applications

Abstract. Our aim is to prove the classical Paley inequality in the context of the Weyl transform. As the Weyl transform maps function spaces to bounded operator, we could prove several versions of this inequality. As for some applications, we prove a version of the Hörmander's multiplier theorem to discuss L^p-L^q boundedness of the Weyl multipliers. We also proved the Pitt's inequality for the Weyl transform. In this talk, the results from the following article will be discussed:

References

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K. N. Sridharan, Indian Institute of Technology Delhi

Orlicz space on Groupoids

Abstract. Let G be a locally compact second countable groupoid with a fixed haar system $\lambda = \{\lambda^u\}_{u\in G^0}$ and (Φ, Ψ) be a complementary pair of N-functions satisfying the Δ_2-condition. We introduce the continuous field of Orlicz space (L_0^Φ, Δ_1), and provide a sufficient condition for the space of continuous sections vanishing at infinity E_0^Φ, to be an algebra under a suitable convolution. The condition for a closed $C_b(G^0)$-submodule I of E_0^Φ to be a left ideal is established. We provide a groupoid version of the result that characterizes the space of convolutors of Morse-Transue space for locally compact groups.

This is a joint work with Dr. N. Shravan Kumar.

Ferenc Weisz, Eötvös Loránd University

Hardy-Littlewood-type theorems for higher dimensional Fourier transforms

Abstract. We obtain Fourier inequalities in the weighted L_p spaces for any $1 < p < \infty$ involving the Hardy-Cesàro and Hardy-Bellman operators. We extend these results to product Hardy spaces for $p \leq 1$. Moreover, boundedness of the Hardy-Cesàro and Hardy-Bellman operators in various spaces (Lebesgue, Hardy, BMO) is discussed.

4.2. **Orthogonal Polynomials and Special Functions.**
Organizers: Jacob S. Christiansen (Lund) and Ana F. Loureiro (Kent)

Monday 14:00-16:00 (SIBLT2) chair: Ana Loureiro

14:00-14:25 Arno Kuijlaars
Matrix valued orthogonal polynomials arising from periodic hexagon tilings
14:30-14:55 Thomas Bothner
Universality for random matrices with an edge spectrum singularity
15:00-15:25 Wolter Groenevelt
Orthogonal polynomials and stochastic duality
15:30-15:55 Margit Rösler
Limits of Bessel functions for root systems as the rank tends to infinity

Monday 16:30-19:00 (SIBLT2) chair: Jacob Christiansen

16:30-16:55 Thomas Wolfs
Approximation of Euler's constant using multiple orthogonal polynomials
17:00-17:25 Maxim Yattselev
On smooth perturbations of Chebyshëv polynomials and $\bar{\delta}$-Riemann-Hilbert method
17:30-17:55 Mateusz Piorkowski
Non-hermitian contour orthogonality with rational weights: Applications to random tiling models
18:00-18:25 Daniel Perales
Finite free probability and hypergeometric polynomials

Wednesday 11:40-12:40 (PSR2) chair: Jacob Christiansen

11:40-12:05 Benjamin Eichinger
Universality limits and homogeneous de Branges spaces
12:10-12:35 Brian Simanek
Orthogonal polynomials and mutually unbiased bases

Wednesday 14:00–15:00 (PSR2) chair: Ana Loureiro

14:00-14:25 Grzegorz Świderski
Asymptotic zeros' distribution of orthogonal polynomials with unbounded recurrence coefficients
14:30-14:55 Andy Hone
Elliptic orthogonal polynomials from solutions of the Toda and Volterra lattice

Thursday 14:00-16:00 (PSR2) chair: Jacob Christiansen

14:00-14:25 Peter Clarkson
Classical solutions of the fifth Painlevé equation
14:30-14:55 Ben Mitchell
Special function solutions of the fifth Painlevé equation
15:00-15:25 Kerstin Jordaan
Orthogonal polynomials and symmetric Freud weights
15:30-15:55 Kenta Miyahara
The sinh-Gordon reduction of the Painlevé III

Thursday 16:30–18:30 (PSR2) chair: Ana Loureiro

16:30-16:55 Walter Van Assche
Unique special solution for discrete Painlevé II

17:00-17:25 Michael Voit
Freezing limits of Calogero–Moser–Sutherland particle models

17:30-17:55 Open Problem Session

Thomas Bothner, University of Bristol

Universality for random matrices with an edge spectrum singularity

Abstract. We study invariant random matrix ensembles defined on complex Hermitian matrices with a single root type singularity and one-cut regular density of states. Assuming that the singularity lies within the soft edge boundary layer we compute asymptotics of the model's generating functional by using Riemann-Hilbert problems for orthogonal polynomials and integrable operators. This extends an old result by Forrester and Witte and is based on ongoing joint work with Toby Shepherd (Bristol).

Peter Clarkson, University of Kent

Classical solutions of the fifth Painlevé equation

Abstract. In this talk I will discuss classical solutions of the fifth Painlevé equation ($\mathrm{P_V}$)

$$\frac{\mathrm{d}^2w}{\mathrm{d}z^2} = \left(\frac{1}{2w} + \frac{1}{w-1}\right)\left(\frac{\mathrm{d}w}{\mathrm{d}z}\right)^2 - \frac{1}{z}\frac{\mathrm{d}w}{\mathrm{d}z} + \frac{(w-1)^2(\alpha w^2+\beta)}{z^2 w} + \frac{\gamma w}{z} + \frac{\delta w(w+1)}{w-1}, \tag{$*$}$$

where α, β, γ and δ are constants.

The general solutions of the Painlevé equations are transcendental in the sense that they cannot be expressed in terms of known elementary functions. However, it is well known that all Painlevé equations except the first equation possess rational solutions, algebraic solutions and solutions expressed in terms of the classical special functions for special values of the parameters. These solutions of the Painlevé equations are often called "classical solutions" and frequently can be expressed in the form of determinants.

In the generic case of $\mathrm{P_V}$ ($*$) when $\delta \neq 0$, special function solutions are expressed in terms of Kummer functions and has rational solutions expressed in terms of Laguerre polynomials. In the case of $\mathrm{P_V}$ ($*$) when $\delta = 0$, which is known as deg-$\mathrm{P_V}$ and related to the third Painlevé equation, special function solutions are expressed in terms of Bessel functions and has algebraic solutions expressed in terms of Laguerre polynomials. I shall give some new representations of some of these classical solutions and discuss Bäcklund transformation for $\mathrm{P_V}$ ($*$).

Joint work with Clare Dunning and Ben Mitchell (University of Kent).

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Benjamin Eichinger, TU Wien

Universality limits and homogeneous de Branges spaces

Abstract. Homogeneous de Branges spaces are certain reproducing kernel Hilbert spaces which satisfy an additional rescaling property. We recently showed that the corresponding reproducing kernels appear as limit kernels for regularly varying universality limits. These rescaling limits include in particular bulk universality and hard edge universality limits. In general, the limit kernels form a three parameter family explicitly expressible in terms of confluent hypergeometric functions. In this talk, we discuss structural properties of these spaces and explain why they naturally appear as limit kernels for universality limits.

The talk is based on a joint work with Milivoje Lukić and Harald Woracek

Wolter Groenevelt, TU Delft

Orthogonal polynomials and stochastic duality

Abstract. Duality is a very useful tool in the study of interacting particle processes which allows properties of one process to be studied using properties of the dual process. In this talk I explain how certain families of orthogonal polynomials, for example Krawtchouk polynomials, appear as duality function for certain inclusion and exclusion processes.

Andrew Hone, University of Kent

Elliptic orthogonal polynomials from solutions of the Toda and Volterra lattice

Abstract. We construct a family of elliptic orthogonal polynomials associated with solutions of the Toda lattice that are doubly periodic in time. These arise from a continued fraction expansion, due to van der Poorten, which yields a J-fraction for a family of functions of order 2 on an elliptic curve, and generates Hankel determinant solutions of the Somos-4 recurrence. The orthogonal polynomials may naturally be considered as elliptic analogues of Chebyshev polynomials, and in the real case correspond to polynomials constructed by Akhiezer. The relation with solutions of the Volterra lattice will also be mentioned.

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AH would like to thank the EPSRC for an Established Career Fellowship (2014-2021), and the School of Mathematics & Statistics, UNSW for hospitality and support from the Distinguished Researcher Visitor Scheme during his sabbatical there in 2017-2019..

Kerstin Jordaan, University of South Africa

Orthogonal polynomials and symmetric Freud weights

Abstract. In this talk I will present some results on polynomials orthogonal with respect to exponential weights on the real line, in particular the symmetric Freud weight

$$\omega(x;t,\tau,\rho) = |x|^{\rho} \exp\left\{-x^6 + \tau\, x^4 + t\, x^2\right\}, \qquad x \in \mathbb{R}$$

with τ, t and $\rho > -1$ parameters. For certain specific values of the parameters, the associated moments can be written as finite partition sums of generalised hypergeometric functions. The case where $\rho = 0$ is of particular interest and has been studied in the context of Hermitian one-matrix models and random symmetric matrix ensembles with researchers in the 1990s observing "chaotic, pseudo-oscillatory" behaviour of the recurrence relation coefficients. More recently, this "chaotic phase" was described as a dispersive shockwave in a hydrodynamic chain. I will describe properties of the recurrence coefficients in the three-term recurrence relation associated with the polynomials orthogonal with respect to this weight. Collaborators: Peter Clarkson and Ana Loureiro (University of Kent, UK).

Arno Kuijlaars, KU Leuven

Matrix valued orthogonal polynomials arising from periodic hexagon tilings

Abstract. Periodically weighted random tiling models may be analyzed with the help of matrix valued orthogonal polynomials (MVOP) [1]. I will consider a class of MVOP that arises from periodic lozenge tilings of a hexagon with period three and discuss their asymptotic properties as their degrees tend to infinity. A major role is played by an equilibrium measure on a three sheeted Riemann surface.

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Ben Mitchell, University of Kent

Special Function Solutions of the Fifth Painlevé Equation

Abstract. We explore rational solutions of the fifth Painlevé equation. This equation exhibits special function solutions for specific parameter values, expressed in terms of Kummer functions. It is well-known that the third Painlevé equation has special function solutions represented by Bessel functions. By utilizing connection formulae between Kummer functions and modified Bessel functions, we demonstrate that the fifth Painlevé equation also admits Bessel function solutions. Furthermore, we investigate the structure of the roots of these solutions.

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I would like to thank the EPSRC and University of Kent for their generous support.

Kenta Miyahara, Indiana University Indianapolis

The sinh-Gordon reduction of the Painlevé III

Abstract. We consider the large x asymptotics of any real-valued solutions of the sinh-Godron reduction of the Painlevé III on the real line. One is the one-parameter family of solutions that are smooth near ∞; the other is the two-parameter family of solutions that have accumulated singularities near ∞. We apply the Riemann-Hilbert nonlinear steepest descent method of Deift and Zhou to the sinh-Gordon Painlevé III in view of its Lax integrability and obtain the desired asymptotic formulae. This is joint work with Alexander Its and Maxim Yattselev.

Daniel Perales, Texas A&M University

Finite free probability and hypergeometric polynomials

Abstract. If we consider a generalized hypergeometric series where a parameter on top is a negative integer, then we naturally obtain a polynomial. When we have few paramaters, these hypergeometric polynomials are well known orthogonal families (Laguerre and Jacobi) and in particular are real-rooted for some regions of parameters.

The finite free additive and multiplicative convolutions are binary operations of polynomials that behave well with respect to real roots. We will apply these convolutions to the basic families of polynomials to systematically construct more involved hypergeometric polynomials (with several parameters) that have all real roots.

Furthermore, since the polynomial convolutions can be understood as a finite analogue of free probability that involves only discrete measures, then we can automatically understand the asymptotic root distribution of hypergeometric polynomials as free multiplicative convolutions of the corresponding limiting measures. Our results can be applied to study multiple orthogonal polynomials.

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Margit Rösler, Paderborn University

Limits of Bessel functions for root systems as the rank tends to infinity

Abstract. The asymptotic analysis of multivariate special functions has a long tradition in infinite dimensional harmonic analysis, tracing back to classical work of Olshanski, Vershik, and Kerov. Nowadays, such issues have become of renewed interest in the context of random matrix theory.

In this talk, we consider the asymptotic behaviour of Dunkl-type Bessel functions associated with root systems of type A and type B with positive multiplicities as the rank tends to infinity. In both cases, we characterize the possible limit functions and the sequences of spectral parameters for which such limits exist. In the type A case, we present an alternative and natural approach to recent results by Assiotis and Najnudel related to beta ensembles in random matrix theory. These results generalize classical facts about the approximation of the Olshanski spherical functions on the space of infinite-dimensional Hermitian matrices over $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$ (with the action of the infinite unitary group) by spherical functions of finite-dimensional Hermitian matrix spaces. Our approach naturally carries over to the B-case, where it gives asymptotic results for the spherical functions associated with the Cartan motion groups of non-compact Grassmannians as special cases.

The talk is based on joint work with Dominik Brennecken (Paderborn).

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Brian Simanek, Baylor University

Orthogonal polynomials and mutually unbiased bases

Abstract. Two orthonormal bases of a finite dimensional vector space are called mutually unbiased if the Fourier coefficients of any basis vector in terms of the other basis all have the same magnitude. It is known that an n-dimensional space contains at most $n+1$ mutually unbiased bases and that this bound is sharp. It is unknown if the bound can be saturated for every n. This talk will present an approach to this problem using orthogonal polynomials on the unit circle and a negative result about the prospects of solving the problem with this method.

This is based on joint work with Graeme Reinhart (Baylor).

Grzegorz Świderski, Polish Academy of Sciences

Asymptotic zeros' distribution of orthogonal polynomials with unbounded recurrence coefficients

Abstract. We study spectrum of finite truncations of unbounded Jacobi matrices with periodically modulated entries. In particular, we show that under some hypotheses a sequence of properly normalized eigenvalue counting measures converge vaguely to an explicit infinite Radon measure. Finally, we derive strong asymptotics of the associated orthogonal polynomials in the complex plane, which allows us to prove that Cauchy transforms of the normalized eigenvalue counting measures converge pointwise and which leads to a stronger notion of convergence. This is a joint work with Bartosz Trojan (Wrocław University of Science and Technology).

Walter Van Assche, KU Leuven, Belgium

Unique special solution for discrete Painlevé II

Abstract. We show that the discrete Painlevé II equation with starting value $a_{-1} = -1$ has a unique solution for which $-1 < a_n < 1$ for every $n \geq 0$. This solution corresponds to the Verblunsky coefficients of a family of orthogonal polynomials on the unit circle. The proof uses an idea of Tomas Lasic Latimer [2] who used orthogonal polynomials on the real line. We also give an upper bound for this special solution.

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Michael Voit, Technische Universität Dortmund

Freezing limits of Calogero–Moser–Sutherland particle models

Abstract. One-dimensional Calogero–Moser–Sutherland particle models with N particles can be regarded as diffusions on Weyl chambers or alcoves in $\mathbb{R}^N$ with second order differential operators as generators, which are singular on the boundaries of the state spaces. The most relevant examples are multivariate Bessel and Heckman–Opdam processes which are related to special functions associated with root systems. These models include Dyson's Brownian motions, multivariate Laguerre and Jacobi processes and, for fixed time, β-Hermite, Laguerre,

and Jacobi ensembles. In some cases, they are related to Brownian motions on the classical symmetric spaces.

We review some freezing limits for fixed N when some parameter, an inverse temperature, tends to ∞. The limits are normal distributions and, in the process case, Gaussian processes. The parameters of the limits are described in terms of solutions of ordinary differential equations which are frozen versions of the particle diffusions. We discuss connections of these ODES with the zeros of the classical orthogonal polynomials and polynomial solutions of some one-dimensional inverse heat equations.

The talk is partially based on joint work with Sergio Andraus, Kilian Herrmann, and Jeannette Woerner.

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Thomas Wolfs, KU Leuven

Approximation of Euler's constant using multiple orthogonal polynomials

Abstract. Ever since its first appearance, it has been unknown whether Euler's constant γ is irrational. A common strategy to (try to) prove irrationality of a given constant is by means of rational approximation: if one can construct good enough rational approximants to the constant, its irrationality follows. I will discuss a construction based on the multiple orthogonal polynomials associated with the exponential integral investigated in [3]. Although the approximants will not be good enough to prove that Euler's constant is irrational, we will be able to improve the quality of the approximants studied in [1] and [2]. Afterwards, I will show that other, but related, constants are more susceptible to rational approximation and that certain Bessel-like multiple orthogonal polynomials can be used to explicitly prove their irrationality.

This work is part of my PhD with Walter Van Assche.

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Maxim Yattselev, Indiana University Indianapolis

On smooth perturbations of Chebyshëv polynomials and $\bar{\delta}$-Riemann-Hilbert method

Abstract. Rates of converges of the recurrence coefficients of orthogonal polynomials $P_n(z)$ satisfying orthogonality relations

$$\int_{-1}^{1} x^l P_n(x) \frac{\rho(x)dx}{\sqrt{1-x^2}} = 0, \quad l \in \{0, \dots, n-1\},$$

where $\rho(x)$ is a positive m times continuously differentiable function on $[-1, 1]$, $m \geq 3$, will be discussed.

4.3. **Jordan Algebra Structures and Operator Theory.**
Organizers: Maria Cueto Avellaneda (Lisbon), Bas Lemmens (Kent), and Antonio M. Peralta (Granada)

Tuesday 14:00-16:00 (SIBLT2) chair: Antonio Peralta

14:00-14:25 Harald Upmeier
Hilbert Modules and Jordan Algebras
14:30-14:55 Matthew Neal
Metric linear characterizations of algebraic properties in operator spaces
15:00-15:25 Shiho Oi
Isometries between groups of invertible elements in Fourier-Stieltjes algebras
15:30-15:55 María Cueto Avellaneda
Something old, something new, something borrowed, something...Jordan?

Tuesday 16:30-18:30 (SIBLT2) chair: Bas Lemmens

16:30-16:55 Gerardo Martin Escolano
Lie–Trotter formulae in Jordan–Banach algebras with applications to the study of spectral-valued multiplicative functionals
17:00-17:25 Kieran Power
The horofunction compactification of symmetric cones equipped with the Hilbert metric
17:30-17:55 Lina Oliveira
Matrix operator algebras of JB^*-triples
18:00-18:25 Jorge J. Garcés Pérez
Bilinear maps with product property

Wednesday 11:40-12:40 (SIBLT2) chair: María Cueto Avellaneda

11:40-12:05 Michael Mackey
Fixed points of holomorphic automorphisms in spin factors
12:10-12:35 Pauline Mellon
The Wolff hull of a compact holomorphic map

Wednesday 14:00–15:00 (SIBLT2) chair: María Cueto Avellaneda

14:00-14:25 Bas Lemmens
Horofunction compactifications of Hermitian symmetric spaces and JB^*-triples
14:30-15:00 Cormac Walsh
Isometric embeddings of Hermitian symmetric spaces

Thursday 14:00-16:00 (SIBLT2) chair: Bas Lemmens

14:00-14:25 Lajos Molnár
*Characterizations of Jordan *-isomorphisms between C^*-algebras by relative entropy preserving maps*
14:30-14:55 Jan Hamhalter
Maps preserving Jordan decompositions of functionals in preduals of JBW algebras
15:00-15:25 Antonio M. Peralta
Cartan factors as an ideal mathematical model for Wigner's theorem
15:30-15:55 Michiya Mori
Nonexpansive and noncontractive mappings on the set of quantum pure states

Thursday 16:30–18:30 (SIBLT2) chair: Anonio Peralta

16:30-16:55 Dániel Virosztek
Quantum Wasserstein distances: metric properties and isometries

17:00-17:25 Tamás Titkos
Lattice properties of strength functions

17:30-17:55 Curt Healey
Every Symmetric Kubo-Ando connection has the order determining property

4.3.1. *Abstracts.*

Gerardo Martín Escolano, University of Granada

Lie–Trotter formulae in Jordan–Banach algebras with applications to the study of spectral-valued multiplicative functionals

Abstract. We establish some Lie–Trotter formulae for unital complex Jordan–Banach algebras, showing that for each couple of elements a, b in a unital complex Jordan–Banach algebra $\mathfrak{A}$ the identities

$$\lim_{n\to\infty} \left(e^{\frac{a}{n}} \circ e^{\frac{b}{n}}\right)^n = e^{a+b}, \ \lim_{n\to\infty} \left(U_{e^{\frac{a}{n}}}\left(e^{\frac{b}{n}}\right)\right)^n = e^{2a+b}, \text{ and}$$

$$\lim_{n\to\infty} \left(U_{e^{\frac{a}{n}},e^{\frac{c}{n}}}\left(e^{\frac{b}{n}}\right)\right)^n = e^{a+b+c}$$

hold. These formulae are employed in the study of spectral-valued (non-necessarily linear) functionals $f : \mathfrak{A} \to \mathbb{C}$ satisfying $f(U_x(y)) = U_{f(x)}f(y)$, for all $x, y \in \mathfrak{A}$. We prove that for any such a functional f, there exists a unique continuous (Jordan-)multiplicative linear functional $\psi\colon \mathfrak{A} \to \mathbb{C}$ such that $f(x) = \psi(x)$, for every x in the connected component of set of all invertible elements of $\mathfrak{A}$ containing the unit element. If we additionally assume that $\mathfrak{A}$ is a JB*-algebra and f is continuous, then f is a linear multiplicative functional on $\mathfrak{A}$. The new conclusions are appropriate Jordan versions of results by Maouche, Brits, Mabrouk, Shulz, and Touré. This is a joint work with Antonio M. Peralta and Armando R. Villena.

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María Cueto Avellaneda, IST University of Lisbon

Something old, something new, something borrowed, something...Jordan?

Abstract. This talk is aimed to recap in a light way the description of linear isometries on Jordan structures. We shall do it by invoking celebrated results as well as applying well-known Jordan arguments to go a step further, highlighting new contributions on the topic.

Jorge J. Garcés Pérez, Universidad Politécnica de Madrid

Bilinear maps with product property

Abstract. In this talk we shall present some new trends in the study of linear preserves, such as the study of maps that behave like a homomorphism or a derivation at a fixed point z. In the case $z = 0$ these are the well-known zero-product preservers (or derivable maps at zero). More recently, there has been a lot of interest in studying the case when $z \neq 0$ for some special values of z (for instance for z invertible or a projection). In a recent work in collaboration with Mykola Khrypchenko we develop a unified approach to these type of problems by considering bilinear maps that "have product property at a fixed element". We shall present some of our results and their applications.

Jan Hamhalter, Czech Technical University in Prague

Maps preserving Jordan decompositions of functionals in preduals of JBW algebras

Abstract. Let M be a JBW algebra with predual M_*. Any element $\varrho \in M_*$ has a canonical Jordan decomposition, $\varrho = \varrho_+ - \varrho_-$, where ϱ_+ and ϱ_- are mutually orthogonal positive functionals. Linear maps between preduals preserving these decompositions, called orthogonally decomposable homomorphisms, have been studied in the framework of von Neumann algebras by Araki, Bunce and Wright, and Lau and Wong [1,3]. We continue this line of the research in the more general framework of JBW algebras, which requires some new approaches [2]. We also sharpen hitherto known results for von Neumann algebras and C*-algebras. Among others we characterize continuous orthogonally decomposable homomorphism $\Phi : M_* \to N_*$ in term of its dual map, Φ^*, by showing that Φ^* is a Jordan isomorphism from the support $N_\Phi \subset N$ of the range of Φ onto certain direct summand of M followed by multiplication by a central positive element.

All descriptions of orthogonally decomposable maps [1,3], known so far, assume their continuity. In contrast to this, our new results on order topology on preduals of Jordan algebras (which are of independent interest) imply that in case of order decomposable bijections continuity obtains automatically. As a consequence bijective order decomposable maps arise as Jordan isomorphisms multiplied by central invertible elements. This means that the structure of Jordan decompositions in preduals is a complete invariant for JBW algebras. Various consequences (notably for duals of JB algebras) will be discussed.

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Curt Healey, University of Malta

Every Symmetric Kubo-Ando connection has the order determining property

Abstract. Recently, in [1], [3] and [4] considerable attention was given to the study of extending a surjective map ϕ which preserves the norm of a mean σ to a Jordan $*$-isomorphism. Namely, $\phi : \mathcal{A}^{++} \to \mathcal{B}^{++}$ acting on the positive definite cones of the unital C^*-algebras $\mathcal{A}, \mathcal{B}$ such that $||A\sigma B|| = ||\phi(A)\sigma\phi(B)||$ where $A, B \in \mathcal{A}^{++}$. The means in consideration fall under the general category of *Kubo-Ando means*. A binary operation σ on the positive definite cone $\mathcal{B}(H)^+$ of $\mathcal{B}(H)$ is called a *Kubo-Ando connection* if it satisfies the following properties:

(i) If $A \leq C$ and $B \leq D$, then $A\sigma B \leq C\sigma D$.
(ii) $C(A\sigma B)C \leq (CAC)\sigma(CBC)$.
(iii) If $A_n \downarrow A$ and $B_n \downarrow B$ strongly, then $A_n\sigma B_n \downarrow A\sigma B$ strongly.

A Kubo-Ando mean is a Kubo-Ando connection with the normalization condition $I\sigma I = I$.

Before proving that ϕ can be extended, the weaker condition that ϕ is an *order isomorphism* is first proven. The most direct way in proving the above is showing that σ is Order Determining (**OD**). Let $A, B \in \mathcal{A}^{++}$, then σ is (**OD**) if

$$A \leq B \iff ||A\sigma X|| \leq ||B\sigma X||, \quad \forall\, X \in \mathcal{A}^{++}.$$

It is easy to show that if σ is (**OD**), then a bijective map ϕ preserving the norm of σ is an order isomorphism. In our presentation, we shall show that every symmetric Kubo-Ando connection is (**OD**) [2, Theorem 6].

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Bas Lemmens, University of Kent

Horofunction compactifications of Hermitian symmetric spaces and JB*-triples

Abstract. From seminal works by Kaup and Loos we know that the open unit balls of finite dimensional JB*-triples are precisely the noncompact type Hermitian symmetric spaces. In this talk I will discuss the horofunction compactification of the open unit ball D in a finite dimensional JB*-triple V equipped with Carathéodory distance. The Carathéodory distance is a length metric with a Finsler structure, where the norm in the tangent space corresponds to the JB*-triple norm. Among other things we shall see that the geometry and global topology of the horofunction compactification coincides with the closed dual unit ball of the JB*-triple V. Moreover, we shall give explicit descriptions of the horofunction compactifications of D and of the JB*-triple V as a normed space, and see that the exponential map $\exp_0 \colon V \to D$ extends as a homeomorphism to the horofunction compactification of these spaces. The talk is based on joint work with Cho-Ho Chu and Maria Cueto Avellaneda

References

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Michael Mackey, University College Dublin

Fixed points of holomorphic automorphisms in spin factors

Abstract. A result of Hayden and Suffridge provides that biholomorphic automorphisms of the open unit ball of a Hilbert space have a fixed point when extended to the closed ball. Spin factors form a key test space in the generalisation of this result. The approach used for Hilbert space fails because non-linear automorphisms of a spin factor are never weakly continuous and so different techniques are required. We will present some conditions which guarantee that an automorphism of the unit ball of a spin factor has a fixed point.

This is joint work with Pauline Mellon.

Pauline Mellon, University College Dublin

The Wolff hull of a compact holomorphic map

Abstract. Let Z be a complex Banach space with open unit ball B and $f : B \to B$ be a compact holomorphic fixed-point free map. For example, for Z the complex plane and B the unit disc Δ, the classic Wolff Theorem gives that $(f^n)_n$ converges to (a Wolff point) ξ on the

boundary of the disc. However, $(f^n)_n$ does not generally converge even in finite dimensions (and for the infinite dimensional Hilbert ball compactness of f is necessary for convergence).

The more general aim therefore must be to locate the accumulation points $\Gamma(f)$ of $(f^n)_n$ for a suitable topology, or indeed to ask for $g \in \Gamma(f)$, where does $g(B)$ lie? This information is captured if we could locate the target set of f, namely,

$$T(f) = \bigcup_{g \in \Gamma(f)} g(B).$$

To this end, we introduce the concept of the *Wolff hull*, $W(f)$, of f and prove that this hull $W(f)$ is proximal in a precise way to $T(f)$. In particular, the Wolff hull generalises the concept of a Wolff point, where such a point can no longer be uniquely determined, and it coincides with the Wolff point if Z is a Hilbert space. We do this for the large class of Banach spaces known as the JB^*-triples.

This is joint work with Michael Mackey.

Lajos Molnár, University of Szeged

*Characterizations of Jordan *-isomorphisms between C^*-algebras by relative entropy preserving maps*

Abstract. We consider several concepts of (not only numerical valued) relative entropies on positive cones in C^*-algebras. We present recent results showing that bijective transformations between such cones that respect any of those quantities necessarily originate from Jordan *-isomorphisms between the underlying full algebras. Therefore, each of the considered relative entropies to some extent determines the full Jordan structure of C^*-algebras.

Michiya Mori, University of Tokyo

Nonexpansive and noncontractive mappings on the set of quantum pure states

Abstract. Wigner's theorem characterizes isometries of the set of all rank one projections on a Hilbert space. In metric geometry, nonexpansive maps and noncontractive maps are well studied generalizations of isometries. I will explain that under certain conditions Wigner symmetries can be characterized as nonexpansive or noncontractive maps on the set of all projections of rank one. The assumptions required for such characterizations are injectivity or surjectivity and they differ in the finite and the infinite-dimensional case. Motivated by a recently obtained optimal version of Uhlhorn's generalization of Wigner's theorem, I also give a description of nonexpansive maps which satisfy a condition that is much weaker than surjectivity. Such maps do not need to be Wigner symmetries. The optimality of the results is shown by (counter)examples.

This talk is based on a joint work [1] with Peter Šemrl (University of Ljubljana).

References

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The speaker is supported by JSPS KAKENHI Grant Number 22K13934.

Matthew Neal, Denison University

Metric linear characterizations of algebraic properties in operator spaces

Abstract. In the last several decades, many algebraic structures on operator spaces have been characterized in terms of the matrix norm and linear structure of the operator space, which we call the "metric-linear" or "geometric" structure. One famous early result is the characterization of unital operator algebras up to complete isometry among operator spaces that are Banach

algebras. Recently, David Blecher and I have given many other such characterizations, including the first metric-linear characterizations of C*-algebras up to complete isometry. All of these metric-linear characterizations will be explained in the talk. Our ultimate goal is to give a metric-linear characterization of JC*-triples among operator spaces. In addition to this, we will give some bonus results on holomorphic characterizations of algebraic structures on operator spaces, inspired of course by the holomorphic characterization of JB*-triples. We will also throw in one affine geometric characterization of JB*-triples among Banach spaces that references the dual space (inspired by the results of Alfsen and Shultz).

Shiho Oi, Niigata University, Japan

Isometries between groups of invertible elements in Fourier-Stieltjes algebras

Abstract. In this talk, we describe the structure of isometric real algebra isomorphisms between Fourier-Stieltjes algebras. By applying it, we see that if open subgroups of the groups of invertible elements in two Fourier-Stieltjes algebras are isometric as metric spaces, the underlying locally compact groups are topologically isomorphic.

This talk is supported by JSPS KAKENHI Grant Numbers JP24K06754.

Antonio M. Peralta, Universidad de Granada

Cartan factors as an ideal mathematical model for Wigner's theorem

Abstract. In 2002, L. Molnár established an analogue to the celebrated Piron-Wigner theorem by showing that, for each complex Hilbert space H with $\dim(H) \geq 3$, every bijective transformation on the lattice of partial isometries on H which preserves the partial ordering and orthogonality between partial isometries in both directions, and is norm continuous at a single non-zero partial isometry, extends to a real-linear triple isomorphism on $B(H)$ [2]. The novelty consisted in replacing the lattice of projections in $B(H)$ by the lattice of partial isometries.

The Banach space $B(H)$ is a particular case of type 1 Cartan factors. Cartan factors are useful to represent, via a Gelfand-Naimark type theorem, each abstract JB*-triple as a JB*-subtriple of an ℓ_∞-sum of Cartan factors (the latter objects are called atomic JBW*-triples). This talk will be devoted to presenting some recent extensions to the case of bijective transformations between lattices of atomic JBW*-triples preserving the natural partial ordering in both directions and orthogonality (in one direction) and enjoying a mild continuity property [1].

We shall also present some recent advances on preservers of triple transition pseudo-probabilities between minimal tripotents of two atomic JBW*-triples [3,4].

References

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Supported by MICINN Spain PID2021-122126NB-C31, MOST China G2023125007L and Junta de Andalucía FQM375

Kieran Power, University of Kent

The horofunction compactification of symmetric cones equipped with the Hilbert metric

Abstract. The horofunction compactifaction of a metric space is a well known concept that goes back to Gromov, with many applications in dynamics, geometry, and complex variables. In general the horofunction compactification is hard to compute, and its global geometry and topology is not well understood. In certain classes of finite dimensional normed spaces a duality phenomenon has been observed. In work with Bas Lemmens we have shown that this duality phenomenon occurs for a variety of classes of Finsler metric spaces. In this talk we will focus specifically on symmetric cones under the Hilbert metric, which originated as a way to generalise the metric for Klein's model of hyperbolic geometry. We exploit the interplay between the Hilbert metric and the JB-algebra structure of the cone in order to explicitly construct the horofunction compactification for these spaces. We begin the talk with a brief discussion of hyperbolic models of geometry and introduce the horofunction compactification. We proceed to introduce the machinery needed to show an explicit homeomorphism from the horoboundary onto the closed unit ball of the dual of the tangent space at the base point.

References

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Lina Oliveira, University of Lisbon

Matrix operator algebras of JB-triples*

Abstract. In this talk we describe the Tits–Kantor–Koecher Lie algebras of JB*-triples and characterise their representations as matrix operator algebras.

Tamás Titkos, Corvinus University and Rényi Institute

Lattice properties of strength functions

Abstract. We investigate an important functional representation of the cone of bounded positive semidefinite operators. It is known that the representation by strength functions turns the Löwner order into the pointwise order. However, very little is known about the structure of strength functions. We are going to show that the representation behaves naturally with the infimum and supremum operations. More precisely, we show that the pointwise minimum of two strength functions f_A and f_B is a strength function if and only if the infimum of A and B exists. The cornerstone of each argument in this talk is a recent discovery of Molnár and Ramanantoanina, namely that the strength function of the parallel sum $A : B$ (which is half of the harmonic mean) equals the parallel sum of the strength functions f_A and f_B. As a byproduct of this fact, in some special cases, we describe the strength function of the so-called (generalized) short.

This is a joint work with Andriamanankasina Ramanantoanina.

Harald Upmeier, University of Marburg

Hilbert Modules and Jordan Algebras

Abstract. Commuting tuples of non-selfadjoint operators can often be modelled by Hilbert modules of holomorphic functions on bounded domains D in $E = \mathbb{C}^n$. If D is a bounded symmetric domain, then E carries the structure of a Jordan algebra or, more generally, a hermitian

Jordan triple. Based on the Peter-Weyl decomposition of polynomials on E, under the Jordan triple automorphism group K, we construct a Hilbert module with reproducing kernel function, for each fixed integer partition of length r, the rank of E. As our main result, we determine the so-called eigenbundle of this Hilbert module, which is a singular vector bundle with respect to a natural stratification on the domain D, in terms of the Peirce decomposition under certain tripotents. This eigenbundle determines the Hilbert module up to unitary equivalence and corresponds to a coherent analytic sheaf, in contrast to the real-analysis methods in the self-adjoint case. Generalizations to the vector-valued case are also discussed shortly.

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Dániel Virosztek, Alfréd Rényi Institute of Mathematics

Quantum Wasserstein distances: metric properties and isometries

Abstract. Although the theory of classical optimal transport has been playing an important role in mathematical physics (especially in fluid dynamics) and probability since the late 80s, concepts of optimal transportation in quantum mechanics have emerged only very recently. First, we briefly review two such concepts: one relying on quantum channels (pioneered by De Palma and Trevisan) and one relying on quantum couplings (pioneered by Caglioti, Golse, Mouhot, and Paul). Then, we report on our progress in proving a conjecture of De Palma and Trevisan, saying that a smart modification of channel-based quantum Wasserstein distances gives rise to genuine metrics on quantum state spaces [1]. Finally, we describe the isometries of the qubit state space endowed with distinguished quantum Wasserstein distances [2].

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Cormac Walsh, INRIA Saclay and CMAP École Polytechnique Paris

Isometric embeddings of Hermitian symmetric spaces

Abstract. We study the rigidity of maps between Hermitian symmetric spaces that preserve the Carathéodory (or equivalently the Kobayashi) distance. We show that for such maps the rank of the codomain must be at least as great as the rank of the domain. When the two ranks are the same, and the domain is irreducible, we show that the map is either holomorphic or antiholomorphic. These results generalise those of Kim and Seo, who worked in the setting of C^1-smooth maps. Our metric geometry techniques allow us to dispense with any smoothness assumptions on the map. The main tool we use is the horofunction boundary, which for Hermitian symmetric spaces was described by Chu, Cueto-Avellaneda, and Lemmens.

This is joint work with Bas Lemmens.

4.4. **Differential Operators and Mathematical Physics.**
Organizers: Sabine Bögli (Durham), Jean-Claude Cuenin (Loughborough), Petr Siegl (Graz)

Monday 14:00-16:00 (SIBLT1) chair: SB/JCC

14:00-14:25 Ivica Nakic
Spectrum of operators on equilateral metric graphs
14:30-14:55 Marzieh Baradaran
Spectrum of quantum graphs with time-reversal non-invariant vertex couplings
15:00-15:25 Jon Harrison
A Discrete Analog of Quantum Unique Ergodicity on Circulant Graphs
15:30-15:55 Bryn Davies
Subwavelength spectra for highly contrast coefficients: an asymptotic framework for metamaterial design

Monday 16:30-19:00 (SIBLT1) chair: SB/JCC

16:30-16:55 Yuri Latushkin
The Duistermaat index and eigenvalue interlacing for self-adjoint extensions of a symmetric operator
17:00-17:25 Oliver Fürst
Regularized Index of non-Fredholm Callias Operators
17:30-17:55 Sukrid Petpradittha
Lieb-Thirring type inequalities for multidimensional Schrödinger operators with complex-valued potentials
18:00-18:25 Rongwei Yang
Yang-Mills equations in C-algebras*

Tuesday 14:00-16:00 (SIBLT1) chair: SB/JCC

14:00-14:25 Piero D'Ancona
Dispersion estimates for Dirac equations with Aharonov–Bohm magnetic fields
14:30-14:55 Łukasz Rzepnicki
Dirac system with an integrable potential and asymptotic behavior of its solutions on the plane
15:00-15:25 Jakob Reiffenstein
Eigenvalue density of limit circle Jacobi operators
15:30-15:55 Giovanni Bracchi
The propagator for the operator curl

Thursday 14:00-16:00 (SIBLT1) chair: SB/JCC

14:00-14:25 Bernhard Aigner
Well-posedness for a generalisation of a model for cell migration

14:30-14:55 Andreas Buchinger
Strong Operator Convergence in Homogenization of PDEs with Nonlocal Coefficients

15:00-15:25 Davide Macera
Disordered Dyson chains and disordered dimer models

15:30-15:55 Esmanur Yıldız Akil
Transition Dynamics of Reaction-Diffusion Equations at the kc-th Eigenvalue

Thursday 16:30–18:30 (SIBLT1) chair: SB/JCC

16:30-16:55 Filippo Santi
Galilei covariance of the theory of Thouless pumps

17:00-17:25 Elmira Nabizadeh Morsalfard
Continuity properties and Bargmann mappings of quasi-Banach Orlicz modulation spaces

17:30-17:55 Mohammed Sanduk
Is the complex harmonic oscillator a transformation due to a problem of partial observation? Rolling Circles Theory

Bernhard Aigner, TU Bergakademie Freiberg

Well-posedness in H^1 of a model for stem cell growth

Abstract. Delay differential equations are notoriously difficult to tackle with standard approaches. Recent results from [1] for ordinary state-dependent delay differential equations, that are based on the theory of evolutionary equations (cf. [2,3]), show that a traditional approach using exponentially weighted Sobolev-spaces is possible though. I will apply these results, in particular a generalisation of the Picard-Lindelöf theorem for H^1, to the setting of ordinary state-dependent delay differential equations to give an easier proof for well-posedness of a generalized population model from cell-biology. This approach requires less assumptions than previous well-posedness results (cf. [4,5]), that relied upon classical $\mathcal{C}$- or $\mathcal{C}^1$-theory such as [6,7]. The talk is based upon joint research with Marcus Waurick in [8].

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I would like to thank the state of Saxony for funding my PhD research via a stipend.

Esmanur Yıldız Akıl, Yeditepe University

Transition Dynamics of Reaction-Diffusion Equations at the k_c-th Eigenvalue

Abstract. Dynamic transition theory has been developed to understand transitions in nonlinear sciences, examining transitions between stable states of a system and classifying these dynamics [1]. The applications of dynamic transition theory in nonlinear sciences span across various physics, biology, and chemistry models [2, 3]. In this study, under certain assumptions, the dynamic transitions of the basic solution of a partial differential reaction-diffusion equation with high-order nonlinearity, given by $u_t = L_\lambda u + g(u, u_x)$, will be classified. Here, our first fundamental assumption is that the eigenvectors of the linear operator L form a basis of sine functions. The second fundamental assumption is that the critical value of the first eigenvalue where the transition occurs is taken at $\lambda_c = k_c$. In this study, the dynamics at the k_c-th eigenvalue of the one spatial dimensional reaction-diffusion equation with higher order nonlinearity will be examined.

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Marzieh Baradaran, University of Hradec Králové

Quantum graphs with time-reversal non-invariant vertex coupling

Abstract. Motivated by the application of quantum graphs to model the anomalous Hall effect, we discuss spectral properties of magnetic and non-magnetic quantum graphs assuming a preferred-orientation coupling at the graph vertices. The used vertex coupling violates the time reversal invariance, and its high-energy behavior depends on the vertex degree parity. Special attention is paid to the asymptotic behavior of the spectral bands in the high-energy regime. We see that the Band-Berkolaiko universality holds as long as the graph edge lengths are incommensurate. The talk is based on joint works with Pavel Exner and Jiří Lipovský [1-3].

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Support by the Czech Science Foundation grant no. 22-18739S is acknowledged.

Giovanni Bracchi, UCL

Hyperbolic propagator for the operator curl

Abstract. In this talk, I will describe an algorithm for computing Weyl coefficients of the operator curl $= *\mathrm{d}$ on a connected oriented closed Riemannian 3-manifold. This approach hinges on careful examination of the propagator of curl, *i.e.* the solution of the initial value problem $(-i\partial_t + \mathrm{curl})U(t) = 0$, $U(0) = \mathrm{Id}$, as a Fourier integral operator in the limit as $t \to 0^+$. In the end, I will provide formulae for the first three Weyl coefficients of curl.

Andreas Buchinger, TU Bergakademie Freiberg

Strong Operator Convergence in Homogenization of PDEs with Nonlocal Coefficients

Abstract. In this talk, we will revisit the classical notion of homogenization of div-grad-systems (H-convergence) and its operator-theoretic description that allows for more general systems with possibly nonlocal coefficients (nonlocal H-convergence provided by M. Waurick). We will introduce a convergence theorem for the corresponding solution operators, and we will discuss its sharpness in the sense of weak vs. strong operator convergence. This is joint work with S. Franz, N. Skrepek and M. Waurick.

Piero D'Ancona, Sapienza University of Rome

Dispersion estimates for Dirac equations with Aharonov–Bohm magnetic fields

Abstract. We examine the dispersive properties of a two dimensional Dirac operator perturbed by a critical Aharonov–Bohm potential. The flow can be split into a dispersive part which decays like in the unperturbed case, plus a singular component with weaker decay. For a partial range of indices, we deduce sharp Strichartz estimates for the flow. This is a joint work with Federico Cacciafesta, Zhiqing Yin and Junyong Zhang.

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Supported by the MIUR PRIN project 2020XB3EFL, "Hamiltonian and Dispersive PDEs", and by the Gruppo Nazionale per l'Analisi Matematica, la Probabilita' e le loro Applicazioni (GNAMPA)

Bryn Davies, Imperial College London

Subwavelength spectra for high-contrast coefficients: an asymptotic framework for metamaterial design

Abstract. High-contrast heterogeneous media are an attractive platform for microscopic wave control, thanks to their ability to support resonance at subwavelength scales. We have developed an asymptotic method for characterising these high-contrast scattering problems, which uses boundary integral representations to derive a characterisation of subwavelength resonance in terms of eigenstates of the generalised capacitance matrix (GCM). The GCM provides a concise framework to explore some of the important applications and exotic phenomena related to these materials. In this talk, we will introduce this approximation strategy and survey some of its applications to metamaterial design problems. These results are collaborations with Habib Ammari and Erik Orvehed Hiltunen.

I would like to thank the EPSRC for their support under grant number EP/X027422/1.

Oliver Fürst, University of Bonn

Trace and generalized index of Callias operators

Abstract. A Callias operator D is a Dirac-Schrödinger operator over a non-compact manifold M with an operator valued potential A admitting decay of its derivative in some sense. Usually it is required that the potential is invertible outside some compact region, allowing the operator D to be Fredholm. In this talk, this assumption is suspended, leading to a new trace formula for the heat-semigroups associated to D for $M = \mathbb{R}^d$. We will also see that the trace formula leads to a new index formula, where the Fredholm index needs to be replaced by the Witten index. We will calculate the Witten index for $(d+1)$-massless Dirac-Schrödinger operators in $\mathbb{R}^{d+1}$, and show that it may attain any real number on that class of operators, in contrast to the Fredholm index.

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Jon Harrison, Baylor University

Discrete Quantum Unique Ergodicity on Circulant Graphs

Abstract. A discrete analog of quantum unique ergodicity was proved for Cayley graphs of quasirandom groups by Magee, Thomas and Zhao [1]. They show that for large graphs there exists an orthonormal basis of Eigenfunctions of the adjacency matrix such that quantum probability measures of the eigenfunctions put approximately the correct proportion of their mass on subsets of the vertices that are not too small. We investigate this property for families of circulant graphs with prime order. We see that the property holds for an orthonormal Eigenfunction basis but fails if the basis is also required to be real. The equivalent result for a real basis holds for the Cayley graphs of quasirandom groups. This is work with Clare Pruss at Baylor University.

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Yuri Latushkin, University of Missouri

The Duistermaat index and eigenvalue interlacing for self-adjoint extensions of a symmetric operator

Abstract. Eigenvalue interlacing is a useful tool in linear algebra and spectral analysis. In its simplest form, the interlacing inequality states that a rank-one positive perturbation shifts each eigenvalue up, but not further than the next unperturbed eigenvalue. We prove a sharp version of the interlacing inequalities for "finite-dimensional perturbations in boundary conditions," expressed as bounds on the spectral shift between two self-adjoint extensions of a fixed symmetric operator with finite and equal defect numbers. The bounds are given in terms of the Duistermaat index, a topological invariant describing the relative position of three Lagrangian planes in a symplectic space. Two of the Lagrangian planes describe the self-adjoint extensions being compared, while the third corresponds to the Friedrichs extension, which acts as a reference point.

This is a joint work with G. Berkolaiko, G. Cox and S. Sukhtaiev.

Davide Macera, Durham University

On a dimer model with random weights

Abstract. After a brief outline on dimer models and their relevance in statistical physics, I will introduce a dimer model on the half-lattice whose weights are random variables which are independent in the horizontal direction and equal in the vertical one. I will then explain how to relate the rate of convergence of dimer-dimer correlations in such a model to the spectral properties of a disordered linear chain. This is a joint work (in progress) with Sunil Chhita.

Elmira Nabizadeh Morsalfard, Linnaeus University

Continuity properties and Bargmann mappings of quasi-Banach Orlicz modulation spaces

Abstract. We deduce continuity, compactness and invariance properties for quasi-Banach Orlicz modulation spaces on $\mathbf{R}^d$. We characterize such spaces in terms of Gabor expansions and by their images under the Bargmann transform.

Ivica Nakić, University of Zagreb

Spectrum of operators on equilateral metric graphs

Abstract. It is well known that the spectrum of the Laplacian on an equilateral metric graph is essentially determined by the spectrum of the corresponding Laplacian matrix. Similar results of this type are also known for the transport equations on metric graphs and in some other specific cases. In this talk we consider general operators on equilateral metric graphs with the property that their actions on all edges coincide. I will show that the spectrum of such operators can be elegantly described in terms of the spectrum of the underlying discrete graph and the spectrum of the restriction of the operator on one edge.

This is a joint work with Marjeta Kramar Fijavž.

Sukrid Petpradittha, Durham University

Lieb-Thirring type inequalities for multidimensional Schrödinger operators with complex-valued potentials

Abstract. The purpose of this research is to investigate a conjecture that was stated by Demuth, Hansmann and Katriel in 2013. We study a possible generalization of Lieb-Thirring type inequalities for eigenvalues of non-selfadjoint Schrödinger operators, with complex-valued potentials, acting on $L^2(\mathbb{R}^d)$ where $d \geq 2$. In particular, we find the asymptotic behavior for the discrete spectra of Schrödinger operators with a one-parameter family of rapidly decaying complex-valued potentials and present a disproof of this conjecture. This is a joint work with Sabine Bögli (Durham) and František Štampach (Prague).

I would like to thank the Isaac Newton Institute for Mathematical Sciences (INI) and the Engineering and Physical Sciences Research Council (EPSRC) grant (Ref. EP/V521929/1) via the UK Spectral Theory Network for their support.

Jakob Reiffenstein, Stockholm University

Eigenvalue density of limit circle Jacobi operators

Abstract. The eigenvalues of a limit circle Jacobi operator can be described on a quantitative level in terms of the growth of its Nevanlinna matrix. The most prominent result in this direction is a theorem of Berezanskii which gives a sufficient condition for limit circle case to take place and states that the exponential order of the Nevanlinna matrix is equal to the convergence exponent of the off-diagonal Jacobi sequence. However, its assumptions are very restrictive.
It turns out that the same assertion holds in significantly more general settings, which may lead to the intuition that the order of the Nevanlinna matrix should generically be equal to the above-mentioned convergence exponent. We will show that, for *any* limit circle Jacobi operator, the order is not less than the convergence exponent, and present a situation where it is actually larger.

Łukasz Rzepnicki, Nicolaus Copernicus University, Toruń

Dirac system with an integrable potential and asymptotic behavior of its solutions on the plane

Abstract. The main focus of this talk is a Dirac-type system considered on the interval $[0, 1]$ with a potential from L_p space, where $1 \leq p < 2$. We propose a new approach to study asymptotic behaviour of its solutions with respect to spectral parameter $\mu \in \mathbb{C}$ and $\mu \to \infty$. Our method gives results not only for μ in a horizontal stripe but for $\mathrm{Im}\mu \geq -c$, where $c > 0$. This allows us to obtain asymptotic identities valid in the whole complex plane which generalize those for the stripe. Moreover, using our approach one can get asymptotic formulas for spectral problems with eigenvalues lying outside a horizontal stripe.

Presented results are joint work with Alexander Gomilko.

Mohammed Sanduk, University of Surrey

Is the complex harmonic oscillator a transformation due to a problem of partial observation? Rolling Circles Theory

Abstract. The present project aims to establish a physical foundation for the complex harmonic oscillator. It is based on two main steps:

First: In the Bohr model, the particle was considered a classical particle. However, a classical particle does not exhibit the electron's spin. In the present project, the particle is considered an extended body with an internal clock, as hypothesized by Dirac and de Broglie. The resulting system is termed the Modified Bohr Model (MBM). This model depicts a system of two rolling circles (the orbit and the extended object). The MBM is a classical hypothetical model.

Second: In 1925, Heisenberg noted that some quantities in the Bohr model are observable, while others are not. In the present project, that Heisenberg's observation is interpreted as a partial observation process, where some quantities are observable and others are unobservable due to the resolving power. For mathematical application, this observation can be considered a measurement process. Substituting the quantities from the partial observation (as those of the quantum mechanics) into the mathematical forms MBM produces the mathematical forms of the observable MBM (OMBM).

These mathematical forms of the OMBM show similarities with those of quantum mechanics. In this context, the complex harmonic oscillator can be attributed to a classical model (rolling circles) and a physical process (partial observation).

Rongwei Yang, University at Albany, the State University of New York

Pluri-harmonic solutions to the Maxwell's equations and Yang-Mills equations

Abstract. Abstract: This talk provides a view of Maxwell's equations and Yang-Mills equations from the perspective of complex analysis, operator theory, and C^*-algebras. Based on pluri-harmonic differential forms, we will present a new class of instanton (self-dual or anti-self-dual) solutions to the equations. It is a joint work with Marius Beceanu and Sachin Munshi.

4.5. **Multivariable Operator Theory.**
Organizers: Sanne ter Horst (NWU), Tirtha Bhattacharyya (Indian Institute of Science), Nicholas Young (Newcastle & Leeds)

Tuesday 16:30-19:00 (SIBLT1) chair: Nicholas Young

16:30-16:55 Hugo Woerdeman
Optimal interpolation in Hardy and Bergman spaces: a reproducing kernel Banach space approach
17:00-17:25 Lijia Ding
The biholomorphic invariance of essential normality

Wednesday 11:40-12:40 (SIBLT1) chair: Nicholas Young

11:40-12:05 Piotr Pikul
The Szász inequality for matrix polynomials and functional calculus
12:10-12:35 Jeet Sampat
Jointly cyclic polynomials and maximal domains

Wednesday 14:00–15:00 (SIBLT1) chair: Sanne ter Horst

14:00-14:25 Raul Curto
Slantification of Hankel Operators on the Hardy Space of the n–torus
14:30-14:55 Dariusz Bugajewski
On composition of formal power series of multiple variables with applications

Thursday 14:00-16:00 (SIBSR7) chair: Sanne ter Horst

14:00-14:25 Greg Knese
Boundary local Integrability of rational functions in two variables
14:30-14:55 Connor Evans
Types of singularity on the boundary for Schur-Agler class functions on $\mathbb{D}^d$
15:00-15:25 Kenta Kojin
Some relations between Schwarz-Pick inequality and von Neumann's inequality
15:30-15:55 Robert T.M. Martin
A Fejér–Riesz theorem for non-commutative rational functions

Thursday 16:30–18:30 (SIBSR7) chair: Sanne ter Horst

16:30-16:55 Baruch Solel
Isometric dilation for representations of product systems
17:00-17:25 Amit Maji
Wold decomposition for isometries with equal range
17:30-17:55 Victor Bailey
Frames Generated by Unilateral Iterations of Bounded Commuting Operators

4.5.1. *Abstracts.*

Victor Bailey, University of Oklahoma

Frames Generated by Unilateral Iterations of Bounded Commuting Operators

Abstract. Recent work in Dynamical Sampling has been centered on characterizing frames obtained from the orbit of a vector under a bounded operator [1]. In this talk, we will provide a necessary and sufficient condition for a frame in a separable infinite-dimensional Hilbert space to be generated by unilateral iterations of a pair of bounded commuting operators on a vector. Applying the theory of shift-invariant subspaces of the Hardy Space on the bidimensional torus, we characterize these frames and provide several properties of frames of this form. This is joint work with Carlos Cabrelli.

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Dariusz Bugajewski, Adam Mickiewicz University, Poznań

On composition of formal power series of multiple variables with applications

Abstract. Formal power series as well as formal Laurent series play an important role in various branches of mathematics. In particular, they are frequently used in solving various type of equations, like ordinary differential equations or partial differential equations. Other applications of formal power series can be found in combinatorics, in Riordan groups and also in the proofs of some classical results, like the Cayley-Hamilton Theorem.

During the talk I am going to focus mainly on a necessary and sufficient condition for the existence of the composition of formal power series in the case when the outer series is a series of one variable while the inner one is a series of multiple variables. I am planning to discuss some ambiguities connected with the Right Distributive Law for formal power series of one variable, and next to present analogues of that law in the multivariable case. In the second part of the talk (depending on time), I am going to discuss the famous J.C.P. Miller formula which provides a recurrence algorithm for the composition of the formal binomial series and a formal power series f being a nonunit. I will present the general J.C.P. Miller formula which eliminates the requirement of nonunitness of f, formulating a necessary and sufficient condition for the existence of such composition.
The results presented in this talk come mainly from the papers [1] and [2].

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Raúl E. Curto, The University of Iowa

Slantification of Hankel Operators on the Hardy Space of the n–torus

Abstract. In the case of Toeplitz operators on the Hardy space $H^2(\mathbb{T})$ of the unit circle, the notion of slant Toeplitz operator was introduced by Mark Ho in 1996. Subsequently, slantification has been extended to Hankel operators on $H^2(\mathbb{T})$ and to Toeplitz operators acting on $H^2(\mathbb{T}^n)$, the Hardy space of the n–torus. In this talk, we will first introduce the notion of slantification of a Hankel operator acting on the space $H^2(\mathbb{T}^n)$. We will then study various properties of these operators, including hyponormality, isometric behavior, co-isometric behavior, and compactness.

For our purposes, the appropriate definition of Hankel operator is as follows. Given a symbol $\phi \in L^\infty(\mathbb{T}^n)$, the Hankel operator on $H^2(\mathbb{T}^n)$ induced by ϕ is given as

$$H_{\phi,n} := PJ_nM_\phi|_{H^2(\mathbb{T}^n)};$$

here J_n is the flip operator defined on $L^2(\mathbb{T}^n)$ by

$$(J_nf)(z_1,\dots,z_n) := f(\bar{z}_1,\dots,\bar{z}_n) \qquad (f \in L^2(\mathbb{T}^n))$$

and P is the orthogonal projection of $L^2(\mathbb{T}^n)$ onto $H^2(\mathbb{T}^n)$.

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Lijia Ding, Zhengzhou University

The biholomorphic invariance of essential normality

Abstract. The study of p-essentially normal Hilbert modules originated from Arveson's seminal works circa 2000, where he applied the results to investigate the dilation theory and the geometric invariant theory of the commuting operator tuple. Recently, there has been an increasing focus on the p-essential normality of Hilbert modules determined by subvarieties. In this talk, I shall discuss the biholomorphic invariance of p-essential normality of analytic Hilbert modules and give some intrinsic characterizations of p-essential normality. Furthermore, I will provide an application that extends the recent results on the equivalence between essential normality and hyperrigidity.

Connor Evans, Newcastle University, UK

Types of singularity on the boundary for Schur-Agler class functions on $\mathbb{D}^d$

Abstract. Classical results for analytic functions on $\mathbb{D}$ often have non-trivial generalizations, so how can we find analogous results for the polydisc $\mathbb{D}^d$? Recent success in the case of the bidisk $\mathbb{D}^2$ was found by Agler, McCarthy and Young, [1], through the use of Hilbert space models on $\mathbb{D}^2$, specifically, let φ be a function on $\mathbb{D}^2$, then $(\mathcal{H}, u)$ is said to be a model for φ if $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$ is a Hilbert space, $\mathcal{H}_1$ and $\mathcal{H}_2$ are orthogonally complementary subspaces of $\mathcal{H}$ and $u = (u_1, u_2)$ is a pair of analytic maps from $\mathbb{D}^2$ to $\mathcal{H}_1, \mathcal{H}_2$ respectively such that, for all $\lambda, \mu \in \mathbb{D}^2$

$$1 - \overline{\varphi(\mu)}\varphi(\lambda) = \left\langle (1 - \overline{\mu_1}\lambda_1)u_1(\lambda), u_1(\mu)\right\rangle_{\mathcal{H}_1} + \left\langle (1 - \overline{\mu_2}\lambda_2)u_2(\lambda), u_2(\mu)\right\rangle_{\mathcal{H}_2}.$$

Every Schur class function φ on $\mathbb{D}$ and $\mathbb{D}^2$ has such a model, but for $d \geq 3$ this is not true. We instead remedy this by considering the Schur-Agler class, $\mathcal{SA}_d$, this is the set of holomorphic functions $\varphi : \mathbb{D}^d \to \mathbb{C}$ such that, for all commuting d-tuples $(T_1, \dots, T_d)$ of contractions on a Hilbert space $\mathcal{H}$,

$$\|\varphi\|_{(d,\infty)} = \sup_{r<1} \|\varphi(rT_1, \ldots, rT_d)\|_\infty \leq 1.$$

Every function $\varphi \in \mathcal{SA}_d$ does have a Hilbert space model.

In this talk, we consider singularities $\tau \in \partial\mathbb{D}^d$ for functions $\varphi \in \mathcal{SA}_d$ and give specific criteria for the study of such singularities by means of Hilbert space models.

This talk is based on joint work with Dr. Zinaida Lykova and Prof. Nicholas Young.

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Greg Knese, Washington University in St. Louis

Boundary local Integrability of rational functions in two variables

Abstract. Motivated by studying boundary singularities of rational functions in two variables that are analytic on a domain, we investigate local integrability on $\mathbb{R}^2$ near $(0,0)$ of rational functions with denominator non-vanishing in the bi-upper half-plane but with an isolated zero (with respect to $\mathbb{R}^2$) at the origin. Building on work of Bickel-Pascoe-Sola, we give a necessary and sufficient test for membership in a local $L^p(\mathbb{R}^2)$ space and we give a complete description of all numerators Q such that Q/P is locally in a given L^p space. As applications, we prove that every bounded rational function on the bidisk has partial derivatives belonging to L^1 on the two-torus. In addition, we give a new proof of a conjecture started in work of Bickel-Knese-Pascoe-Sola and completed by Kollár characterizing the ideal of Q such that Q/P is locally bounded. A larger takeaway from this work is that a local model for stable polynomials we employ is a flexible tool and may be of use for other local questions about stable polynomials.

Kenta Kojin, Nagoya University

Some relations between Schwarz-Pick inequality and von Neumann's inequality

Abstract. I will talk about a Schwarz-Pick type inequality for the Schur-Agler class $SA(B_\delta)$ on a polynomial polyhedron B_δ, where δ is a matrix of polynomials in d-variables. We define a pseudo-distance on B_δ by

$$d_\delta(z,w) := \Big\| (I - \delta(w)\delta(w)^*)^{-\frac{1}{2}} (\delta(z) - \delta(w)) \times (I - \delta(w)^*\delta(z))^{-1} (I - \delta(w)^*\delta(w))^{\frac{1}{2}} \Big\| \quad (z, w \in B_\delta).$$

This is a generalization of the pseudo-hyperbolic distance on the open unit disk $\mathbb{D}$ defined by

$$d_{\mathbb{D}}(z,w) = \left| \frac{z-w}{1-\overline{w}z} \right| \quad (z, w \in \mathbb{D}).$$

Then, we can prove that

$$\sup_{f \in SA(B_\delta)} d_{\mathbb{D}}(f(z), f(w)) = d_\delta(z,w)$$

holds for any pair $z, w \in B_\delta$. As an application of this result and Jim Agler's deep observations on relations between operator theory and complex geometry [1], we can give a quantitative sufficient condition on a diagonalizable commuting tuple T acting on $\mathbb{C}^2$ for B_δ to be a complete spectral domain for T. We apply this sufficient condition to generalizing von Neumann's inequalities studied by Drury and by Hartz-Richter-Shalit.

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Amit Maji, Indian Institute of Technology Roorkee, India

Wold decomposition for isometries with equal range

Abstract. The main aim of this talk is the *Wold decomposition* of a large class of tuples of isometries on Hilbert spaces. More specifically, let $n \geq 2$, and $V = (V_1, \ldots, V_n)$ be an n-tuple of isometries acting on a Hilbert space $\mathcal{H}$. We say that V is an n-tuple of *isometries with equal range* if $V_i^{m_i} V_j^{m_j} \mathcal{H} = V_j^{m_j} V_i^{m_i} \mathcal{H}$ and $V_i^{*m_i} V_j^{m_j} \mathcal{H} = V_j^{m_j} V_i^{*m_i} \mathcal{H}$ for $m_i, m_j \in \mathbb{Z}_+$, where $1 \leq i < j \leq n$.

We prove that each n-tuple of *isometries with equal range* admits a unique *Wold decomposition*. We further obtain analytic models of the above class, and as a consequence, we show that the wandering data are complete unitary invariants for n-tuples of *isometries with equal range*. Our results unify all prior findings on the decomposition for tuples of isometries in the existing literature.

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I would like to thank MATRICS (SERB), the Government of India and IIT Roorkee for the financial support.

Robert T. W. Martin, University of Manitoba

A Fejér–Riesz theorem for non-commutative rational functions

Abstract. We prove a Fejér–Riesz theorem for bounded, positive semi-definite Toeplitz operators on the full Fock space with 'non-commutative (NC) rational symbols'. Here, the full Fock space can be identified with the 'free Hardy space' of all square–summable power series in several non-commuting variables $\{\mathfrak{z}_1, \cdots, \mathfrak{z}_d\}$. A bounded linear operator, T, on this space is left-Toeplitz, if $L_j^* T L_k = \delta_{j,k} T$, where $L_j = M_{\mathfrak{z}_j}^L$ are the left free shifts, the isometries of left multiplication by any of the formal NC variables, $\mathfrak{z}_j$. A non-commutative rational function is (essentially) any valid expression that can be obtained by applying the arithmetic operations of summation, multiplication and inversion to the free algebra of all free (non-commutative) polynomials with complex coefficients. Any element of the free Hardy space can be viewed as a non-commutative analytic function on the NC unit row-ball consisting of all row-contractive $d-$tuples of $n \times n$ complex matrices of any fixed size $n \in \mathbb{N}$, and an NC rational function defines a bounded left and right multiplier of this space, if and only if its domain includes a row-ball of radius greater than one.

Our NC rational Fejér–Riesz theorem then states that if $\mathfrak{h}$ is any bounded NC rational multiplier of the free Hardy space, and if $T = \operatorname{Re} M_{\mathfrak{h}}^R$ is positive semi-definite, where $M_{\mathfrak{h}}^R$ denotes right multiplication by $\mathfrak{h}$, then it factors as $T = (M_{\mathfrak{r}}^R)^* M_{\mathfrak{r}}^R$, for some bounded NC rational multiplier, $\mathfrak{r}$. The Fejér–Riesz theorem for free polynomials of G. Popescu is recovered as a special case.

This is joint work with Michael T. Jury (U. Florida).

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Piotr Pikul, Jagiellonian University in Kraków

The Szász inequality for matrix polynomials and functional calculus

Abstract. Szász inequality is a classical result providing a bound for polynomials with zeros in the upper half of the complex plane in terms of its low-order coefficients. Some generalisations of this result to multivariable polynomials were done by Borcea, Brändén and Knese. In the talk there will be presented inequalities of this kind for matrix polynomials in scalar variable and scalar polynomials in one and multiple matrix variables.

Joint work with Michał Wojtylak nad Oskar J. Szymański.

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Jeet Sampat, Technion

Jointly cyclic polynomials and maximal domains

Abstract. This talk is based on recent joint work with M. Mironov.

Let $\mathcal{X}$ be a topological vector space of holomorphic functions on an open set $\Omega \subset \mathbb{C}^d$ for which the polynomials $\mathcal{P}_d$ are dense. A family $\mathcal{F} \subset \mathcal{P}_d$ is called *jointly cyclic* if the shift invariant subspace they generate is $\mathcal{X}$. The *maximal domain* Ω_{max} is the set of all $w \in \mathbb{C}^d$ at which the evaluation functional $\Lambda_w : P \mapsto P(w)$ on $\mathcal{P}_d$ extends continuously to $\mathcal{X}$.

For $d = 1$, we completely determine when a family $\mathcal{F} \subset \mathcal{P}_d$ is jointly cyclic using Ω_{max}. For $d = 2$, we show that this can be reduced to determining the cyclicity of $GCD(\mathcal{F})$. We also examine the topology of Ω_{max} and show that if $\mathcal{X}$ is metrizable then Ω_{max} is an F_σ set. Lastly, we construct Hilbert function spaces on the unit disk $\mathbb{D}$ with $\Omega_{max} = \mathbb{D} \cup \Gamma$, where $\Gamma \subseteq \partial\mathbb{D}$ is both F_σ and G_δ.

Baruch Solel, Technion, Haifa, Israel

Isometric dilation for representations of product systems

Abstract. We discuss representations of product systems (of W^*-correspondences) over the semigroup $\mathbf{Z}_+^n$ and show that, under certain pureness and Szegö positivity conditions, a completely contractive representation can be dilated to an isometric representation. For $n = 1, 2$ this is known to hold in general (without assuming the conditions) but, for $n \geq 3$, it does not hold in general (as is known for the special case of isometric dilations of a tuple of commuting contractions). Restricting to the case of tuples of commuting contractions, our result reduces to a result of Barik, Das, Haria and Sarkar. Our dilation is explicitly constructed and we present some applications.

This is a joint work with S. Barik and M. Bhattacharjee.

Hugo J. Woerdeman, Drexel University

Optimal interpolation in Hardy and Bergman spaces: a reproducing kernel Banach space approach

Abstract. After a review of the reproducing kernel Banach space framework and semi-inner products, we apply the techniques to the setting of Hardy spaces H^p and Bergman spaces A^p, $1 < p < \infty$, on the unit ball in $\mathbb{C}^n$, as well as the Hardy space on the polydisk and half-space. In particular, we show how the framework leads to a procedure to find a minimal norm element f satisfying interpolation conditions $f(z_j) = w_j$, $j = 1, \ldots, n$. We also explain the techniques in the setting of ℓ^p spaces where the norm is defined via a change of variables and provide numerical examples. This talk is based on joint work with Gilbert Groenewald and Sanne ter Horst.

4.6. **Operator Theory and Hypercomplex Analysis.**
Organizers: Daniel Alpay (Chapman), Uwe Kaehler (Aveiro), Irene Sabadini (Milan)

Monday 14:00-16:00 (SIBSR7) chair: Uwe Kähler

14:00-14:25 Fabrizio Colombo
The fine structures on the S-spectrum
14:30-14:55 David Eelbode
The power of the outer exponential
15:00-15:25 Paula Cerejeiras
Fock spaces - a general framework
15:30-15:55 Arran Fernandez
Fractional d-bar derivatives and fractional polyanalyticity

Monday 16:30-19:00 (SIBSR7) chair: Irene Sabadini

16:30-16:55 Milton Ferreira
The Teodorescu and the Π *operators in octonionic analysis and applications*
17:00-17:25 Dmitrii Legatiuk
Discrete octonionic analysis: a non-associative play
17:30-17:55 Qinhai Huo
Octonionic Hilbert spaces and para-linear operators
18:00-18:25 Ming Jin
Weakly Slice Analysis on Non-Symmetric Domains in Several Quaternionic Variables

Tuesday 14:00-16:00 (SIBSR7) chair: Daniel Alpay

14:00-14:25 Raul Quiroga-Barranco
The geometry of slice regular Möbius transformations on the quaternionic unit ball
14:30-14:55 Irene Sabadini
On some classes of infinite order differential operators in hypercomplex analysis
15:00-15:25 Swanhild Bernstein
The q-Dirac operator on the quantum vector space
15:30-15:55 Martha Zimmermann
The q-deformed Dirac operator and the q-Hamiltonian

Tuesday 16:30-19:00 (SIBSR7) chair: Milton Ferreira

16:30-16:55 Cristina Diogo
Characterizing quaternionic numerical range through complex numerical range
17:00-17:25 Peter Schlosser
The H^∞-functional calculus for the quaternionic fine structure of the S-spectrum
17:30-17:55 Duvan Cardona
Control theory for the heat equation for non-local elliptic pseudo-differential operators on compact Lie groups
18:00-18:25 Uwe Kähler
Ternary Grassmann algebras and white noise space analysis

Wednesday 11:40-12:40 (SIBSR7) chair: Dmitrii Legatiuk

11:40-12:05 Elena Luna
Integral Theorems in the Theory of Bicomplex Holomorphic Functions and their relations with hyperbolic curves

12:10-12:35 Daniel Alpay
Scaled global operators and Fueter variables on non-zero scaled hypercomplex numbers

4.6.1. *Abstracts.*

Daniel Alpay, Chapman University

Scaled global operators and Fueter variables on non-zero scaled hypercomplex numbers

Abstract. We present a new family of Fueter-like variables associated to the global operator, and view this in a general setting which encompasses quaternions and split quaternions. Indefinite metric spaces (Pontryagin and Krein spaces) occur in a natural way.

The talk is based on joint works with Ilwoo Cho, Kamal Diki and Mihaela Vajiac.

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Versions of both papers are available on arxiv.

Swanhild Bernstein, TU Bergakademie Freiberg

The q-Dirac operator on the quantum vector space

Abstract. We consider q-commuting variables x_i, x_j such that $x_i x_j = q x_j x_i, i \neq j$. This is the standard setting in quantum calculus. Furthermore, we use the symmetric q-difference (which has some physical meaning [1]) and the multiplication from the right- and left-hand side because the structure is noncommutative. The $U(\mathfrak{o})$-invariant Laplacian [2,3] has the form

$$\Delta_q = q^{n-1}\partial_1^2 + q^{n-2}\partial_2^2 + \ldots + \partial_n^2.$$

We define the Dirac operator as the factorization of the Laplacian, i.e.

$$D_q = \sqrt{q}^{n-1}\partial_1 e_1 + \sqrt{q}^{n-2}\partial_2 e_2 + \ldots + \partial_n e_n.$$

We will discuss some properties of the Dirac operator and give examples of how to construct monogenic polynomials from harmonic polynomials in the case of $n = 2$.

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Duván Cardona, Ghent University

Control theory for the heat equation for non-local elliptic pseudo-differential operators on compact Lie groups

Abstract. In this talk we discuss our recent results about spectral inequalities for eigenvalues and their applications to control theory. In [1] We extend the estimates proved by Donnelly and Fefferman, and by Lebeau and Robbiano for sums of eigenfunctions of the Laplacian (on a

compact manifold) to estimates for sums of eigenfunctions of any positive and elliptic pseudo-differential operator of positive order on a compact Lie group. Our criteria are imposed in terms of the positivity of the corresponding matrix-valued symbol of the operator. As an application of these inequalities, we obtain the null-controllability for diffusion models for elliptic pseudo-differential operators on compact Lie groups. General results are also discussed on compact manifolds. Joint work with Michael Ruzhansky and Julio Delgado.

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[2] H. Donnelly, C. Fefferman, Nodal domains and growth of harmonic functions on noncompact manifolds. *J. Geom. Anal.* **2(1)**, (1992), 79–93.

[3] H. Donnelly, C. Fefferman, Nodal sets of eigenfunctions on Riemannian manifolds, *Invent. Math.* **93**, (1988), 161–183.

Paula Cerejeiras, University of Aveiro, Portugal

Fock spaces - a general framework

Abstract. We consider pairs of weighted shift operators on a weighted ℓ^2 space with weights associated with an entire function. When considered as operators over a general Fock space the commutators of these pairs of weighted shift operators are diagonal operators. We establish a calculus for the algebra of these commutators. As examples, we present the general case of Gelfond-Leontiev derivatives. This construction allows us to establish a general framework, which goes beyond the classic.

This is a joint work with D. Alpay, U. Kähler, and T. Kling.

Fabrizio Colombo, Politecnico di Milano

The fine structures on the S-spectrum

Abstract. Holomorphic functions play a crucial role in operator theory and the Cauchy formula is a very important tool to define functions of operators. The Fueter-Sce-Qian extension theorem is a two steps procedure to extend holomorphic functions to the hyperholomorphic setting. The first step gives the class of slice hyperholomorphic functions; their Cauchy formula allows to define the so-called S-functional calculus for noncommuting operators based on the S-spectrum. In the second step this extension procedure generates monogenic functions; the related monogenic functional calculus, based on the monogenic spectrum, contains the Weyl functional calculus as a particular case. In this talk we show that the extension operator from slice hyperholomorphic functions to monogenic functions admits various possible factorizations that induce different function spaces. The integral representations in such spaces allows to define the associated functional calculi based on the S-spectrum. The function spaces and the associated functional calculi define the so called *fine structure of the spectral theories on the S-spectrum*. Among the possible fine structures there are the harmonic and poly-harmonic functions and the associated harmonic and poly-harmonic functional calculi. In this talk we present the state of the art of this new branch of operator theory based on the S-spectrum.

Cristina Diogo, ISCTE-IUL and

Characterizing quaternionic numerical range through complex numerical range

Abstract. The numerical range, defined as the image of the unit sphere under a certain quadratic form, reveals different geometric structures depending on whether the ground field is the complex numbers $\mathbb{C}$ or the skew field of Hamilton's quaternions $\mathbb{H}$. The complex numerical range has been extensively studied in the literature, contrary to its quaternionic counterpart. In this talk, we explore the coexistence of both notions, and we bring to the quaternionic setting some results from the complex setting. In fact, for the class of complex operators acting on a quaternionic Hilbert space, we characterize the quaternionic numerical range in terms of the complex one and two suitable real numbers. Moreover, we provide a characterization of the quaternionic numerical range for normal operators.

Joint work with Luís Carvalho and Sérgio Mendes.

David Eelbode, University of Antwerp

The power of the outer exponential

Abstract. One of the most important (geometrical) properties of a bivector B in a (Euclidean) Clifford algebra is the fact that it can be decomposed as a sum $B = b_1 + \ldots + b_k$ of simple and mutually orthogonal bivectors b_j (where k is at most half the dimension of the underlying space). This result goes back to the infamous Cartan-Dieudonné theorem and a conjecture made by Marcel Riesz, but recently regained interest in the work of De Keninck and Roelfs (see [1]). The first thing we will do in this talk is to explain how the so-called outer exponential (introduced by P. Lounesto in his seminal book) can give some insight into the nature of these decomposing bivectors b_j, which then appear naturally as images of certain trigonometric functions. If time permits, we will also explain why these observations matter and how the bivectors b_j can be used to construct spinors as clearly defined geometrical objects.

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[2] P. Lounesto, *Clifford Algebras and Spinors*, Cambridge University Press, London Mathematical Society Lecture Note Series (286), 2001.
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This is joint work with S. De Keninck and M. Roelfs.

Arran Fernandez, Eastern Mediterranean University

Fractional d-bar derivatives and fractional polyanalyticity

Abstract. The d-bar derivative, and the closely associated Dirac operator, are differential operators of vital importance in hypercomplex analysis, ranging from complex to quaternionic to Clifford analysis. We consider different possible ways to define fractional powers of these operators, including fractional derivatives of both Riemann–Liouville and Caputo type, in order to obtain a richer and more general theory. A naturally arising question, once fractional d-bar derivatives have been defined, is what are the kernels of these operators, the functions whose fractional d-bar derivatives are zero? We answer this question by giving a complete characterisation of the so-called fractionally polyanalytic functions in $\mathbb{C}$.

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I would like to thank the Eastern Mediterranean University for their financial support via a BAP-C grant with number BAPC-04-24-01.

Milton Ferreira, Polytechnic of Leiria, Portugal

The Teodorescu and the Π *operators in octonionic analysis and applications*

Abstract. In the octonionic function theory, the non-associativity together with the non-commutativity play a key role. For instance, the Stokes formula contains an additional associator term originated by the non-associative property. To take the non-associativity into account, particular intrinsic weight factors are implemented in the definition of octonion-valued inner products to ensure the existence of a reproducing Bergman kernel. This Bergman projection plays a pivotal role in the L_2-space decomposition for octonion-valued functions. We will address this question. Furthermore, we study an octonionic Teodorescu transform and show how it is related to the unweighted version of the Bergman transform and establish some operator relations between these transformations. We apply two different versions of the Borel-Pompeiu formulae that naturally arise in the context of the non-associativity.

Finally, we use the octonionic Teodorescu transform to establish a suitable octonionic generalization of the Ahlfors-Beurling Π-operator. We prove an integral representation formula that presents a unified representation for the Π-operator arising in all hypercomplex function theories, and describe some of its mapping properties. Applications of the Π-operator associated with the octonionic Beltrami equation and the hyperbolic octonionic Dirac operator will be shown.

I would like to thank CIDMA – Center for Research and Development in Mathematics and Applications, and FCT – Fundação para a Ciência e a Tecnologia, for their support within project UIDB/04106/2020 (https://doi.org/10.54499/UIDB/04106/2020).

Qinghai Huo, Hefei University of Technology

Octonionic Hilbert spaces and para-linear operators.

Abstract. The theory of octonionic Hilbert spaces was first introduced by Goldstine and Horwitz in 1964 but subsequently remained dormant. Recently, it has been demonstrated that one of the axioms within this theory is not independent of the others. This revelation has led to the introduction of a novel concept known as octonionic para-linearity, which has emerged as the central focus of octonionic functional analysis. By substituting linearity with paralineartiy, the definition of Hilbert spaces over complex numbers, quaternions and octonions can be unified. The Riesz representation theorem also applies to paralinear functionals naturally. Furthermore, we have introduced the octonionic version of dual operators based on the Riesz representation theorem. Unexpectedly, by establishing a new polar identity for paralinear operators, we find a new characterization of octonionic paralinear self-adjoint operators. The relationships between the kernel and range of a paralinear operator and its dual operator also undergo significant changes. A novel notion of fractional subspaces is introduced to refine the classical relationships.

Ming Jin, Macau University of Science and Technology

Weakly Slice Analysis on Non-Symmetric Domains in Several Quaternionic Variables

Abstract. In this talk, we focus on the algebraic structure of weakly slice regular functions on several quaternionic variables in the frame of weakly slice analysis based on slice topology. Slice analysis is a generalization of function theory of one complex variable(or several complex variables) in high-dimensional spaces. With its development, the theory is further divided into strongly slice analysis and weakly slice analysis. Weakly slice analysis was only proposed in the past three years and the introduction of a new topology(i.e. slice topology) brings a new perspective to slice analysis. For the algebraic structure in weakly slice analysis, we introduced a $*$-product that preserves the slice-regular property. For this purpose, we proposed precise definitions for open neighborhoods of a path and the holomorphism of stem functions.

Uwe Kähler

Ternary Grassmann algebras and white noise space analysis

Abstract. Classic supersymmetry is based on $\mathbb{Z}_2$-graded algebras, like Clifford and Grassmann algebras which still allows us to consider a Fock space of monogenic function and build most of the necessary ingredients for a theory of entire functions. But more general settings like quarks need a more general type of supersymmetry based on a $\mathbb{Z}_3$-grading (also called hypersymmetry). In this talk we present the groundwork for an Itô/Malliavin stochastic calculus and Hida's white noise analysis in the context of a supersymmetry with $\mathbb{Z}_3$-graded algebras. To this end, we establish a ternary Fock space and the corresponding strong algebra of stochastic distributions and present its application in the study of stochastic processes in this context.

Dmitrii Legatiuk, Universität Erfurt

Discrete octonionic analysis: a non-associative play

Abstract. In this talk, we discuss ideas on discretisation of the classical octonionic analysis. In particular, we present the ideas related to a direct discretisation of partial derivatives by help of finite differences. We discuss the influence of the non-associativity of octonionic multiplication and present an explicit derivation of the associator in the discrete setting. Further, we present basics of discrete octonionic function theory, including Borel-Pompeiu formulae and Cauchy transforms.
This is a joint work with Rolf Sören Kraußhar and Anastasiia Legatiuk.

M. Elena Luna-Elizarrarás, Holon Institute of Technology, Holon, Israel

Integral Theorems in the Theory of Bicomplex Holomorphic Functions and their relations with hyperbolic curves

Abstract. In this talk we will analyze some geometric aspects of the well known Integral Theorems for holomorphic bicomplex functions. In particular we will reveal some connections between hyperbolic curves and those (1-dimensional) curves over which the integrals of such theorems are evaluated.

Raul Quiroga-Barranco, Centro de Investigación en Matemáticas, Mexico

The geometry of slice regular Möbius transformations on the quaternionic unit ball

Abstract. Let us denote by $\mathbb{B}$ the quaternionic unit ball centered at the origin. It has been developed in [1,2] a rich theory of hyperholomorphic quaternionic valued functions on $\mathbb{B}$ based on the notion of slice regularity. This includes, among many other topics, the introduction of slice regular Möbius transformations, which constitute a family that will be denoted by $\mathcal{M}(\mathbb{B})$.

We will present the construction, based on the notion of slice regularity, of some geometric structures that can be associated to both $\mathbb{B}$ and $\mathcal{M}(\mathbb{B})$. Our development generalizes the well-known situation for the complex case and the unit disk, but has to overcome some obstructions inherent to the non-commutative quaternionic case. Our results can be found in [3,4].

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Irene Sabadini, Politecnico di Milano

On some classes of infinite order differential operators in hypercomplex analysis

Abstract. Infinite-order differential operators appear in different fields of mathematics and physics and in the past decade they turned out to be of fundamental importance in the study of the evolution of superoscillations In this talk we discuss we investigate the continuity of a class of infinite-order differential operators acting on spaces of two different classes of entire hyperholomorphic functions. In both cases, entire hyperholomorphic functions with exponential bounds play a crucial role in the continuity of infinite-order differential operators. This is particularly remarkable since the exponential function is not in the kernel of the Dirac operator, and so it does not belong to this class of hyperholomorphic functions, but it still plays an important role.

Peter Schlosser, Politecnico di Milano

The H^∞-functional calculus for the quaternionic fine structure of the S-spectrum

Abstract. For slice-hyperholomorphic functions f, the Cauchy-formula

$$f(T) := \frac{1}{2\pi} \int_{\partial(\mathbb{C}_J \cap U)} S_L^{-1}(s,T) ds_J f(s), \tag{9}$$

also called the S-functional calculus, is a way to define functions of operators in quaternionic Banach spaces. However, there are two restrictions:

i) f has to be slice-hyperholomorphic,
ii) f has to admit some decay at ∞ in order to ensure the convergence of the integral.

In this talk I will present methods how to relax both conditions.

i) By changing the integral kernel $S_L^{-1}(s,T)$, we are able additionally treat harmonic, poly-harmonic and axially monogenic functions f in an integral of the form (9). This method results in the so-called fine structure of the S-spectrum.
ii) The decay at ∞ on the other hand can be relaxed, by using regularizer functions e. With them one can define the so called *H^∞-functional calculus*, i.e. $f(T)$ is defined as

$$f(T) := e(T)^{-1}(ef)(T),$$

where on the right hand side, $e(T)$ and $(ef)(T)$ are both defined via (9).

Martha Zimmermann, TU Bergakademie Freiberg

The q-deformed Dirac operator and the q-Hamiltonian

Abstract. We consider Clifford algebras in the context of Jackson calculus. We start with establishing the foundations for this q-deformed setting and introduce q-deformed partial derivatives $\partial_i^q = \frac{f(x_1,\dots,qx_i,\dots,x_n)-f(x_1,\dots,x_n)}{(q-1)x_i}$. The q-partial derivatives allow us to then define required operators such as the q-Dirac operator $D_x^q = \sum_{i=1}^n e_i\partial_i^q$, q-Euler operator $E^q = \sum_{i=1}^n x_i\partial_i^q$ and q-Gamma operator $\Gamma^q = \sum_{i<j} e_ie_j(x_i\partial_j^q - x_j\partial_i^q)$ in our new setting. Further, we will regard some of the relations between those operators.
The Dirac and Euler operator are related to the definition of a Hamiltonian. We transfer this to the q-deformed setting to obtain a q-Hamiltonian. This will also lead us to q-deformed raising and lowering operators and their relations to each other. A central object related to the Hamiltonian is the exponential function. As there are several different possibilities for a q-deformed exponential, part of this talk will focus on the right choice of exponential function for the considered q-Hamiltonian. Further, we will regard some properties of this q-exponential function.

4.7. **Free Analysis and Convexity.**
Organizers: Robert Martin (Manitoba), James Pascoe (Drexel, Philadelphia), Eli Shamovich (Ben Gurion)

Monday 16:30-19:00 (SIBLT3) chair: Eli Shamovich

16:30-16:55 Bill Helton
Solving Quantum Max Cut via Swap Operators, I
17:00-17:25 Igor Klep
Solving Quantum Max Cut via Swap Operators, II
17:30-17:55 William Slofstra
Positivity is undecidable in free product algebras
18:00-18:25 Tea Štrekelj
Duality and extreme points for Γ*-convex sets*
18:30-18:55 Jiří Spurnỳ
Boundary integral representation of multipliers of fragmented affine functions

Tuesday 14:00-16:00 (SIBLT3) chair: Rob Martin

14:00-14:25 Jurij Volčič
Waring problems for noncommutative rational functions
14:30-14:55 Matt Kennedy
Noncommutative Majorization
15:00-15:25 Jeet Sampat
Biholomorphisms between subvarieties of noncommutative operator balls
15:30-15:55 Hridoyananda Saikia
A non-commutative boundary for the dilation order

4.7.1. *Abstracts.*

Bill Helton, University of California San Diego

Solving Quantum Max Cut via Swap Operators, I

Abstract. The Quantum Max Cut (QMC) problem has emerged as a test-problem for designing approximation algorithms for local Hamiltonian problems in quantum physics. In this talk we attack this problem using the algebraic structure of QMC; we will explore the relationship between QMC and the representation theory of the symmetric group.

The first major contribution is an extension of noncommutative Sum of Squares optimization techniques championed by Helton and McCullough to give a new hierarchy of relaxations to Quantum Max Cut. The hierarchy we present is based on polynomials in the swap operators. To prove completeness of this hierarchy, we give a finite presentation of the algebra generated by the swap operators. We find that level-2 of this new hierarchy is exact (up to tolerance 10^7) on all QMC instances with uniform edge weights on small graphs.

The second major contribution of this talk is a polynomial-time algorithm that exactly computes the maximum eigenvalue of the QMC Hamiltonian for certain graphs, including graphs that can be "decomposed" as a signed combination of cliques. A special case of the latter are complete bipartite graphs with uniform edge-weights, for which exact solutions are known from the work of Lieb and Mattis (1962).

The talk is based on joint work with Adam Bene Watts, Anirban Chowdhury, Aidan Epperly, and Igor Klep.

Matthew Kennedy, University of Waterloo

Noncommutative Majorization

Abstract. I will introduce a notion of noncommutative majorization and discuss some application to operator algebras, including a multivariate generalization of the Schur-Horn theorem for finite von Neumann algebras. I will also briefly discuss some applications to quantum information theory. This is joint work with Laurent Marcoux and Paul Skoufranis.

Igor Klep, University of Ljubljana

Solving Quantum Max Cut via Swap Operators, II

Abstract. The Quantum Max Cut (QMC) problem has emerged as a test-problem for designing approximation algorithms for local Hamiltonian problems in quantum physics. In this talk we attack this problem using the algebraic structure of QMC; we will explore the relationship between QMC and the representation theory of the symmetric group.

The first major contribution is an extension of noncommutative Sum of Squares optimization techniques championed by Helton and McCullough to give a new hierarchy of relaxations to Quantum Max Cut. The hierarchy we present is based on polynomials in the swap operators. To prove completeness of this hierarchy, we give a finite presentation of the algebra generated by the swap operators. We find that level-2 of this new hierarchy is exact (up to tolerance 10^7) on all QMC instances with uniform edge weights on small graphs.

The second major contribution of this talk is a polynomial-time algorithm that exactly computes the maximum eigenvalue of the QMC Hamiltonian for certain graphs, including graphs that can be "decomposed" as a signed combination of cliques. A special case of the latter are complete bipartite graphs with uniform edge-weights, for which exact solutions are known from the work of Lieb and Mattis (1962).

The talk is based on joint work with Adam Bene Watts, Anirban Chowdhury, Aidan Epperly, Bill Helton, and work in progress with Tea Štrekelj and Jurij Volčič.

Hridoyananda Saikia, University of Manitoba

A non-commutative boundary for the dilation order

Abstract. Arveson's hyperrigidity conjecture focuses on the unique extension property (UEP) of representations of C^*-algebras with respect to a generating operator system. The states that are maximal in the dilation order fully encapsulate the cyclic representations of a C^*-algebra with the UEP. The set of all maximal states form a norm-closed set which remains stable under absolute continuity. In this talk, we will discuss an equivalent characterization of the dilation maximal states in terms of a boundary projection. Subsequently, we will state a reformulation of Arveson's hyperrigidity conjecture in terms of the non-commutative topological properties of this boundary projection. This is a joint work with Raphaël Clouâtre.

Jeet Sampat, Technion

Biholomorphisms between subvarieties of noncommutative operator balls

Abstract. This talk is based on recent joint work with O. M. Shalit.

Let $\mathbb{M}^d$ be the graded union of d-tuples of complex matrices of arbitrary size. Given any d-dimensional operator space $\mathcal{E}$ with basis $\{Q_1, \dots, Q_d\}$, we define the corresponding noncommutative (nc) operator ball as

$$\mathbb{D}_Q := \left\{ X \in \mathbb{M}^d : \left\| \sum_{j=1}^{d} Q_j \otimes X_j \right\| < 1 \right\}.$$

We shall discuss the problem of extending a nc map between subvarieties $f : \mathfrak{V}_1 \to \mathfrak{V}_2$ to a map $F : \mathbb{D}_{Q_1} \to \mathbb{D}_{Q_2}$. We show that such an extension cannot be guaranteed unless $\mathcal{E}_2$ is an injective operator space. In particular, this holds for the nc unit polydisk $\mathfrak{D}_d$ and the nc unit row-ball $\mathfrak{B}_d$. We shall use this extension property to identify nc biholomorphisms between subvarieties $\mathfrak{V}_j \subseteq \mathbb{D}_{Q_j}$ and show that when $\mathfrak{V}_j$ is 'nice enough,' such a biholomorphism can be modified to be the restriction of a linear isomorphism between $\mathbb{D}_{Q_j}$.

William Slofstra, University of Waterloo

Positivity is undecidable for products of free algebras

Abstract. For free *-algebras, free group algebras, and related algebras, it is possible to decide if an element is positive (in all representations) using results of Helton, Bakonyi-Timotin, Helton-McCullough, and others. In this talk, I'll discuss joint work with Arthur Mehta and Yuming Zhao showing that this problem becomes undecidable for a product of free *-algebras. I'll also discuss how results of this type could be aided by having a Higman embedding theorem for algebras with states, as well as work in progress on this question.

Jičí Spurný, Charles University

Boundary integral representation of multipliers of affine functions

Abstract. We develop a theory of abstract intermediate function spaces on a compact convex set X and study the behaviour of multipliers and centers of these spaces. In particular, we provide some criteria for coincidence of the center with the space of multipliers and a general theorem on boundary integral representation of multipliers. We apply the general theory in several concrete cases, among others to strongly affine Baire functions, to the space $A_f(X)$ of fragmented affine functions, to the space $(A_f(X))^\mu$, the monotone sequential closure of $A_f(X)$, to their natural subspaces formed by Borel functions, or, in some special cases, to the space of all strongly affine functions. In addition, we prove that the space $(A_f(X))^\mu$ is determined by extreme points and provide a large number of illustrating examples and counterexamples.

References

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Tea Štrekelj, Institute of Mathematics, Physics and Mechanics

Duality and extreme points for Γ*-convex sets*

Abstract. In this talk we discuss several generalizations of (matrix) convexity, e.g., partial convexity or biconvexity, which are summed up in the term Γ-convexity. Here Γ is a tuple of free symmetric polynomials determining the geometry of a Γ-convex set. We introduce the notions of Γ-operator systems and Γ-ucp maps and establish a Webster-Winkler type categorical duality between Γ-operator systems and Γ-convex sets. Next, we introduce a notion of an extreme point of a Γ-convex set extending the concept of a free extreme point. To ensure existence of such points, matrix convex sets are extended to include an operator level. The so called Γ-extreme points of an operator Γ-convex set K are in bijective correspondence with the free extreme points of the operator convex hull of $\Gamma(K)$. From this result, a Krein-Milman theorem for Γ-convex sets follows.

Jurij Volčič, Drexel University

Waring problems for noncommutative rational functions

Abstract. Noncommutative polynomials and noncommutative rational functions are elements of the free associative algebra and free skew field, respectively. One may view them as multivariate functions in matrix arguments; this perspective is common in noncommutative function theory and free real algebraic geometry. This talk concerns the images of noncommutative rational functions on large matrices. Firstly, every nonconstant noncommutative rational function attains values with pairwise distinct eigenvalues on sufficiently large matrix tuples. Secondly, one can then apply this to noncommutative variants of the Waring problem. In particular, given a nonconstant noncommutative rational function, every large enough trace-zero matrix is a difference of its values, and every large enough nonscalar determinant-one matrix is a quotient of its values. Based on joint work with Matej Brešar.

4.8. **Quantum Information.**
Organizers: Igor Klep (Ljubljana), David Gross (Cologne), William Slofstra (Waterloo)

Monday 14:00-16:00 (PSR4) chair: Igor Klep

14:00-14:25 Reinhard Werner
Von Neumann algebraic quantum information
14:30-14:55 Jurij Volcic
Post-hoc self-testing of quantum measurements
15:00-15:25 Arthur Mehta
New Approaches to Complexity via Quantum Graphs
15:30-15:55 Anna Skripka
Statistical inference for fermionic quantum time series

Monday 16:30-19:00 (PSR4) chair: Anand Natarajan

16:30-16:55 Bruno Nachtergaele
Ground state gap stability in the GNS representation
17:00-17:25 Hamza Fawzi
Certified algorithms for equilibrium states of local quantum Hamiltonians
17:30-17:55 Angela Capel
Quantum Markov Semigroups and Modified Logarithmic Sobolev Inequalities
18:00-18:25 Omar Fawzi
Capacities of quantum Markovian noise for large times

Tuesday 14:00-16:00 (PSR4) chair: Anna Skripka

14:00-14:25 Anand Natarajan
Bounding the quantum value of compiled nonlocal games: from CHSH to BQP verification
14:30-14:55 Yantian Zhang
Succinct arguments for QMA from standard assumptions via compiled nonlocal games
15:00-15:25 Simon Schmidt
On the quantum value of compiled nonlocal games
15:30-15:55 Tobias Fritz
An approach to homological algebra up to ε

Tuesday 16:30-19:00 (PSR4) chair: Tobias Fritz

16:30-16:55 Tim Netzer
Beyond Operator Systems
17:00-17:25 James Pascoe
Noncommutative change of variables
17:30-17:55 David E. Roberson
Quantum isomorphism and the NPA hierarchy
18:00-18:25 Josse van Dobben de Bruyn
Asymmetric graphs with quantum symmetry

Wednesday 11:40-12:40 (SIBSR2) chair: David Gross

11:40-12:05 Bill Helton
3XOR Games: Critical Thresholds for Solvability and Quantum Solvability

12:10-12:35 Seyed Sajjad Nezhadi
Quantum Perfect Matching Games

Wednesday 14:00–15:00 (SIBSR2) chair: David Gross

14:00-14:25 Miguel Navascués
First-order optimality conditions for non-commutative optimization problems

14:30-14:55 Sander Gribling
Bounding the separable rank via polynomial optimization

4.8.1. *Abstracts.*

Angela Capel, University of Cambridge

Quantum Markov Semigroups and Modified Logarithmic Sobolev Inequalities

Abstract. A dissipative evolution of an open quantum many-body system weakly coupled to an environment can be modelled by a quantum Markov semigroup, and its mixing time can be bounded using optimal constants of certain quantum functional inequalities, such as the modified logarithmic Sobolev constant. In this talk, we will review the mathematical formalism of dissipative evolutions governed by Lindbladians, and we will summarize the current state of the art on mixing times when the system has an associated commuting Hamiltonian.

References

[1] J. Kochanowski, Á. M. Alhambra, Á. Capel and C. Rouzé, *Rapid thermalization of dissipative many-body dynamics of commuting Hamiltonians*, **arXiv: 2404.16780**, 2024,
[2] I. Bardet, Á. Capel, L. Gao, A. Lucia, D. Pérez-García and C. Rouzé, *Entropy decay for Davies semigroups of a one dimensional quantum lattice*, **Communications in Mathematical Physics**, 405, 42, 2024 .

Omar Fawzi, Inria, ENS Lyon

Capacities of quantum Markovian noise for large times

Abstract. Given a quantum Markovian noise model, we study the maximum dimension of a classical or quantum system that can be stored for arbitrarily large time. We show that, unlike the fixed time setting, in the limit of infinite time, the classical and quantum capacities are characterized by efficiently computable properties of the peripheral spectrum of the quantum channel. In addition, the capacities are additive under tensor product. Based on joint work with Mizanur Rahaman and Mostafa Taheri.

Tobias Fritz, University of Innsbruck

An approach to homological algebra up to ε

Abstract. A theorem of Kazhdan on approximate representations of groups is based on a proof which seems to use cohomological methods "up to ε". This means that being a cocycle or a coboundary is not a yes/no-property of a cochain C, but rather a quantitative statement where one measures how strongly C deviates from being either. Based on Grandis's work on nonabelian homological algebra, I will present a framework for such quantitative homological algebra and sketch the intuition behind the resulting definitions of kernel and cokernel.

Unfortunately, the resulting category does not satisfy the axioms required of homological categories in Grandis's sense. Our main result solves this problem by showing that an arrow category is a homological category already under very weak assumptions. It follows that versions of derived functors and long exact sequences can be constructed for arrow categories quite generally, and this applies to quantitative homological algebra in particular.

References

[1] T. Fritz, *Non-abelian and ε-curved homological algebra with arrow categories*, J. Pure Appl. Algebra **227**, (2023), 107314.
[2] M. Grandis, *Homological Algebra: In Strongly Non-Abelian Settings*, World Scientific, Singapore, 2013.
[3] D. Kazhdan, *On ε-representations*, Israel J. Math. **43**, (1982), 315–323.

Sander Gribling, Tilburg University

Bounding the separable rank via polynomial optimization

Abstract. In this talk we consider the set $\mathcal{SEP}_d$ consisting of the linear maps ρ acting on $\mathbb{C}^d \otimes \mathbb{C}^d$ that can be written as a convex combination of rank-one matrices of the form $xx^* \otimes yy^*$, i.e., the set of separable states. We first revisit the well-known Doherty-Parrilo-Spedalieri hierarchy from the perspective of moment techniques, giving a new proof of convergence. We then introduce a new hierarchy of outer approximations based on lower bounding the *separable rank* of the state ρ.

References

[1] S. Gribling, M. Laurent, A. Steenkamp, Bounding the separable rank via polynomial optimization, *Lin. Alg. Appl.* **648**, (2022), 1–55.

Hamza Fawzi, University of Cambridge

Certified algorithms for equilibrium states of local quantum Hamiltonians

Abstract. Predicting observables in equilibrium states is a central yet notoriously hard question in quantum many-body systems. In the physically relevant thermodynamic limit, certain mathematical formulations of this task have even been shown to result in undecidable problems. Using a finite-size scaling of algorithms devised for finite systems often fails due to the lack of certified convergence bounds for this limit. In this work, we design certified algorithms for computing expectation values of observables in the equilibrium states of local quantum Hamiltonians, both at zero and positive temperature. Importantly, our algorithms output rigorous lower and upper bounds on these values. This allows us to show that expectation values of local observables can be approximated in finite time, contrasting related undecidability results. When the Hamiltonian is commuting on a 2-dimensional lattice, we prove fast convergence of the hierarchy at high temperature and as a result for a desired precision ϵ, local observables can be approximated by a convex optimization program of quasi-polynomial size in $1/\epsilon$.

Based on joint work with Omar Fawzi and Samuel Scalet (arXiv:2311.18706).

Bill Helton, UC San Diego

3XOR Games: Critical Thresholds for Solvability and Quantum Solvability

Abstract. Most of the talk will be devoted to proving that a particular function h of six variables is nonpositive. The approach envolves many cases and calculations which reduce the problem to analysis of one dimensional functions. These can be successfully bounded above by interval arithmetic.

The motivation for this is to determine if a 3 XOR game (asymptotically) has a sharp phase transition between solvability and nonsolvability as the number of constraints vs unknowns increase. Also what is the critical threshold for the phase transition? Our work parallels the approach by Dubois-Mandler 2003 which solved the classical 3 SAT problem (and has some connection to spin glass models). However, the formulas are much more complicated.

To back up further, while the talk is on purely classical mathematics, it arose from ways to find perfect quantum strategies to games, in work with Adam Bene Watts, Igor Klep and Zehong Zhang.

Arthur Mehta

New Approaches to Complexity via Quantum Graphs

Abstract. Problems based on the structure of graphs — for example finding cliques, independent sets, or colourings — are of fundamental importance in classical complexity. In this work, we introduce and study the clique problem for quantum graphs presented as quantum channels. We show that, by varying the collection of channels in the language, these give rise to complete problems for the classes NP, MA, QMA, and QMA(2). In this way, we exhibit a classical complexity problem whose natural quantisation is QMA(2), rather than QMA, which is commonly assumed. This talk is based on work with Eric Culf.

Bruno Nachtergaele, University of California, Davis

Ground state gap stability in the GNS representation

Abstract. Quantum many-body systems describe a wide variety of fascinating phenomena in the physical world. The last decade has seen impressive progress in our understanding of these systems. Notable topics are the classification of symmetry protected and symmetry enhanced topological phases, the fractional quantum Hall effect, and the stability of spectral gaps for many-body quantum systems. As a rule, these topics call for the analysis of arbitrarily large systems or infinite systems. While many works restrict themselves to obtain results for finite systems that are uniform in the system size, studies of infinite systems are increasingly common and are often preferred. Analysis of infinite many-body systems leads one to transition from the standard algebraic setting to the Hilbert space description provided by the GNS representation. In this talk I will explain a recent result proving stability of the ground state gap of infinite quantum spin systems (joint work with Robert Sims and Amanda Young [1]).

References

[1] B. Nachtergaele, Robert Sims, and Amanda Young, *Stability of the bulk gap for frustration-free topologically ordered quantum lattice systems*, Lett. Math. Phys. **114**, 24 (2024).

Anand Natarajan, Massachusetts Institute of Technology

Bounding the quantum value of compiled nonlocal games: from CHSH to BQP verification

Abstract. In the classical world, an extremely fruitful technique for constructing interactive protocols is "compiling" a multiprover game, using cryptography to simulate the separation between the provers. In the quantum world, the study of compiled nonlocal games was introduced by Kalai et al. (STOC'23), who defined a compilation procedure that applies to any nonlocal game and preserves the *classical* value; however, they did not show any bounds on the *quantum* value of their protocols. In this work, we make progress towards a full understanding of the quantum value of compiled nonlocal games. For the special case of the CHSH game, we show that the Tsirelson bound holds for the compiled game in two ways: by extending the "macroscopic locality" argument of Rohrlich, and by showing that strategies for the compiled game yield feasible solutions to the Tsirelson SDP. We conjecture that the latter argument can be extended to all XOR games. Using our SDP argument, we are able to recover a strong version of the "rigidity" property that makes CHSH so useful in applications; specifically, we show that compiled CHSH is a "computational self-test" in the sense of Metger and Vidick. As an application, we give a classical verification protocol for BQP based on a compiled nonlocal game and prove soundness. Our protocol replicates the functionality of Mahadev '18 but with two advantages: (1) the soundness analysis is much simpler, and directly follows the analysis of the nonlocal case, and (2) the soundness does not "explicitly" use the assumption of a TCF or an adaptive hardcore bit, and only requires QFHE as a black box (though currently the only known constructions of QFHE use TCFs).

Miguel Navascués

First-order optimality conditions for non-commutative optimization problems

Abstract. We consider the problem of optimizing the state average of a polynomial of non-commuting variables, over all states and operators satisfying a number of polynomial constraints, and over all Hilbert spaces where such states and operators are defined. Such non-commutative polynomial optimization (NPO) problems are routinely solved through hierarchies of semidefinite programming (SDP) relaxations. By phrasing the general NPO problem in Lagrangian form, we heuristically derive, via small variations on the problem variables, state and operator optimality conditions, both of which can be enforced by adding new positive semidefinite constraints to the SDP hierarchies. State optimality conditions are satisfied by all Archimedean (that is, bounded) NPO problems, and allow enforcing a new type of constraints: namely, restricting the optimization over states to the set of common ground states of an arbitrary number of operators. Operator optimality conditions are the non-commutative analogs of the Karush–Kuhn–Tucker (KKT) conditions, which are known to hold in many classical optimization problems. In this regard, we prove that a weak form of non-commutative operator optimality holds for all Archimedean NPO problems; stronger versions require the problem constraints to satisfy some qualification criterion, just like in the classical case. We test the power of the new optimality conditions by computing local properties of ground states of many-body spin systems and the maximum quantum violation of Bell inequalities.

References

[1] Mateus Araújo, Igor Klep, Andrew J. P. Garner, Tamás Vértesi, Miguel Navascués, First-order optimality conditions for non-commutative optimization problems, *arXiv:2311.18707*

Tim Netzer, University of Innsbruck

Beyond Operator Systems

Abstract. We generalize the setup of finite-dimensional operator systems to so-called *cone systems*, and prove the most important results in this general setup. This has several interesting consequences. We obtain a vector-valued Krein extension theorem, prove that the free mapping cones of positive maps and completely positive maps are not finitely generated, and deduce that there exists a self-dual functorial tensor products for finite-dimensional cones and for finite-dimensional operator systems.

This is joint work with Gemma De les Coves and Mirte van der Eyden.

James Eldred Pascoe, Drexel University

Noncommutative change of variables

Abstract. We give an overview of the change of variables theory for natural functions of several operator variables with emphasis on contributions from a variety of authors. Specializing, Augat's Ax-Grothendieck theorem for free polynomials as functions on tuples of matrices of arbitrary size gives that an invertible free polynomial map has a free polynomial inverse. If one instead evaluates over all linear maps on complex vector spaces, Augat's theorem is grossly facilitated and extensions to rational maps are available.

Seyed Sajjad Nezhadi, University of Maryland

Quantum Perfect Matching Games

Abstract. I will talk about the notion of a quantum or no-signaling perfect matching (as well as fractional matching and bipartite matching) for graphs. These are defined via nonlocal games inspired by the homomorphism game which are classically winnable with probability 1 if and only if the graph has the matching properties. I will talk about when these games have quantum and no-signaling advantages. In particular, for bipartite and fractional matching I will provide an algorithm that checks if the game has a perfect no-signaling strategy, and show that while there can be a quantum advantage for these games there is no possibility for pseudo-telepathy. For perfect matching of general graphs I will show that all three models define distinct properties and provide some results that characterize when a game is quantum or no-signaling winnable.

Simon Schmidt, Ruhr University Bochum

On the quantum value of compiled nonlocal games

Abstract. Nonlocal games are a foundational tool for understanding entanglement and constructing quantum protocols in settings with spatially separated quantum devices. Recently, Kalai et al. (STOC '23) defined a cryptographic compilation procedure for nonlocal games. It translates a (two-player) nonlocal game into a single-player game, using cryptography to simulate the spatial separation between the players. This talk will be about the quantum value of such compiled nonlocal games. We will discuss techniques for bounding the quantum value and see when the value is preserved under the compilation procedure.

Anna Skripka, University of New Mexico

Statistical inference for fermionic quantum time series

Abstract. We will discuss asymptotic equivalence of quantum statistical experiments pertaining to certain gauge invariant quasifree states on a fermionic Fock space to certain classical experiments. The respective asymptotic equivalence is understood in the sense of the quantum Le Cam distance measuring the least trace-norm error incurred while mapping one model into another via quantum channels. The obtained result is applied to construct an asymptotically optimal estimator of an unknown state parameter. The talk is based on joint work with M. Nussbaum.

Josse van Dobben de Bruyn, Technical University of Denmark (DTU)

Asymmetric graphs with quantum symmetry

Abstract. The quantum symmetries of a graph form a C^*-algebraic compact quantum group, called the *quantum automorphism group* of the graph. In recent years, various authors have asked variations of the following question: how far can the classical and quantum automorphism group of a graph be apart? In this talk, after presenting some background on C^*-algebraic quantum groups, I will answer one of these questions by presenting a class of graphs which have trivial (classical) automorphism group and non-trivial quantum automorphism group. The construction is inspired by solution groups of binary linear systems, as defined by Cleve, Liu and Slofstra [1] in the context of linear constraint system games. This talk is based on joint work with David E. Roberson and Simon Schmidt [2].

References

[1] R. Cleve, L. Liu, W. Slofstra, Perfect commuting-operator strategies for linear system games, *J. Math. Phys.* **58** (2017), 012202.
[2] J. van Dobben de Bruyn, D. E. Roberson, S. Schmidt, *Asymmetric graphs with quantum symmetry*, Preprint, arXiv:2311.04889 (2023).

I would like to thank the Carlsberg Foundation for their generous support.

Jurij Volčič, Drexel University

Post-hoc self-testing of quantum measurements

Abstract. Self-testing is the strongest form of quantum functionality verification, which allows one to deduce the quantum state and measurements of an entangled system from its classically observed statistics. From a mathematical perspective, self-testing is an intriguing uniqueness phenomenon, pertaining to functional analysis, moment problems, convexity and representation theory. This talk restricts itself to self-testing in bipartite systems. It focuses on a criterion for extending self-testing results when measurements are added to the system. This criterion in particular implies that every collection of real projective-valued measurements can be self-tested in a bipartite scenario. Based on joint work with Ranyiliu Chen and Laura Mančinska.

Reinhard F. Werner, Leibniz Universität Hannover, Germany

Von Neumann algebraic quantum information

Abstract. General von Neumann algebras appear as the observable algebras of systems with infinitely many degrees of freedom, as in quantum field theory, statistical mechanics in the thermodynamic limit, or as idealized resources in quantum information theory. Their structure has a direct relevance for the type of entanglement possible in such situations. I will report some old and some recent results in this regard.

References

[1] L. van Luijk, A. Stottmeister, R. F. Werner, and H. Wilming, Embezzlement of entanglement, quantum fields, and the classification of von Neumann algebras, arXiv:2401.07299
[2] L. van Luijk, A. Stottmeister, and H. Wilming, Critical Fermions are universal embezzlers, arXiv:2406.11747
[3] S.J. Summers and R.F. Werner, Maximal violation of Bell's inequalities for algebras of observables in tangent spacetime regions. *Ann. Inst. H. Poincaré A* **49** (1988) 215-243.

Tina Zhang, MIT

Succinct arguments for QMA from standard assumptions via compiled nonlocal games

Abstract. We construct a succinct classical argument system for QMA, the quantum analogue of NP, from generic and standard cryptographic assumptions. Previously, building on the prior work of Mahadev (FOCS '18), Bartusek et al. (CRYPTO '22) also constructed a succinct classical argument system for QMA. However, their construction relied on post-quantumly secure indistinguishability obfuscation, a very strong primitive which is not known from standard cryptographic assumptions. In contrast, the primitives we use (namely, collapsing hash functions and a mild version of quantum homomorphic encryption) are much weaker and are implied by standard assumptions such as LWE. Our protocol is constructed using a general transformation which was designed by Kalai et al. (STOC '23) as a candidate method to compile any quantum nonlocal game into an argument system. Our main technical contribution is to analyze the soundness of this transformation when it is applied to a succinct self-test for Pauli measurements on maximally entangled states, the latter of which is a key component in the proof of MIP*=RE in quantum complexity.

References

[1] Tony Metger, Anand Natarajan, Tina Zhang, S̈uccinct arguments for QMA from standard assumptions via compiled nonlocal games, https://arxiv.org/abs/2404.19754.

4.9. **Noncommutative Geometry and Operator Spaces.**
Organizers: Evgenios Kakariadis (Newcastle), Haluk Şengün (Sheffield)

Monday 14:00-16:00 (PSR2) chair: Haluk Şengün

14:00-14:25 Walter van Suijlekom
Operator systems and noncommutative geometry
14:30-14:55 Jens Kaad
Spectral metrics on quantum projective spaces
15:00-15:25 Michael Rosbotham
Trace-class operators on Hilbert modules and the Haagerup tensor product
15:30-15:55 Yufan Ge
SU(2)-symmetries of C^*-algebras: from bricks to buildings

Monday 16:30-19:00 (PSR2) chair: Jens Kaad

16:30-16:55 Ivan Todorov
Quantum game values
17:00-17:25 Shintaro Nishikawa
Exploring reductive symmetric spaces through Noncommutative Geometry
17:30-17:55 Malte Leimbach
Peter–Weyl truncations of compact quantum groups
18:00-18:25 Haluk Şengün
Local theta correspondence via C^*-algebras of groups

Tuesday 14:00-16:00 (PSR2) chair: Evgenios Kakariadis

14:00-14:25 Chris Bruce
C-envelopes of semigroup operator algebras: beyond group-embeddable semigroups*
14:30-14:55 Christian Bonicke
Dynamic asymptotic dimension and its applications to C^*-algebras
15:00-15:25 Bhumi Amin
Completely positive maps: pro-C^*-algebras and Hilbert modules over pro-C^*-algebras
15:30-15:55 Christian De Nicola Larsen
The Haagerup property via Jones' technology

Tuesday 16:30-19:00 (PSR2) chair: Christian Bonicke

16:30-16:55 Matt Kennedy
Intermediate subalgebras for reduced crossed products of discrete groups
17:00-17:25 Kevin Aguyar Brix
C-envelopes and left-cancellative small categories*
17:30-17:55 Evgenios Kakariadis
Operator algebras of product systems

4.9.1. *Abstracts.*

Bhumi Amin, Indian Institute of Technology Hyderabad

Completely Positive Maps: Pro-C^*-algebras and Hilbert Modules over Pro-C^*-algebras

Abstract. We present a construction for induced representations on Hilbert modules over pro-C^*-algebras for a given continuous *-morphism between pro-C^*-algebras. Subsequently, we describe the structure of completely positive maps between two pro-C^*-algebras using Paschke's GNS construction for CP-maps on pro-C^*-algebras. Furthermore, through our devised construction, we establish a structure theorem for a $\phi-$map between two Hilbert modules over pro-C^*-algebras, where the map ϕ is a continuous CP-map between pro-C^*-algebras. We also discuss the minimality of these representations.

References

[1] M. B. Asadi, "Stinespring theorem for Hilbert C*-modules", J. Operator Theory 62, (2009), 235-238.
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[5] M. Joita, "Hilbert modules over locally C*-algebras", University of Bucharest Press, 2006.
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[8] Maliev, I.N., Pliev, M.A.," A stinespring type representation for operators in hilbert modules over local C*-algebras.", Russ Math. 56, 43–49 (2012).
[9] W. L. Paschke, "Completely Positive Maps on U^*-Algebras", Proceedings of the American Mathematical Society 34, no. 2 (1972).
[10] W. L. Paschke, "Inner product modules over B^*-algebras", Trans. Amer. Math. Soc. 182 (1973), 443-468.
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Christian Bönicke, Newcastle University

Dynamic asymptotic dimension and its applications to C**-algebras*

Abstract. Dynamic asymptotic dimension is a dimension theory for topological dynamical systems introduced by Guentner, Willett, and Yu in 2017. It makes sense for global dynamical systems in the form of actions of discrete groups on compact Hausdorff spaces, but it is most naturally formulated in the language of topological groupoids. This makes the theory applicable in a variety of different contexts. Since its inception, the concept has found several applications

to the structure theory of C*-algebras as well as homology and K-theory. In this talk I will give an introduction to the theory and provide an overview over some of its applications relevant to operator algebraists.

Kevin Aguyar Brix, University of Lund

C-envelopes and left-cancellative small categories*

Abstract. The C*-envelope provides a noncommutative analogue of the Shilov boundary in function theory and it establishes a deep and interesting connection between C*-algebras and non-self adjoint operator algebras. I will discuss joint work with Chris Bruce and Adam Dor-On in which we prove that any normal coaction on an operator algebra extends to its C*-envelope. As a consequence, we can identify the C*-envelope in many example classes of operator algebras; I will particularly discuss the case coming from groupoid-embeddable categories. Chris Bruce will discuss other applications of our results.

Chris Bruce, Newcastle University

C-envelopes of semigroup operator algebras: beyond group-embeddable semigroups.*

Abstract. The problem of computing the C*-envelope of semigroup operator algebras has been studied for several decades. For group-embeddable semigroups, it was resolved in full generality by Sehnem, after the works of many others. I will present on joint work with Kevin Aguyar Brix and Adam Dor-On in which we compute the C*-envelope for all cancellative right LCM monoids, a class that contains many non group-embeddable semigroups, where previously no information about the C*-envelope was known. Our approach relies on a uniqueness theorem by Starling and on our general theorem that every normal coaction of a discrete group on an operator algebra extends to a (unique) normal coaction on the C*-envelope of the operator algebra. This general result on coactions which will be presented in the talk by K.A. Brix.

Christian De Nicola Larsen, UNSW Sydney

The Haagerup property via Jones' Technology

Abstract. Vaughan Jones developed a powerful technology for constructing representations of groups such as Richard Thompson's groups $F \subseteq T \subseteq V$ while exploring the links between subfactors and conformal field theory.

We will discuss new techniques for using Jones' technology to prove that Thompson's group V has the Haagerup property (first proven by Farley geometrically in 2003). This unifies previous analytical proofs for F and T by Jones and Brothier.

We will also discuss an application of our techniques to certain wreath products with Thompson's groups. These wreath products are the first non-trivial examples of finitely presented wreath products with the Haagerup property, and we obtain explicit formulae for some interesting positive definite functions on them. This significantly simplifies the proof due to Brothier that they have the Haagerup property. We end the talk with some future directions for this approach.

This is joint work with Dilshan Wijesena.

Yufan Ge, Leiden University

$SU(2)$-symmetries of C^*-algebras: from bricks to buildings.

Abstract. In this talk, we will consider subproduct systems coming from $SU(2)$-representations and discuss the associated C*-algebras. We will first review results concerning irreducible representations from Arici–Kaad, then provide some further results about more general cases. More specifically, we will discuss the structure of the $SU(2)$-subproduct systems associated to isotypic representations and multiplicity-free representations as well as the induced Toeplitz algebras. Finally, we will provide results about the K-theory groups of their Toeplitz algebras. This is joint work in progress with Francesca Arici.

Jens Kaad, University of Southern Denmark

Spectral metrics on quantum projective spaces

Abstract. A spectral metric space is a unital spectral triple satisfying that the coordinate algebra becomes a compact quantum metric space via the seminorm which measures the size of first order derivatives.

In this talk, we investigate the spectral metric properties of quantum projective spaces. The geometric framework for this investigation is provided by the unital spectral triples introduced by D'Andrea and Dabrowski in their CMP paper from 2010. We shall see that these unital spectral triples are in fact spectral metric spaces and, if time permits, indicate how this can be proved.

This result makes it possible to investigate the spectral metric continuity properties of quantum projective spaces under variations of the deformation parameter q. It can moreover be viewed as a first step for understanding the higher Vaksman-Soibelman spheres from the point of view of spectral metric spaces.

The talk is based on joint work with Max Holst Mikkelsen.

References

[1] M. H. Mikkelsen, J. Kaad, Spectral metrics on quantum projective spaces, *J. Funct. Anal.* **287**, (2024).
[2] F. D'Andrea, L. Dabrowski, Dirac operators on quantum projective spaces, *Comm. Math. Phys.* **295**, (2010), no. 3, 731–790.

Evgenios Kakariadis, Newcastle University

Operator algebras of product systems

Abstract. Product systems provide a common language and context to encode geometric structures such as semigroups, graphs, dynamics etc. Their operator algebras have been under thorough study in the past 40 years with much success giving rise to two objects: (a) the maximal Fock-covariant C*-algebra, and (b) the minimal strong covariant C*-algebra. There are further links between these objects by using the nonselfadjoint operator algebra generated in the Fock representation. In this talk I will present results concerning the parametrisation of gauge-invariant ideals for product systems over $\mathbb{Z}_+^d$ (joint work with Joe Dessi) and the reduced Hao-Ng isomorphism problem for general product systems by a discrete action (joint work with Apollonas Paraskevas).

Matthew Kennedy, University of Waterloo

Intermediate subalgebras for reduced crossed products of discrete groups

Abstract. We obtain sufficient and (almost always) necessary conditions for a complete characterization of intermediate subalgebras of reduced crossed products of discrete groups. I will discuss this result and briefly mention some of the key underlying ideas. This is joint work with Dan Ursu.

Malte Leimbach, Radboud University Nijmegen

Peter–Weyl truncations of compact quantum groups

Abstract. A compact quantum group (A, Δ) can be compressed to an operator system PAP, for a projection P onto direct summands of the Peter–Weyl decomposition. The comultiplication Δ induces a coaction of the C*-algebra A on the compression PAP. When equipped with suitable Lip-norms, A and PAP become compact quantum metric spaces. We consider convergence $PAP \to A$ in quantum Gromov–Hausdorff distance, as the range of the projection P increases appropriately. In particular, we discuss the relevance of invariance of the involved Lip-norms for the coactions and a Lip-norm estimate of slice maps using the Monge–Kantorovich distance on the state space of A.

Shintaro Nishikawa, University of Southampton

Exploring Reductive Symmetric Spaces through Noncommutative Geometry

Abstract. A homogeneous space G/H is called a reductive symmetric space if G is a (real) reductive Lie group and H is a symmetric subgroup of G, fixed by some involution on G. The representation theory of reductive symmetric spaces was extensively studied in the 1990s by Erik van den Ban, Patrick Delorme, Henrik Schlichtkrull, and others, particularly achieving the Plancherel formula for the L2-space of G/H. This work generalises the group case established by Harish-Chandra in the early 70s, where G = G' x G' and H is the diagonal subgroup. In our collaborative research with A. Afgoustidis, N. Higson, P. Hochs, and Y. Song, we explore this topic from the perspective of noncommutative geometry. I will present these exciting new developments, highlighting the novel aspects and differences from the traditional group case, especially concerning the reduced group C*-algebra of G.

Michael Rosbotham, University of Maine

Trace-class Operators on Hilbert Modules and the Haagerup Tensor Product

Abstract. In 2021, Stern and van Suijlekom used frames to define and study the trace class, and other Schatten classes, for operators on Hilbert modules [1]. They showed that, when the modules are over a commutative C*-algebra, many key features carry over from the Hilbert space case.

In this talk, I discuss joint work with Tyrone Crisp [2] where we show that the space of trace-class operators on a Hilbert module over a commutative C*-algebra is completely isometrically isomorphic to a Haagerup tensor product of the module with its operator-theoretic adjoint. This generalises a well-known property of Hilbert spaces.

References

[1] A. B. Stern and W. D. van Suijlekom, Schatten classes for Hilbert modules over commutative C*-algebra, *J. Funct. Anal.*, 281(4):Paper No. 109042, 35, 2021.
[2] T. Crisp, M. Rosbotham, Trace-class operators on Hilbert modules and the Haagerup tensor product, https://arxiv.org/abs/2403.00449.

Guy Salomon, Weizmann Institute of Science

Noncommutative ergodic theory and SAT actions

Abstract. I will give an introduction to noncommutative ergodic theory and discuss a new framework of (commutative) ergodicity based on SAT (strongly approximate transitive) actions. These will give rise to a noncommutative generalization of a theorem of Nevo and Zimmer about factors of certain nonsingular actions that naturally arise when studying lattices of higher rank Lie groups. I will explain the main ideas behind its proof, which are new even in the commutative case.
The talk is based on joint work with Uri Bader.

Haluk Şengün, University of Sheffield

Local theta correspondence via C^*-algebras of groups

Abstract. Theta correspondence is a major theme in the theory of automorphic forms and in representation theory. The local version of the correspondence sets up a bijection between certain subsets of admissible duals of suitable pairs of reductive groups.

In joint work with Bram Mesland (Leiden), we show that in certain cases, there is a Morita equivalence of certain C^*-algebras that underlies the local theta correspondence. There are interesting applications and some global questions that follow this result, some of which will be discussed if time remains.

Ivan Todorov, University of Delaware

Quantum game values

Abstract. In this talk I will present a tensor norm expression for the quantum commuting value of non-local games with quantum inputs and quantum outputs. This expression complements well-known tensor norm formulas for the quantum value of a quantum non-local game, obtained previously by Cooney-Junge-Palazuelos-Perez-Garcia. Our approach places quantum game values in a general framework that allows us to view quantum game values as a special case of values of probabilistic hypergraphs. The talk is based on a joint work with Jason Crann and Lyudmila Turowska.

Walter van Suijlekom, Radboud University Nijmegen

Operator systems and noncommutative geometry

Abstract. We give an overview of the recent interactions between operator systems and noncommutative differential geometry. We will find that the traditional role played by C*-algebras is taken over by so-called operator systems. Essentially, this is the minimal structure required on a space of operators to be able to speak of positive elements, states, pure states, etc. We consider C*-envelopes and introduce a propagation number for operator systems, which we show to be an invariant under stable equivalence and use it to compare approximations of the same space. Examples are spectral truncations associated to a Dirac or Laplace operator on a Riemannian (spin) manifold.

4.10. **Random Matrices and Free Probability.**
Organizers: Ian Charlesworth (Cardiff), Håkan Hedenmalm (KTH Stockholm), Torben Krüger (FAU Erlangen-Nürnberg), Jani Virtanen (Reading)

Tuesday 14:00-16:00 (SIBSR6) chair: TBC

14:00-14:25 Daniel Perales
Finite free probability and the S-transform
14:30-14:55 Jacob Campbell
Even hypergeometric polynomials and finite free probability
15:00-15:25 Charles-Philippe Diez
TBA
15:30-15:55 Daniel Munoz
Asymptotic limit of cumulants of complex Wigner matrices

Wednesday 11:40-12:40 (PSR4) chair: Jani Virtanen

11:40-12:05 Tom Claeys
Large deviations for the log-Gamma polymer
12:10-12:35 Arno Kuijlaars
Orthogonal polynomials in a normal matrix model with two insertions

Wednesday 14:00–15:00 (PSR4) chair: Jani Virtanen

14:00-14:25 Santeri Miihkinen
Double-scaling limits of Toeplitz determinants with emerging Fisher-Hartwig singularities
14:30-14:55 Håkan Hedenmalm
Soft Riemann-Hilbert problems and random matrix theory

Thursday 14:00-16:00 (PSR4) chair: Ian Charlesworth

14:00-14:25 Paul Skoufranis
Bi-Free Independence and the Asymptotics of Tensor Random Matrices
14:30-14:55 Patrycja Hęćka
The Boolean quadratic forms and tangent law
15:00-15:25 Claus Köstler
Jones-Temperley-Lieb algebras from the viewpoint of distributional invariance principles
15:30-15:55 Natasha Blitvic
TBA

Thursday 16:30–18:30 (PSR4) chair: Håkan Hedenmalm

16:30-16:55 Yacin Ameur
The Coulomb gas near a spectral outpost
17:00-17:25 Oleksii Kolupaiev
Loschmidt echo for deformed Wigner matrices
17:30-17:55 Volodymyr Riabov
Eigenstate Thermalization for Wigner-type Matrices
18:00-18:25 Sang-Jun Park
Tensor-freeness and central limit theorem

Friday 14:00-16:00 (SIBLT3) chair: Torben Krüger

14:00-14:25 Johannes Alt
TBA

14:30-14:55 Félix Parraud
Interpolation between random matrices and free operators

15:00-15:25 Hong Chang Ji
Spectral edge of non-Hermitian random matrices

15:30-15:55 David Renfrew
Eigenvalues of minors of random matrices and roots of derivatives of random polynomials

4.10.1. *Abstracts.*

Yacin Ameur, University of Lund

The Coulomb gas near a spectral outpost

Abstract. We consider two-dimensional Coulomb systems in the regime when the droplet is connected, while the coincidence set for the obstacle problem contains an analytic Jordan curve outside of the droplet. A nontrivial (Heine-distributed) number of particles will tend to fall in the vicinity of this curve, which we denote "spectral outpost". Under the process of Laplacian growth, the outpost grows into a new ring-shaped component of the droplet, and the study of outposts is therefore closely related to the regime of disconnected droplets. We study among other things fluctuations of linear statistics. The talk is based on joint works with Joakim Cronvall and Christophe Charlier.

Jacob Campbell, University of Virginia

Even hypergeometric polynomials and finite free probability

Abstract. In 2015, Marcus, Spielman, and Srivastava realized that expected characteristic polynomials of sums and products of randomly rotated matrices behave like finite versions of Voiculescu's free convolution operations. In 2022, I obtained a similar result for commutators of such random matrices; one feature of this result is the special role of even polynomials, in parallel with the situation in free probability.

It turns out that a certain family of special polynomials, called hypergeometric polynomials, arises naturally in relation to convolution of even polynomials and finite free commutators. I will explain how these polynomials can be used to approach questions of real-rootedness and asymptotics for finite free commutators, and how they provide a systematic framework for the main examples. Based on arXiv:2209.00523 and ongoing joint work with Rafael Morales and Daniel Perales.

Tom Claeys, UCLouvain

Large deviations for the log-Gamma polymer

Abstract. I will present a conjecture about large deviations of the partition function of the log-Gamma polymer. I will show that the conjectured large deviation rate function matches with that of last passage percolation with exponential weights in the zero-temperature limit, and with the lower tail of the Tracy-Widom distribution for moderate deviations. We can nearly prove our result but there is one step in the proof for which we only have heuristic evidence. The talk will be based on joint work with Julian Mauersberger.

Patrycja He cka, Wrocław University of Science and Technology

The Boolean quadratic forms and tangent law

Abstract. Ejsmont and Lehner [3] study the limit sums of free commutators and anticommutators and show that the generalized tangent function

$$\frac{\tan z}{1 - x \tan z}$$

describes the limit distribution. This is the generating function of the higher order tangent numbers of Carlitz and Scoville [1] which arose in connection with the enumeration of certain permutations. I will talk about the limit of weighted sums of Boolean commutators and anticommutators and I will show that the shifted generalized tangent function appears in a limit theorem. In order to do this, I shall provide an arbitrary cumulants formula of the quadratic

form. I will also apply this result to show several theorems in a Boolean probability theory. The talk is based on joint work [2] with Wiktor Ejsmont.

References

[1] L. Carlitz, R. Scoville, Tangent numbers and operators, *Duke Math. J.* **39** (1972), 413–429.
[2] W. Ejsmont, P. Hećka, The Boolean quadratic forms and tangent law, *Random Matrices: Theory and Applications* **13** (2024), no. 1, Paper No. 2450004.
[3] W. Ejsmont, F. Lehner, The free tangent law, *Adv. Appl. Math.* **121** (2020), 102093.

Håkan Hedenmalm, KTH and Reading

Soft Riemann-Hilbert problems and random matrix theory

Abstract. It has been known since the 2007 paper by Its and Takhtajan that the orthogonal polynomials with respect to an exponentially varying weight in the plane are characterized in terms of a 2x2 matrix dbar-problem. In the recent breakthrough by Hedenmalm and Wennman the asymptotics of these planar orthogonal polynomials was found (Acta Math, 2021). However, the connection with the 2x2 dbar-problem was left open for further investigation. It turns out that there is a nice algorithm to find the asymptotics of the planar orthogonal polynomials in terms of the dbar-problem, which also has the benefit of supplying better error terms. This algorithm will be presented here. The work was published in CPAM 2024.

References

[1] H. Hedenmalm, Soft Riemann-Hilbert problems and planar orthogonal polynomials, *Comm. Pure Appl. Math.* **77**, (2024), 2413–2451.

Hong Chang Ji, Institute of Science and Technology Austria

Spectral edge of non-Hermitian random matrices

Abstract. We report recent progresses on spectra of so-called deformed i.i.d. matrices. They are square non-Hermitian random matrices of the form $A+X$ where X has centered i.i.d. entries and A is a deterministic bias, and A and X are on the same scale so that their contributions to the spectrum of $A+X$ are comparable. Under this setting, we present two recent results concerning universal (over A) patterns arising in eigenvalue statistics of $A+X$ around its boundary, on macroscopic and microscopic scales. The first result shows that the macroscopic eigenvalue density of $A+X$ typically has a jump discontinuity around the boundary, which is a distinctive feature of the eigenvalue density of X by *circular law*. The second result shows universality for the local eigenvalue statistics of $A+X$ around a typical (jump) boundary point, i.e. that the local statistics match with that of a Ginibre matrix X with i.i.d. standard Gaussian entries. Based on joint works with A. Campbell, G. Cipolloni, and L. Erdős.

References

[1] L. Erdős and H.C. Ji, Density of Brown measure of free circular Brownian motion, *arXiv:2307.08626.*
[2] A. Campbell, G. Cipolloni, A. Campbell, and H.C. Ji, On the spectral edge of non-Hermitian random matrices, *arXiv:2404.17512* .

Oleksii Kolupaiev, Institute of Science and Technology Austria

Loschmidt echo for deformed Wigner matrices

Abstract. We will discuss recent results on sensitivity of a quantum evolution to perturbations. Consider two self-adjoint Hamiltonians $H_1 \approx H_2$ and an initial quantum state ψ_0. First, evolve ψ_0 under the Hamiltonian H_1 from time zero to t, and then consider the backward evolution from t to zero under H_2, resulting in ψ_0'. The question is how precisely the initial state is recovered. One possible way to measure this revival is the Loschmidt echo $|\langle \psi_0, \psi_0' \rangle|^2$. We model H_1, H_2 by deformed Wigner matrices and show that the Loschmidt echo follows a universal behavior as a function of time. Our proof relies on a two-resolvent global law, i.e. a concentration estimate for products of resolvents of H_1 and H_2. The talk is based on a joint work with Laszlo Erdős and Joscha Henheik.

Arno Kuijlaars, KU Leuven, Belgium

Orthogonal polynomials in a normal matrix model with two insertions

Abstract. The talk is about the asymptotic behavior of polynomials $P_{n,N}$ with orthogonality in the complex plane

$$\int_{\mathbb{C}} P_{n,N}(z)\overline{z}^k |z^2+a^2|^{2cN} e^{-N|z|^2} dA(z) = 0, \quad k = 0, \dots, n-1,$$

with $c, a > 0$ and $dA(z)$ denotes planar Lebesgue measure. These polynomials are connected with a normal matrix model with external potential $N|z|^2 - 2cN \log|z^2+a^2|$ which is a modification of the Ginibre ensemble with two logarithmic singularities. The eigenvalues of the random matrices fill out a bounded region in the complex plane as $n, N \to \infty$ with $n/N \to t > 0$. We prove, that for a certain regime of parameters a, c, t, the zeros of the orthogonal polynomials tend to an interval on the real line, with an asymptotic density that is characterized by a vector equilibrium problem.

Our analysis essentially relies on the reformulation of the planar orthogonality as non-Hermitian multiple orthogonality and on a steepest descent analysis of the associated Riemann-Hilbert problem of size 3×3.

This is based on joint work with Mario Kieburg and Sampad Lahiry.

References

[1] M. Kieburg, A.B.J. Kujlaars, and S. Lahiri, Orthogonal polynomials in the normal matrix model with two insertions, preprint.

Santeri Miihkinen, University of Reading

Double-scaling limits of Toeplitz determinants with emerging Fisher-Hartwig singularities

Abstract. Asymptotic behaviour of determinants $D_n(f)$ of Toeplitz matrices generated by Fisher-Hartwig (F-H) symbols f has been under active investigation since the late 1970s by many authors. An interesting trend is to consider so-called double-scaling limits of Toeplitz determinants $D_n(f_t)$ with the F-H symbol f_t depending on some external parameter t (corresponding to e.g. temperature in applications.) A goal is to find an asymptotic formula for $D_n(f_t)$ when n is large and t is close to some critical value. In this talk, I review some background on this topic and present our recent results on computing the double-scaling limits of Toeplitz determinants with $m \geq 1$ fixed Fisher-Hartwig singularities located on the unit circle away from $z = 1$. They describe the transition between the asymptotic regimes of m singularities and $m+1$ singularities, where one singularity emerges at $z = 1$ as $t \to 0$. We consider both cases for the seminorm associated to the singularities of f_t being less than one or equal to one. This talk is based on joint work with Reham Alahmadi and Jani Virtanen (University of Reading).

Daniel Munoz, City University of Hong Kong

Asymptotic limit of cumulants of complex Wigner matrices

Abstract. During this talk we will look at the asymptotic limit of normalized cumulants of Traces of powers of complex Wigner matrices. We will prove that as the dimension of the matrices goes to infinity the large limit converges and its limit can be characterized by some graphs that result to be trees. If we regard this limit as moment sequences indexed by the powers of the Wigner matrices then another natural question is to find their corresponding higher order free cumulants. This question has already been answered in the first order case by the well known Wigner semicircle law and for second and third order in [1] and [2] respectively. We will see that the trees that show up in the limit can be counted using the set of non-crossing partitioned permutations which permit us to find the higher order free cumulants up to order 4. If time permits we will see that our methods can be used to find a convergence rate for the trace of a polynomial on GUE variables to the normal distribution.

References

[1] M. Camille, J.A. Mingo, S. Péché, R. Speicher, Joint Global Fluctuations of complex Wigner and deterministic Matrices, *Random Matrices: Theory Appl.* Vol. 11, No. 02, 2250015 (2022).
[2] J.A. Mingo, D. Munoz, Third order cumulants of Complex Wigner matrices, submitted. Arxiv preprint. 2205.13081
[3] J.A. Mingo, D. Munoz, Asymptotic limit of cumulants and higher order free cumulants of Complex Wigner matrices. Arxiv preprint 2407.17608

Sang-Jun Park, CNRS, Laboratoire de Physique Théorique

Tensor-freeness and central limit theorem

Abstract. Voiculescu's notion of asymptotic free independence applies to a wide range of random matrices, including those that are independent and unitarily invariant. In this talk, we generalize this notion by considering random matrices with a tensor product structure that are invariant under the action of local unitaries. We show that, given the existence of 'local moments' described by tuples of permutations, an independent family of locally unitarily invariant random matrices satisfies a new kind of freeness in the limit, which we will call 'local freeness'. This can be defined via vanishing mixed 'local-free cumulants', allowing the joint local moments of local-free elements to be described in terms of that of individual elements. Furthermore, we propose a local-free version of the central limit theorem, which extends and recovers several previous results. This is a joint work with Ion Nechita.

Daniel Perales, Texas A& M University

Finite free probability and the S-transform

Abstract. The finite free additive and multiplicative convolutions are binary operations of polynomials that behave well with respect to the roots. On one hand, they can be understood as a finite analogue of free probability that involves only discrete measures. On the other hand, these operations can be realized as expected characteristic polynomials of adding (or multiplying) two randomly rotated matrices.

In the first half of this talk we will introduce these polynomial convolutions and mention their basic properties.

For the second part of the talk we will focus on an ongoing project where we define a new finite S-transform. Among other things, this transform provides a simple way to obtain the limiting spectral distribution of a sequence of polynomials (with increasing degree) directly using their coefficients. As one of many applications, we can easily compute the limiting S-transform of hypergeometric polynomials. These large class of polynomial contains many of the important families of polynomials that naturally appear in finite free probability, such as Laguerre, Hermite and Jacobi.

References

[1] Octavio Arizmendi, Katsunori Fujie, Daniel Perales, Yuki Ueda, S-transform in finite free probability (work in progress).

David Renfrew, University of Binghamton

Eigenvalues of minors of random matrices and roots of derivatives of random polynomials

Abstract. I will describe the Brown measure of the free compression of R-diagonal operators, which also gives limiting behavior of the eigenvalues of minors of large bi-unitarily random matrices, and the roots of derivatives of polynomials with independent, random coefficients. These processes are given by fractional convolutions powers of the additive convolution of R-diagonal operators introduced by Kösters and Tikhomirov. This is joint work with Andrew Campbell and Sean O'Rourke.

References

[1] A. Campbell, S. O'Rourke, D. Renfrew, The fractional free convolution of R-diagonal elements and random polynomials under repeated differentiation, *IMRN* , (2024).

Volodymyr Riabov, ISTA

Eigenstate Thermalization for Wigner-type Matrices

Abstract. Following the typicality paradigm, random matrix models are ubiquitously used in studying properties of chaotic complex quantum systems. One such property is the Eigenstate Thermalization Hypothesis (ETH) positing that the eigenfunctions of a sufficiently chaotic quantum system become uniformly distributed in the phase space. Following the recent surge in understanding of the multi-resolvent local laws, the validity of ETH was rigorously established for the simplest model of a chaotic quantum system - the Wigner random matrix [1,2]. However, standard Wigner matrices (with essentially i.i.d. entries) only model quantum systems with no underlying spatial structure.

To mimic physically more realistic systems, we consider a much more general class of random matrices - the Wigner-type ensembles. The entries of a Wigner-type matrix follow a non-trivial variance profile, with potentially large vanishing blocks, which allows to encode a non-trivial spatial structure.

In this talk, we present our result on the Eigenstate Thermalization Hypothesis in the bulk spectrum of general Wigner-type matrices. Based on [3], joint work with László Erdős.

References

[1] G. Cipolloni, L. Erdős, and D. Schröder. Eigenstate Thermalization Hypothesis for Wigner Matrices. *Commun. Math. Phys.* **388**, (2021), 1005–1048.

[2] G. Cipolloni, L. Erdős, and J. Henheik, Eigenstate Thermalisation at the Edge for Wigner Matrices, arXiv:2309.05488, (2023).

[3] L. Erdős, R., Eigenstate Thermalization Hypothesis for Wigner-type Matrices, arXiv:2403.10359, (2024).

Supported by the ERC Advanced Grant “RMTBeyond” No. 101020331.

Paul Skoufranis, York University

Bi-Free Independence and the Asymptotics of Tensor Random Matrices

Abstract. It has been 10 years since the first paper on bi-free independence was published by Voiculescu. Since then, the theory of bi-free probability has been well developed. In this talk, a basic introduction to the main concepts of bi-free probability will be provided and work connecting bi-free probability to the asymptotics of tensor products of random matrices will be discussed.

References

[1] C. Lacien, P. O. Santos, P. Youssef, *Central Limit Theorem for tensor products of free variables*, preprint at arXiv:2404.19662, (2024), 25 pages.
[2] P. Skoufranis, Central Limit Theorem for Tensor Products of Free Variables via Bi-Free Independence, preprint at arXiv:2405.19216, (2024), 20 pages.

I would like to thank the Natural Sciences and Engineering Research Council of Canada for their generous support.

4.11. **Finite and Infinite Dimensional Moment Problems.**
Organizers: Salma Kuhlmann (Konstanz), Maria Infusino (Cagliari)

Monday 14:00-16:00 (SIBSR5) chair: Maria Infusino

14:00-14:25 Lawrence Fialkow
A core variety approach to the truncated moment problem on $y = x^d$

14:30-14:55 Dragu Atanasiu
Moment problems related to the intrinsic characterization of moment functionals in the compact case

15:00-15:25 Moritz Schick
Separation of the sums of squares cone and the sums of nonnegative circuit polynomials cone

15:30-15:55 Philipp di Dio
K-Positivity Preserver and their Generators

Tuesday 16:30-19:00 (SIBSR5) chair: Maria Infusino

16:30-16:55 Raul Curto
Mellin Transform and Exponential Polynomial Methods in the Study of the Square Root Problem for Positive Measures

17:00-17:25 Lorenzo Baldi
Trucated moment problems for genus one curves

17:30-17:55 Aljaž Zalar
Positive polynomials and the truncated moment problem on plane cubics

18:00-18:25 Pawel Pietrzycki
Hyperrigidity and the property of rigidity at zero

Wednesday 11:40-12:40 (W1-SR5, Woolf) chair: Maria Infusino

11:40-12:05 Andreas Debrouwere
The Stieltjes moment problem in Gelfand-Shilov spaces

12:10-12:35 Pier Luigi Novi Inverardi
Method of moments: a maxentropic re-examination of the Fréchet-Shohat theorem

4.11.1. *Abstracts.*

Dragu Atanasiu, University of Borås

Moment problems related to the intrinsic characterization of moment functionals

Abstract. In [2] is established an intrinsic characterization of moment functionals on compacts of characters of an unital commutative real algebra.
In this talk some applications of this characterization to the moment problem will be presented. Moreover, using results from [1] and [3], other intrinsic characterizations of moment functionals in the compact case will be obtained.
For example, with the definitions from [2], a short proof of the following theorem is given using the main result from [1].

Theorem. Let A be an unital commutative $\mathbb{R}$-algebra and $L : A \to \mathbb{R}$ be linear with $L(A^2) \subseteq [0, \infty)$ and $L(1) = 1$.

Then there exists a representing Radon measure for L with compact support if and only if the quadratic module Q_L is *Archimedean*.

References

[1] D. Atanasiu, Un théorème du type Bochner-Godement et le problème des moments, *J. Funct. Anal.* **92**, (1990), 92–102.
[2] M. Infusino, S. Kuhlmann, T. Kuna, P. Michalski, An intrinsic characterization of moment functionals in the compact case *Int. Math. Research Notes.*, **3**, (2023), 2281–2303.
[3] P. H. Maserick, Moments of measures on convex bodies, *Pacific J. Math.* **68**, (1977), 135–152.

Lorenzo Baldi, Universität Leipzig

Trucated moment problems for genus one curves

Abstract. The study of Carathéodory numbers of the truncated moment cone is a classical topic, but exact results are rare in the literature. This is the number of Dirac measures necessary to represent all positive Borel measures supported on a fixed domain when restricted to polynomials of bounded degree.

In this talk, we study the Carathéodory number for projective genus one curves, and show how it is exactly determined by the topology of the curve. As an application, we study the truncated moment problem for affine plane cubics, and solve it proving sharp degree bounds on the flat extension degree.

References

[1] L. Baldi, G. Blekherman, R. Sinn, Nonnegative Polynomials and Moment Problems on Algebraic Curves, *arXiv preprint*, 2407.06017.

Raúl E. Curto, The University of Iowa

Mellin Transform and Exponential Polynomial Methods in the Study of the Square Root Problem for Positive Measures

Abstract. For recursively generated shifts, we provide definitive answers to two outstanding problems in the theory of unilateral weighted shifts: the Subnormality Problem (**SP**) (related to the Aluthge transform) and the Square Root Problem (**SRP**) (which deals with Berger measures of subnormal shifts). We use the Mellin Transform and the theory of exponential polynomials to establish that (**SP**) and (**SRP**) are equivalent if and only if a natural functional equation

holds for the canonically associated Mellin transform. For p-–atomic measures with $p \leq 6$, our main result provides a new and simple proof of the above-mentioned equivalence. Subsequently, we obtain an example of a 7-–atomic measure for which the equivalence fails. This provides a negative answer to a problem posed by G.R. Exner in 2009, and to a recent conjecture formulated by R.E. Curto et al in 2019.

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Research partially supported by U.S. NSF DMS-2247167.

Andreas Debrouwere, Vrije Universiteit Brussel (VUB)

The Stieltjes moment problem in Gelfand-Shilov spaces

Abstract. In this talk, I will discuss the unrestricted Stieltjes moment problem in the setting of weighted spaces of smooth functions defined via weight sequences (= Gelfand-Shilov spaces). I will present a characterization of the surjectivity and the existence of a continuous linear right inverse of the Stieltjes moment mapping

$$f \mapsto \left(\int_0^\infty x^n f(x) dx \right)_{n \in \mathbb{N}}$$

on Gelfand-Shilov spaces in terms of their defining weight sequence. Furthermore, I will explain the connection between this problem and Borel type problems in ultradifferentiable classes.

Philip J. di Dio, University of Konstanz

K-Positivity Preserver and their Generators

Abstract. *joint work with Konrad Schmüdgen*

We study K-positivity preservers for closed sets $K \subseteq \mathbb{R}$, i.e., linear maps $T : \mathbb{R}[x_1, \ldots, x_n] \to \mathbb{R}[x_1, \ldots, x_n]$ such that $T\mathrm{Pos}(K) \subseteq \mathrm{Pos}(K)$ holds where $\mathrm{Pos}(K)$ is the set of all polynomials non-negative on K [1-3]. We give a full description of K-positivity preservers for general closed K [2]. Borcea gave a complete classification for $K = \mathbb{R}^n$ [4] and Netzer only gave a partial description for arbitrary closed K [5].

We use the technique of regular Fréchet Lie groups and Lie algebras to define what e^{tA} means for a linear operator $A : \mathbb{R}[x_1, \ldots, x_n] \to \mathbb{R}[x_1, \ldots, x_n]$. We characterize all such maps A such that e^{tA} is a K-positivity preserver on $\mathbb{R}$ [1,2].

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The author and this project are supported by the Deutsche Forschungsgemeinschaft DFG with the grant DI-2780/2-1 and his research fellowship at the Zukunftskolleg of the University of Konstanz, funded as part of the Excellence Strategy of the German Federal and State Government.

Lawrence Fialkow, SUNY

A core variety approach to the truncated moment problem on $y = x^d$

Abstract. We discuss a framework for studying the truncated moment problem on the curve $y = x^d$ based on the core variety. This permits us to give a short new proof to the solution for $d = 3$ first proved in [TAMS 363(2011), 3133-3165].

Pier Luigi Novi Inverardi, University of Trento

Method of moments: a maxentropic re-examination of the Fréchet-Shohat theorem

Abstract. This talk reconsiders the Fréchet-Shohat theorem (1931) in the special case where the random variables involved are absolutely continuous with the entire real line as support (also known as the Hamburger moment problem). Using recent results from information theory, maximum entropy and convergence in the entropy of maximum entropy densities, we obtain, as expected, stronger modes of convergence than in distribution. As a first result, an alternative proof of such a theorem is given. Furthermore, due to the flexibility of the maximum entropy formalism, the new proof can be easily extended to the case of the Stieltjes moment problem.

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Paweł Pietrzycki, Jagiellonian University

Hyperrigidity and the property of rigidity at zero

Abstract. Motivated both by the fundamental role of the Choquet boundary in classical approximation theory, and by the importance of approximation in the contemporary theory of operator algebras, Arveson introduced hyperrigidity as a form of 'noncommutative' approximation that captures many important operator-algebraic phenomena.

The concept of rigidity at 0 was introduced by Salomon, who studied hyperrigid subsets of Cuntz-Krieger algebras.

In this talk, we will discuss new results on hyperrigid sets that have the property of rigidity at zero.

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Moritz Schick, University of Konstanz

Separation of the sums of squares cone and the sums of nonnegative circuit polynomials cone

Abstract. An n-variate homogeneous polynomial (form) is positive semidefinite (PSD), if it attains nonnegative values over the whole $\mathbb{R}^n$. The set of all such forms of fixed degree $2d$ is a convex cone, called the PSD cone. Studying subcones of the PSD cone is a rich topic in Real Algebraic Geometry with applications in polynomial optimization. Two prominent subcones of the PSD cone, are the sums of squares (SOS) cone and the sums of nonnegative circuit polynomials (SONC) cone. In this talk, we formally introduce the three cones, and study their set-theoretic relations. We then show how to find forms separating two of the cones, respectively. Therefore, we especially focus on methods studying the involved Newton polytopes. Moreover, it was recently shown by Reznick, that the odd powers of the Motzkin form M are not SOS. In this context, we show that M^k is SONC if and only if $k = 1$.

Aljaž Zalar, University of Ljubljana

Positive polynomials and the truncated moment problem on plane cubics

Abstract. Given a linear functional $L : \mathbb{R}[x, y]_{\leq 2k} \to \mathbb{R}$ on all bivariate polynomials of total degree at most $2k$ and a cubic curve C, the truncated moment problem (C–TMP) asks to characterize the existence of a positive Borel measure μ on $\mathbb{R}^2$ with support in C such that $L(p) = \int_{\mathbb{R}^2} p \, d\mu$. In the talk concrete solutions to the C–TMP will be presented for C being one of the following: (i) vanishing set of a polynomial of the form $g(x, y) = y^2 - r(x)$, $\deg r = 3$; (ii) rational curve; (iii) reducible curve of a special form. Certificates for positivity of polynomials with best possible degree bounds are crucial ingredients in the proofs of (i),(ii).

This is joint work with Abhishek Bhardwaj, Mario Kummer and Seonguk Yoo.

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4.12. **Herglotz Functions and Composite Materials.**
Organizers: Victor Vinnikov (Ben Gurion), Joe Ball (Virginia Tech), Aaron Welters (Florida Tech)

Monday 14:00-16:00 (SIBSR3) chair: TBD

14:00-14:25 Aaron Welters
Perspectives on perspective functions, part 1: Effective operators in the theory of composites
14:30-14:55 Aaron Welters
Perspectives on perspective functions, part 2: Kubo-Ando means
15:00-15:25 James Pascoe
The inverse problem for kernel means
15:30-15:55 Ryan Tully-Doyle
Graphs and representations of analytic functions

Monday 16:30-19:00 (SIBSR3) chair: TBD

16:30-16:55 Annemarie Luger
About the integral representation for Herglotz-Nevanlinna functions
17:00-17:25 Joseph Ball
Representations for Herglotz-Agler functions
17:30-17:55 Alexander Kiselev
Negative group velocity in doubly-porous media via sharp norm-resolvent estimates

Tuesday 14:00-16:00 (SIBSR3) chair: TBD

14:00-14:25 Elena Cherkaev
Herglotz functions in forward and inverse homogenization
14:30-14:55 Yury Grabovsky
Optimal recovery of Stieltjes functions from their noisy measurements
15:00-15:25 Oranelle Mattei
On bounding the response in time of viscoelastic composites with a void phase
15:30-15:55 Daniel Alpay
Hyperpositive functions

4.12.1. *Abstracts.*

Daniel Alpay, Chapman University

Hyperpositive functions

Abstract. Given a strictly positive matrix $T \in \mathbb{C}^{n\times n}$, we study matrix-valued functions for which the kernel

$$\frac{F(z)+F(w)^*-T-F(z)TF(w)^*}{z+\overline{w}}$$

is positive definite on the open right half-plane. These functions are associated to absolute stability (the Lurie problem). We will in particular present state-space characterization of these functions through a corresponding Kalman-Yakubovich-Popov Lemma. Links with de Branges $H(S)$ spaces will be discussed.

This is joint work with Izchak Lewkowicz (Ben-Gurion University of the Negev, Beer-Sheva, Israel).

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Joseph Ball, Virginia Tech

Representations for Herglotz-Agler-class functions

Abstract. By the d-variable Herglotz-Agler class we mean functions F mapping the polydisk $\mathbb{D}^d$ holomorphically into the right half plane having the additional property that, for any commuting d-tuple $(T_1,\dots,T_d)$ of Hilbert space contraction operators, the operator $F(T_1,\dots,T_d)$ (defined e.g. via the Riesz-Dunford functional calculus) has positive real part (in the standard Loewner partial order for Hilbert-space operators). We discuss kernel decompositions and representations associated with such functions and indicate how, for the case $d=1$, one can recover the standard Herglotz integral representation for a single-variable Herglotz function.

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Elena Cherkaev, University of Utah

Herglotz functions in forward and inverse homogenization

Abstract. Herglotz and Stieltjes functions play a central role in the homogenization of microstructured media, linking composite microgeometry to the spectral properties of a related

self-adjoint operator. The talk will discuss the Stieltjes/ Herglotz function integral representations for the homogenized transport coefficients of composites and matrix Pade approximations of the resolvents of operators arising in homogenization problems. I will show that the matrix spectral measure in the integral representation of the effective properties of the composite can be uniquely reconstructed; this uniqueness provides a basis for the inverse homogenization problem of recovering information about the microgeometry of the medium. Pade approximations of the spectral measure lead to efficient computational methods for wave propagation in composite materials, allow constructing bounds on the effective properties of composites in forward homogenization, and result in spectrally matched geometries in inverse homogenization, linking it to the inverse spectral problem.

Yury Grabovsky, Temple University

Optimal recovery of Stieltjes functions from their noisy measurements

Abstract. In many applications, such as electrochemical impedance spectroscopy, remote sensing, analysis of composite materials, etc., the response of materials can be described in terms of a Stieltjes function. This function can be measured experimentally at a number of points in the complex plane. The problem of its identification is then of central importance. In this talk I will address the question of quantitative measure of feasibility of such an identification. I will also examine this problem by placing it in a general Hilbert space framework, deriving optimality conditions, error estimates, and proposing an algorithm for its solution that comes with a certificate of optimality. This is a joint work with Dr. Narek Hovsepyan (Rutgers University) and Henry J. Brown (Temple University).

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I would like to thank National Science Foundation for their support through grants NSF-2005538, and NSF-2305832.

Alexander V. Kiselev, University of Bath

Negative group velocity in doubly-porous media via sharp norm-resolvent estimates

Abstract. I will discuss the possibility for doubly-porous media to exhibit negative group velocities, or, in other words, to support metamaterial regimes of wave propagation.

On the one hand, I will argue that the norm-resolvent limits of such media cannot exhibit negative behaviour (see, e.g., [1] for the context of linear elasticity). On the other hand, following the rationale introduced in [2], I will show that the next-order term of the resolvent asymptotics, if taken into account, leads to negative properties of the effective material.

The model I will consider in detail will be based on a periodic graph, which is itself seen as a norm-resolvent limit of a PDE on a thin network (see [3]). Unlike [4], where a metamaterial-type behaviour is demonstrated in a periodic medium under a strong enough magnetic field, or a number of other works, where the effect is essentially due to some special assumptions

on the components constituting the composite, I will consider the simplest possible, although high-contrast, scalar setup.

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Annemarie Luger, Stockholm University

About the integral representation for Herglotz-Nevanlinna functions

Abstract. In this talk we will give an overview on what is known about the integral representation for Herglotz-Nevanlinna functions in several variables.

Ornella Mattei, San Francisco State University

On bounding the response in time of viscoelastic composites with a void phase

Abstract. We are interested in the quasi-static shear antiplane response of those composite materials composed by three phases: a void phase, an elastic phase and a viscoelastic phase. As an example, consider the case of a 3d-printed polymer-based composite reinforced by carbon fibers, where the presence of voids in the microstructure of the composite is either by design or a result of the printing process. By exploiting the analytic properties of the effective tensor as a function of the shear moduli of the phases, we propose bounds on the antiplane shear effective tensor of the composite in the complex plane, in the following two scenarios: (i) no information is provided regarding the microstructure of the composite; (ii) the volume fraction of the three phases is known. The bounds consist in curves in the complex plane that denote the lens-shaped region where the effective tensor takes value: as expected the region corresponding to the second scenario (the volume fractions are known) lies within the region corresponding to the first one (no information is available). Furthermore, we provide bounds in the time domain, when the material is subject to a relaxation experiment. Specifically, we determine the minimum and maximum values of the antiplane shear stress in time in the two aforementioned scenarios. Very interestingly, the bounds incorporating the volume fractions are incredibly tight at certain moments of time, allowing us to use them in an inverse fashion to determine the volume fraction of voids. Furthermore, by comparing such bounds to the ones obtained for the case when there are no voids, one can quantify the effects of the presence of voids in the shear response of the composite.

James Eldred Pascoe, Drexel University

The inverse problem for kernel means

Abstract. We give an overview of the kernel embedding of measures and discuss the inverse problem for kernel means. That is, when is an element of a reproducing kernel Hilbert space an embedded positive measure? We will give insights into the answer over various spaces with varying levels of systematization. Finally, we will discuss obstructions to an integral Herglotz-type representation theory in several variables

Ryan Tully-Doyle, Cal Poly San Luis Obispo

Graphs and representations of analytic functions

Abstract. Given a self-adjoint matrix A and a vector α, the map

$$f(z) = \langle (A - zI)^{-1}\alpha, \alpha \rangle,$$

produces a rational self-map of the complex upper half plane. One interesting class of self-adjoint matrices that can be studied via this function representation is adjacency matrices of simple undirected graphs. Given a simple undirected graph G with adjacency matrix A, one can view the matrix $A - zI$ as the adjacency matrix of a graph that puts self-loops of weight z onto the existing structure of the graph G. In this talk, we'll look extending examples studied by Pascoe, by Bickel, Pascoe, and Sola, and in an honors thesis of Hong, connecting graph structures to the behavior of a two variable analogue of the classical representation, which allows for more complicated boundary behavior to occur. The central matrix becomes $(A - zY - w(I - Y))$, where Y is, for example, a diagonal matrix with diagonal entries 0 and 1. Reformulating these function theoretic ideas in linear algebraic terms turns out to provide straightforward arguments that, for example, extend known one-variable ideas about representation of certain graph products into two-variables. One interesting consequence of this perspective is a method for creating compositions of representations.

I would like to thank the NSF for their generous support.

Aaron Welters, Florida Institute of Technology

Perspectives on perspective functions, part 1: Effective operators in the theory of composites

Abstract. In the first part of my talk, I will give a brief overview of the definition of effective operator for an n-phase isotropic composite in the theory of composites. Then I will highlight their analytic properties as a function of the phases and, in particular, show that they belong to a class of maps known as perspective functions. As a quintessential concrete example, we will consider the problem of periodic conductivity in d-dimensions ($d = 2, 3$) and effective conductivity in that setting.

Aaron Welters, Florida Institute of Technology

Perspectives on perspective functions, part 2: Kubo-Ando means

Abstract. In the second part of my talk, I will use the perspective of perspective functions to bridge a connection between effective operators of 2-phase composites and Kubo-Ando means of pairs of positive operators. Special attention will be given to the arithmetic, harmonic, and geometric means as examples within this context. I will conclude with some recent results I have obtained with my collaborator, Graeme Milton (Univ. of Utah), to illustrate the effectiveness of such perspectives.

4.13. **Linear Algebra and Control Theory.**
Organizers: Andre Ran (Amsterdam), Volker Mehrmann (TU Berlin)

Tuesday 14:00-16:00 (CHLT) chair: André Ran

14:00-14:25 Sanne ter Horst
Some classes of positive linear matrix maps that are also completely positive

14:30-14:55 Froilan Dopico
Strongly minimal linear polynomial system matrices of structured rational matrices

15:00-15:25 Alicia Roca
Row completion of polynomial and rational matrices

15:30-15:55 Anton Arnold
Short- and long-time behavior in evolution equations: the role of the hypocoercivity index

Thursday 14:00-16:00 (CHLT) chair: Alicia Roca

14:00-14:25 André Ran
Unbounded Toeplitz operators: invertibility and Riccati equations

14:30-14:55 Mikael Kurula
Canonical Wiener-Hopf factorization of dichotomous transfer functions on the unit circle

15:00-15:25 Christian Mehl
Computing the eigenvalues of singular Hermitian pencils

15:30-15:55 Patryk Pagacz
On four (quasi-)singular classes of linear pencils

Thursday 16:30–18:30 (CHLT) chair: Christian Mehl

16:30-16:55 Dorothea Hinsen
Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems - Part 1: An Overview

17:00-17:25 Riccardo Morandin
Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems - Part 2: On Time-Varying Storage Functions

17:30-17:55 Hannes Gernandt
Linear port-Hamiltonian DAE systems revisited once more

18:00-18:25 Alexander Wierzba
Towards BIBO stability of port-Hamiltonian systems

4.13.1. *Abstracts.*

Anton Arnold, Vienna University of Technology

Short- and long-time behavior in evolution equations: the role of the hypocoercivity index

Abstract. The *index of hypocoercivity* is defined via a coercivity-type estimate for the self-adjoint/skew-adjoint parts of the generator, and it quantifies 'how degenerate' a hypocoercive evolution equation is, both for ODEs and for evolutions equations in a Hilbert space. We show that this index characterizes the polynomial decay of the propagator norm for short time and illustrate these concepts for the Lorentz kinetic equation on a torus.

This talk is based on joint work with F. Achleitner, E. Carlen, E. Nigsch, and V. Mehrmann.

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Froilán Dopico, Universidad Carlos III de Madrid, Spain

Strongly minimal linear polynomial system matrices of structured rational matrices

Abstract. We consider three classes of rational matrices $R(z)$ that are Hermitian upon evaluation on (a) the real axis, (b) the imaginary axis, or (c) the unit circle. Our goal is to show how to construct linear polynomial system matrices for those $R(z)$ that preserve the corresponding structures and are strongly minimal, a property that guarantee that such polynomial system matrices allow for a complete recovery of the poles, zeros, and minimal indices of $R(z)$. This goal is fully achieved for the Hermitian structures on the real and on the imaginary axes, but for the Hermitian structure on the unit circle some obstacles arise, which require to modify the original problem at some extent and to construct a structured linear polynomial system matrix for the rational function $(1 + z)R(z)$ instead of for $R(z)$. The results presented in this talk are based on the references [1] and [2].

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This work has been partially supported by the Agencia Estatal de Investigación of Spain MCIN/AEI/10.13039/501100011033/ through grants PID2019-106362GB-I00 and RED2022-134176-T.

This is joint work with V. Noferini, M.C. Quintana, and P. Van Dooren.

Hannes Gernandt, University of Wuppertal

Linear port-Hamiltonian DAE systems revisited once more

Abstract. In this talk, we prove a one-to-one correspondence between the geometric formulation of port-Hamiltonian (pH) systems defined by Dirac structures, Lagrange structures, maximal resistive structures, and external ports and a state-space formulation by means of port-Hamiltonian descriptor systems, i.e., differential algebraic equations (DAE) with inputs and outputs. Furthermore, assuming that the Lagrange structure is nonnegative, we are able to show that the resulting pH descriptor system that results from the geometric formulation is positive real. Furthermore, we compare our findings with recent results from [1], [3], and [4].

This talk is based on the preprint [2] with Friedrich Philipp, Till Preuster and Manuel Schaller (TU Ilmenau).

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Dorothea Hinsen, Technische Universität Berlin

Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems - Part 1: An Overview

Abstract. In this talk, we study the relationship between passivity, having a nonnegative supply, and port-Hamiltonian representations for continuous-time linear time-varying systems. The previous results are surveyed and the subtle differences between the concepts are analyzed in detail. Furthermore, the connection to positive semidefinite solutions of the Kalman-Yakubovich-Popov inequality is investigated.

Mikael Kurula, Åbo Akademi University, Finland

Canonical Wiener-Hopf factorization of dichotomous transfer functions on the unit circle

Abstract. The talk is on joint work with Sanne ter Horst and André Ran. We wish to obtain the following result:

Theorem Let F be a Hilbert-space operator-valued function of the form

$$W(z) = I + F(z), \qquad F(z) = D + zC(I - zA)^{-1}B, \qquad \sup_{z\in\mathbb{T}} \|F(z)\| < 1,$$

where the operators A, B, C, D are bounded and A is *dichotomous*, i.e., the resolvent set $\mathrm{res}\,(A)$ of A contains the complex unit circle $\mathbb{T}$.

Then W has a *left canonical Wiener-Hopf factorization*, i.e., $W(z) = V_+(z)\,V_-(z)$, where neither V_+ nor V_+^{-1} has any poles in the closed unit disk $\overline{\mathbb{D}}$, while neither V_- nor V_-^{-1} has any poles in the closed complement $\overline{\mathbb{E}}$ of the unit disk.

Via symmetry, one also gets a *right canonical Wiener-Hopf factorization*, i.e., $W(z) = W_-(z)\,W_+(z)$, where W_+ and W_- have the same properties as V_+ and V_- above, respectively.

We follow the so-called *matching subspaces* approach [1] to Wiener-Hopf factorization, using the Dichotomous Bounded Real Lemma [2] and Kreĭn-space theory [3].

In [1], canonical Wiener-Hopf factorizations are established for F a strictly proper matrix valued function which is contractive on the real line. The theorem above is an extension to Hilbert space operator valued functions which are instead strictly contractive on the complex unit circle.

References

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Christian Mehl, Technische Universität Berlin

Computing the eigenvalues of singular Hermitian pencils

Abstract. The solution of singular eigenvalue problems has been a challenge for many years. In this talk, we show how theoretical results from perturbation theory can be exploited to compute the eigenvalues of singular pencils with symmetry structures. Special focus is laid on singular Hermitian pencils. The developed method for solving the corresponding eigenvalue problem has the advantage that it relies on structure-preserving transformations. As a consequence important structural invariants like the so-called sign characteristic are preserved.

This is joint work with Michiel Hochstenbach and Bor Plestenjak.

Riccardo Morandin, Technische Universität Berlin

Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems – Part 2: On Time-Varying Storage Functions

Abstract. In this talk we study more in detail the characterization of storage functions for linear time-varing systems, and how to use their properties to connect the different dissipativity concepts presented in Part 1.

Patryk Pagacz, Jagiellonian University

On four (quasi-)singular classes of linear pencils.

Abstract. During my talk I will discuss the relation between the following four kinds of linear pencils:

(1) linear pencils with the spectrum equal to the whole complex plane,
(2) linear pencils with the numerical range equal to the whole complex plane,
(3) linear pencils such that $(0, 0)$ belongs to the Taylor spectrum of their coefficients,
(4) linear pencils such that (0,0) belongs to the joint numerical range of their coefficients.

The direct motivation for our research was two questions: "Is it true that the class (1) is equal to (3)?" arise in [1] and "Is it true that the class (2) is equal to (4)?" arise in [2]. The answers of these questions differ depending on the specific assumptions. For example there are no equivalences relations between (1)-(4) for linear pencils of operators. We will also show that linear pencils from class (2) do not have to belong to (4), even if matrix coefficients commute. However, for matrix linear pencils with coefficients that have positive semidefinite hermitian

parts conditions (2) and (4) means the same. Moreover, we will show that the first question has an affirmative answer also for matrix polynomials. The talk is based on the joint works with Vadym Koval.

References

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André Ran, Vrije Universiteit Amsterdam adn North West University, South Africa

Unbounded Toeplitz operators: invertibility and Riccati equations

Abstract. For a class of unbounded block-Toeplitz operators it will be shown how invertibility is connected to factorization of the symbol, and to existence of a particular solution to an unsymmetric algebraic Riccati equation. This is motivated by a similar result for bounded Toeplitz operators due to Rien Kaashoek, Art Frazho and the speaker. In turn, that result was inspired by a paper by Peter Lancaster, Leiba Rodman and the speaker.

This is joint work with Jacob Jaftha (University of Cape Town), Gilbert Groenewald and Sanne ter Horst (both North West University).

References

[1] G.J. Groenewald, S. ter Horst, J.J. Jaftha, and A.C.M. Ran. A Toeplitz-like operator with rational matrix symbol having poles on the unit circle: invertibility and Ricatti equations *Journal of Mathematical Analysis and Applications*, 532 (2024) Paper no. 127925, 15 pp.

The research was partially supported by the National Research Foundation of South Africa (NRF, Grant Numbers 118513, 127364 and 145688) and the DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS).

Alicia Roca, Universitat Politécnica de Valéncia

Row completion of polynomial and rational matrices

Abstract. An important problem in Matrix Theory is the *matrix completion problem*. It consists in characterizing the existence of a matrix with certain properties when a submatrix is prescribed. This work is devoted to the row completion problem for polynomial and rational matrices.

We characterize the existence of a polynomial matrix when its complete structural data (the invariant factors, the invariants orders at infinity, and the column and row minimal indices) and some of its rows are prescribed. This problem was solved in [1] when the polynomial matrix has the same degree as the prescribed submatrix. Here we remove this restriction.

The same problem is also solved for rational matrices.

Obviously, the results obtained hold for the corresponding column completion problems.

References

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This work has been supported by grants PID2021-124827NB-I00 and RED2022-134176-T funded by MCIN/AEI/ 10.13039/501100011033, "ERDF A way of making Europe" funded by the "European Union", and grant GIU21/020 funded by UPV/EHU.

Sanne ter Horst, North-West University

Some classes of positive linear matrix maps that are also completely positive

Abstract. Motivated by a Nevanlinna-Pick type interpolation problem, the question arose whether linear matrix maps of the form $\mathcal{L}_B \circ \mathcal{L}_A^{-1} : \mathbb{C}^{n\times n} \to \mathbb{C}^{n\times n}$ are always completely positive whenever they are positive, for matrices $A, B \in \mathbb{C}^{n\times n}$ with A Lyapunov regular (eigenvalues $\lambda_1, \ldots, \lambda_n$ satisfy $\lambda_i + \overline{\lambda}_j \neq 0$ for all i, j) and B in the double commutant of A. Here for $E \in \mathbb{C}^{n\times n}$, $\mathcal{L}_E$ is the Lyapunov operator $\mathcal{L}_E(X) = XE + E^*X$, $X \in \mathbb{C}^{n\times n}$. While "there are many more positive maps than completely positive maps" [3], we identify certain classes of linear matrix maps for which positivity implies complete positivity, and in doing so provide a positive answer to the question. The talk is based on work with Alma van der Merwe in [1,2].

References

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Alexander A. Wierzba, University of Twente

Towards BIBO stability of port-Hamiltonian systems

Abstract. Port-Hamiltonian systems (pHS) provide a useful tool for modelling physical systems such as e.g. flexible beams within mechanical systems. In this contribution we consider the question of when a distributed port-Hamiltonian system is bounded-input bounded-output (BIBO) stable, continuing recent research on subtleties of this classical notion for infinite-dimensional systems. Analysing and utilizing the special structure of the transfer function of this system class, we provide sufficient conditions for BIBO stability for a sub-class of pHS.

This contribution is based upon joint work with Felix L. Schwenninger.

4.14. **Systems Theory.**
Organizers: Felix Schwenninger (Twente)

Tuesday 16:30-19:00 (CHLT) chair: Felix Schwenninger

16:30-16:55 Andreas Buchinger
Duality for Evolutionary Equations with Applications to Control Theory
17:00-17:25 Merlin Schmitz
Operator Splitting for Optimal Control
17:30-17:55 Philip Preußler
Tests for L^p-admissibility
18:00-18:25 David Seifert
Stability of abstract coupled systems

Wednesday 11:40-12:40 (CHLT) chair: David Seifert

11:40-12:05 Sahiba Arora
Limit-case admissibility of positive systems
12:10-12:35 Mohamed Fkirine
Stochastic Admissibility and Generator Perturbations for Stochastic Cauchy Problems

Wednesday 14:00–15:00 (CHLT) chair: Sahiba Arora

14:00-14:25 Nicolas Vanspranghe
Relaxed admissibility and sharp transfer function estimates for ill-posed systems
14:30-14:55 Karsten Kruse
C-*maximal regularity and* C-*admissibility for semigroups on locally convex spaces*

4.14.1. *Abstracts.*

Sahiba Arora, University of Twente

Limit-case admissibility of positive systems

Abstract. Consider the linear time-invariant system

$$\Sigma(A,B,C) \quad \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), & t \geq 0 \\ y(t) = Cx(t), & t \geq 0 \\ x(0) = x_0; \end{cases}$$

where A generates a C_0-semigroup on a Banach space X, the *control* operator B is a bounded operator from a Banach space U mapping into the extrapolation space X_{-1}, and the *observation* operator $C \in \mathcal{L}(D(A), Y)$ for some Banach space Y. We give sufficient conditions that ensure the positivity of the system $\Sigma(A,B,C)$ automatically yields L^1-admissibility of C and L^∞-admissibility of B respectively.

To be more precise, we look at situations where all given spaces are function spaces, the operators B and/or C are positive (i.e., they map positive functions to positive functions), and the semigroup generated by A is positive (i.e., each semigroup operator is positive).

This is joint work with Jochen Glück, Lassi Paunonen, and Felix Schwenninger.

Andreas Buchinger, TU Bergakademie Freiberg

Duality for Evolutionary Equations with Applications to Control Theory

Abstract. In this talk, we will recall the theory of evolutionary equations (afforded by R. Picard). This theory provides a general well-posedness theorem that covers a vast class of PDEs. In this setting, we will introduce a natural concept of duality that gives rise to "evolutionary control and observation", and we will discuss consistency with classical control theory.

Mohamed Fkirine, Tampere university

Stochastic Admissibility and Generator Perturbations for Stochastic Cauchy Problems

Abstract. In this talk, we delve into the study of evolution equations with white-noise boundary conditions. By rewriting these equations as stochastic Cauchy problems, we establish necessary and sufficient conditions for the existence of solutions using the concept of admissible observation operators and the Yosida extension for such operators. Additionally, we examine the robustness properties of these equations, including well-posedness, absolute continuity, and the existence of invariant measures under various types of unbounded perturbations.

This is joint work with S. Hadd and A. Rhnadi.

References

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[2] M. Fkirine, S. Hadd, A. Rhandi, On evolution equations with white-noise boundary conditions, *Journal of Mathematical Analysis and Applications* **535**(1) (2024), 128087.
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Karsten Kruse, University of Twente

C-maximal regularity and C-admissibility for semigroups on locally convex spaces

Abstract. In this talk we consider C-maximal regularity and C-admissibility for strongly continuous locally equicontinuous semigroups on sequentially complete locally convex Hausdorff spaces. The C here stands for *continuous* and describes the regularity of the inhomogeneity of an abstract Cauchy problem of first order involving the generator of a strongly continuous locally equicontinuous semigroup. We show that C-maximal regularity and the a priori weaker C-admissibility are equivalent for such semigroups on certain classes of locally convex Hausdorff spaces.

This contribution is a joint work with Felix L. Schwenninger [1].

References

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Philip Preußler, University of Twente

Tests for L^p-admissibility

Abstract. The talk will be centered around approaches for checking L^p-admissibility for infinite-dimensional linear control systems, with special focus on the case $p \neq 2$. We compare known methods based on abstract interpolation spaces, Laplace–Carleson embeddings and the p-Weiss property while giving some extensions and reviewing their applicability to infinite-dimensional input spaces. Moreover, we illustrate the theory by means of various examples based on the heat equation.

This is joint work with Felix Schwenninger.

References

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Merlin Schmitz, University of Wuppertal

Operator Splitting for Optimal Control

Abstract. In this talk I will investigate a dissipativity-based time-splitting for finite-dimensional, linear-quadratic optimal control problems. This method is based on a Peaceman-Rachford algorithm that has been generalized in [1] to monotone operators. The main idea is to reformulate the problem as a state and adjoint equation, where the adjoint equation has its initial value at the other end of the time interval. Then, we decompose the time-domain into subintervals and refresh both the initial values in each iteration. Thanks to this "dichotomy" of equations, the proposed splitting algorithm has a monotonly decreasing error bound.
Joint work with B. Farkas, B. Jacob and M. Schaller.

References

[1] P.L. Lions, B. Mercier, Splitting Algorithms for the Sum of Two Nonlinear Operators, *SIAM J. Numer. Anal.* **16**, (1979), 964–979.

David Seifert, Newcastle University

Stability of abstract coupled systems

Abstract. We present an abstract framework for studying the asymptotic behaviour of coupled linear systems. Our approach combines ideas from systems theory with results in the quantitative asymptotic theory of strongly continuous operator semigroups, and it allows us to study composite systems by looking separately at the (often much simpler) constituent components and the properties of a certain "transfer function". We illustrate the power of our abstract results by using them to obtain (typically sharp) rates of energy decay in certain wave-heat systems and for a wave equation with an acoustic boundary condition. The talk is based on joint work with Lassi Paunonen and Serge Nicaise.

Nicolas Vanspranghe, Tampere University

Relaxed admissibility and sharp transfer function estimates for ill-posed systems

Abstract. Despite their prevalence in the abstract literature, admissible control and observation operators exclude a number of classical models in the control theory of PDEs. Notably, for the multidimensional wave equation $(\partial_t^2 - \Delta)w = 0$ posed in $(0,T) \times \Omega$, it was shown by I. Lasiecka and R. Triggiani that general Neumann boundary data $u \in L^2((0,T) \times \partial\Omega)$ fail to produce finite-energy solutions (i.e., at the natural $H^1(\Omega) \times L^2(\Omega)$-level). In an attempt to bridge the gap between semigroup- or system-theoretic and PDE approaches, we investigate relaxed admissibility properties formulated in terms of abstract Sobolev scales based on the functional calculus of the semigroup generator and quadratic interpolation in Hilbert spaces. Such scales allow us to quantify the defect of admissibility and, in the group case, yield an exact correspondence between loss of regularity in the time domain and growth rates at high frequencies of certain operator-valued functions in the Laplace domain. Likewise, under mild assumptions, we are able to translate input-output regularity properties into high-frequency growth rates of the system's transfer function. As an application, we use the resulting estimates to derive non-uniform energy decay rates for the wave equation with Neumann boundary damping under an observability hypothesis on the undamped system.

This is a joint work with Lassi Paunonen (Tampere University) and David Seifert (Newcastle University).

4.15. **Numerical Ranges.**
Organizers: Ilya Spitkovsky (NYU, Abu Dhabi), Tin-Yau Tam (Nevada)

Tuesday 16:30-18:30 (SIBSR6) chair: Tin-Yau Tam

16:30-16:55 Chi-Kwong Li, College of William & Mary, USA
Linear preserver problems and generalized numerical radius
17:00-17:25 Muyan Jiang, University of California, Berkeley, USA
Unified approach to reciprocal matrices with Kippenhahn curves containing elliptical components
17:30-17:55 Anne Greenbaum, University of Washington, USA
On the approximate rank and numerical range of the resolvent
18:00-18:25 Brooke Randell, University of California, Santa Cruz, USA
Exploring the numerical range of block Toeplitz operators

Wednesday 11:40-12:40 (SIBSR6) chair: Rute Lemos

11:40-12:05 Thomas Schulte-Herbruggen, Technische Universitẗ München, Germany
Quantum systems theory meets numerical ranges: New observability results in terms of C-numerical ranges
12:10-12:35 Edward Poon, Embry-Riddle Aeronautical University, USA
The simultaneous zero inclusion property and Birkhoff-James orthogonality

Wednesday 14:00-15:00 (SIBSR6) chair: Rute Lemos

14:00-14:25 Takeaki Yamazaki, Toyo University, Japan
Generalizations of Aluthge transform, numerical ranges and spectral radii
14:30-14:55 Nicholas Young, Newcastle University, UK
Operators with numerical range in an elliptical region

Thursday 14:00-16:00 (SIBSR6) chair: Douglas Farenick

14:00-14:25 Hugo Woerdeman, Drexel University, USA
Partial isospectrality of a matrix pencil and circularity of the c-numerical range
14:30-14:55 Stephan Weis, Czech Technical University in Prague, Czech Republic
Restricting states to a matrix algebra is an open map
15:00-15:25 Rute Lemos, University of Aveiro, Portugal
Algebraic curves associated to centrosymmetric matrices
15:30-15:55 Zinaida Lykova, Newcastle University, UK
The bfd-norm on spaces of analytic functions and the numerical range

Thursday 16:30-18:30 (SIBSR6) chair: Ilya Spitkovsky

16:30-16:55 Douglas Farenick, University of Regina, Canada
Matrix ranges and the Halmos dilation theorem in several variable operator theory
17:00-17:25 Jens de Vries, University of Twente, Netherlands
A Drury-type bound for ρ-contractions
17:30-17:55 Damian Kolaczek, University of Agriculture in Krakow, Poland
Numerical ranges of antilinear operators
18:00-18:25 Helena Soares, ISCTE, Portugal
Quaternionic essential numerical range of complex operators

Friday 14:00-16:00 (SIBSR6) chair: Chi-Kwong Li

14:00-14:25 Petr Blaschke, Silesian University in Opava, Czech Republic
Co-oval description of the boundary curve of the numerical range of a finite matrix

14:30-14:55 Piotr Pikul, Jagiellonian University in Kraków, Poland
On the joint numerical range of triples of 4×4 *Hermitian matrices*

15:00-15:25 Tin-Yau Tam, University of Nevada, Reno, USA
Generalized numerical ranges in Lie framework

4.15.1. *Abstracts.*

Petr Blaschke, Silesian university in Opava

Co-oval description of the boundary curve of the numerical range of a finite matrix

Abstract. It is known that the numerical range of a finite matrix is a convex bounded set, so its boundary is a simple closed curve. Numerical range also contains all the eigenvalues. For a 2 by 2 matrix, the numerical range is just an ellipse with foci exactly at the eigenvalues. Is there a nice description of the boundary curve of a numerical range also for 3 by 3 matrices, 4 by 4 matrices and beyond in terms of eigenvalues? We will give such a description which includes not only eigenvalues but also other interesting points, some of which behave like "anti-eigenvalues".

Jens de Vries, University of Twente

A Drury-type bound for ρ-contractions

Abstract. We discuss a new Drury-type bound for (Sz.-Nagy–Foiaş) ρ-contractions for $\rho \in [1, 2]$. In particular, this result interpolates von Neumann's inequality and Drury's classical bound [1], where the latter states: If A is any square matrix whose numerical radius is at most 1, then

$$\|p(A)\| \leq 1 - |p(0)|^2 + \sqrt{(1 - |p(0)|^2)^2 + |p(0)|^2}$$

for any polynomial p with $\sup_{|z|\leq 1} |p(z)| \leq 1$. This is joint work with Felix Schwenninger.

References

[1] S.W. Drury, Symbolic calculus of operators with unit numerical radius, *Linear Algebra and its Applications*, 428(8-9):2061–2069, 2008.

This research is financed by the Dutch Research Council (NWO), grant OCENW.M20.292.

Douglas Farenick, University of Regina

Matrix ranges and the Halmos dilation theorem in several variable operator theory

Abstract. The well-known theorem of P.R. Halmos [1] concerning the existence of unitary dilations for contractive linear operators acting on Hilbert space is extended to d-tuples of contractive Hilbert space operators satisfying a certain matrix-positivity condition. Such operator d-tuples are called, herein, Toeplitz-contractive, and a characterisation of the Toeplitz-contractivity condition is presented. The matrix-positivity condition leads to the definition of a new metric for operator tuples, and some of the properties of this metric are explored. Lastly, the elements of the closed unit ball in this metric are shown to coincide with the matrix ranges or operator ranges of $(U, U^2, \ldots, U^n)$, where U is a universal unitary operator.

References

[1] P.R. Halmos, Normal dilations and extensions of operators, *Summa Brasil. Math.* **2**, (1950), 125–134.

This work is supported, in part, by the NSERC Discovery Grant program.

Anne Greenbaum, University of Washington

On the approximate rank and numerical range of the resolvent

Abstract. Let A be a square matrix and let λ be a simple eigenvalue of A with unit right and left eigenvectors x and y: $Ax = \lambda x$, $y^*A = \lambda y^*$. If $z \in \mathbb{C}$ is *much* closer to λ than to any other eigenvalue of A, then the resolvent $(A - zI)^{-1}$ is approximately equal to the rank one matrix $(\lambda - z)^{-1}xy^*$. If λ is *ill-conditioned*, then y is almost orthogonal to x, and for z very close to λ, the numerical range of the resolvent is approximately equal to a disk of radius $\frac{1}{2}|(\lambda - z)^{-1}|$ about the point $\frac{1}{2}|(\lambda - z)^{-1}|\ |y^*x|$.

We describe conditions under which $(A-zI)^{-1}$ closely resembles a rank one matrix, with numerical range approximately equal to a disk about a point close to the origin, even when z is much further away from an ill-conditioned eigenvalue λ. In this case, $(A - zI)^{-1} \approx \sigma_1(z)u_1(z)v_1(z)^*$, where $\sigma_1(z)$ is the largest singular value of $(A - zI)^{-1}$ and $u_1(z)$ and $v_1(z)$ are the associated left and right singular vectors. To see this, we start with the matrix $A_0 = A - \lambda I$, which has a 0 singular value whose right and left singular vectors are the same as the corresponding eigenvectors. Assuming that these singular vectors are almost orthogonal to each other (i.e., that λ is an ill-conditioned eigenvalue) and that the second smallest singular value of A_0 is well-separated from 0, we show that $(A_0 - zI)^{-1}$ resembles such a rank one matrix for a wide range of z values.

References

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Muyan Jiang, University of California, Berkeley

Unified approach to reciprocal matrices with Kippenhahn curves containing elliptical components

Abstract. *Reciprocal matrices* are tridiagonal matrices $(a_{ij})_{i,j=1}^n$ with constant main diagonal and such that $a_{i,i+1}a_{i+1,i} = 1$ for $i = 1, \ldots, n-1$. For these matrices, criteria are established under which their Kippenhahn curves contain elliptical components or even consist completely of such. These criteria are in terms of system of homogeneous polynomial equations in variables $(|a_{j,j+1}| - |a_{j+1,j}|)^2$, and established via a unified approach across arbitrary dimensions. The results are illustrated, and specific numerical examples provided, for $n = 7$ thus generalizing earlier work in the lower dimensional setting.

Damian Kołaczek, University of Agriculture in Krakow

Numerical ranges of antilinear operators

Abstract. We study numerical ranges of antilinear operators acting on Hilbert and Banach spaces. We discuss various similarities and differences between numerical radii and numerical ranges in linear and antilinear setting. Our main result is proving that the numerical ranges of antilinear operators on at least two-dimensional space are always discs, which improve previously known results [1,2] stating that such numerical ranges on Hilbert and Banach spaces are always annuli. We also introduce the concept of an *antilinear numerical index* of the space and compare it with ordinary numerical index based on linear operators.

The talk is based on recent research with Vladimir Müller (Institute of Mathematics, Czech Academy of Sciences) [3].

References

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Rute Lemos, University of Aveiro

Algebraic curves associated to centrosymmetric matrices

Abstract. The boundary generating curves and numerical ranges of some centrosymmetric matrices of orders up to 6 are characterized in terms of the matrices entries [1]. These results extend previous ones concerning Kac-Sylvester matrices [2]. The classification of all the possible boundary generating curves for centrosymmetric matrices of higher dimensions remains open. Illustrative figures of the obtained results are presented. This talk is based on a joint work with Natália Bebiano and Graça Soares.

References

[1] N. Bebiano, R. Lemos and G. Soares, Algebraic curves associated with centrosymmetric matrices of orders up to 6. *Adv. Oper. Theory* **9** (2024), 56.
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I would like to thank the Center for Research and Development in Mathematics and Applications (CIDMA), supported through the Portuguese Foundation for Science and Technology (FCT - Fundacão para a Ciência e a Tecnologia), project UIDB/04106/2020 (https://doi.org/10.54499/UIDB/04106/2020).

Chi-Kwong Li, College of William and Mary

Linear preserver problems and generalized numerical radius

Abstract. We discuss linear preserver problems related to the generalized numerical range and generalized numerical radius. Recent results and open problems will be presented.

Zinaida Lykova, Newcastle University

The bfd-norm on spaces of analytic functions and the numerical range

Abstract. Let E be the open region in the complex plane bounded by an ellipse. The B. and F. Delyon norm $\|\cdot\|_{\mathrm{bfd}}$ on the space $\mathrm{Hol}(E)$ of holomorphic functions on E is defined by

$$\|f\|_{\mathrm{bfd}} \stackrel{\mathrm{def}}{=} \sup_{T\in\mathcal{F}_{\mathrm{bfd}}(E)} \|f(T)\|,$$

where $\mathcal{F}_{\mathrm{bfd}}(E)$ is the class of operators T such that the closure of the numerical range $W(T)$ of T is contained in E. The name of the norm recognizes a celebrated theorem of the brothers Delyon, which implies that $\|\cdot\|_{\mathrm{bfd}}$ is equivalent to the supremum norm $\|\cdot\|_\infty$ on H$ol(E)$.

We show that there exists an interesting connection between the bfd norm on $\mathrm{Hol}(E)$ and the supremum norm $\|\cdot\|_\infty$ on the space $\mathrm{H}^\infty(G)$ of bounded holomorphic functions on the symmetrized bidisc, the domain G in $\mathbb{C}^2$ defined by

$$G \stackrel{def}{=} \{(z+w, zw) : |z|<1, |w|<1\}.$$

It transpires that there exists a holomorphic embedding $\tau : E \to G$ having the property that, for any bounded holomorphic function f on E,

$$\|f\|_{\mathrm{bfd}} = \inf\{\|F\|_\infty : F \in \mathrm{H}^\infty(G), F \circ \tau = f\},$$

and moreover, the infimum is attained at some $F \in \mathrm{H}^\infty(G)$.

We also consider connections between operators T with the closure of $W(T)$ contained in E and Douglas-Paulsen operators.

The talk is based on joint work with Jim Agler and Nicholas Young [1].

References

[1] J. Agler, Z. A. Lykova and N. J. Young, On the operators with numerical range in an ellipse, *J. Funct. Anal.*, **2**87(8) Article 110556 (2024) https://doi.org/10.1016/j.jfa.2024.110556.

Piotr Pikul, Jagiellonian University in Kraków

On the joint numerical range of triples of 4×4 *Hermitian matrices*

Abstract. Joint numerical ranges of hermitian matrices are of interest due to their relation to quantum states. The JNR of a triple of hermitian matrices of order 4 represents expected values of three observables applied to all possible two-qubit states.

$$W\big(A_1, A_2, A_3\big) = \Big\{\big(\operatorname{tr}(PA_j)\big)_{j=1}^3 \colon P \in M_4,\ P \geq 0,\ \operatorname{tr}(P) = 1\Big\}.$$

The configuration of flat portions of the boundary (faces) was studied in case of triples of hermitian 3×3 matrices [1]. As could be expected, in dimension 4 both shape of the faces and their configuration can be of higher complexity. In the talk there will be presented current results on such JNRs.

This is an ongoing project in collaboration with Konrad Szymański, Stephan Weis, Karol Życzkowski and Ilya Spitkowsky.

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Edward Poon, Embry-Riddle Aeronautical University

The simultaneous zero inclusion property and Birkhoff-James orthogonality

Abstract. A normed space $\mathcal{X}$ is said to have the Simultaneous Zero Inclusion (S0I) property if, for every invertible bounded linear operator T on $\mathcal{X}$, 0 lies in the spatial numerical range of T if and only if 0 lies in the spatial numerical range of T^{-1}. Previously the only known spaces with the S0I property were inner product spaces, corresponding to the classical numerical range. By connecting the S0I property to Birkhoff-James orthogonality, we show that there are non-inner product spaces based on Radon planes that have the S0I property.

Brooke Randell, University of California, Santa Cruz

Exploring the numerical range of block Toeplitz operators

Abstract. We will discuss the numerical range of a family of Toeplitz operators with symbol function $\phi(z) = A_0 + zA_1$, where A_0 and A_1 are 2×2 matrices with complex-valued entries. A special case of a result proved by Bebiano and Spitkovsky in 2011 states that the closure of the numerical range of the Toeplitz operator $T_{\phi(z)}$ is the convex hull of $\{W(\phi(z)) : z \in \partial\mathbb{D}\}$. Here, $W(\phi(z))$ denotes the numerical range of $\phi(z)$. We combine this result with the envelope algorithm to describe the boundary of the convex hull of $\{W(\phi(z)) : z \in \partial\mathbb{D}\}$. We also place specific conditions on the matrices A_0 and A_1 so that $\{W(\phi(z)) : z \in \partial\mathbb{D}\}$ is a set of potentially degenerate circular disks. The convex hull of $\{W(\phi(z)) : z \in \partial\mathbb{D}\}$ takes on a wide variety of shapes, including the convex hull of limaçons.

Thomas Schulte-Herbrüggen, Technical University of Munich (TUM)

Quantum systems theory meets numerical ranges: New observability results in terms of C-numerical ranges

Abstract. By way of example we connect key-notions of quantum systems and control theory with numerical ranges.

The well-known C-numerical range of A defined as $W(C, A) := \{\mathsf{tr}(C^\dagger UAU^\dagger) \,|\, U \in \mathbf{U}(n)\}$ is the projection of the unitary orbit of A onto C under the Hilbert-Schmidt scalar product. In quantum control A can be seen as an initial state (the reachable set of which under Hamiltonian dynamics equals its unitary orbit) and C as a quantum observable. If system dynamics are limited to proper subgroup orbits $\mathcal{O}_{\mathbf{K}}(A) := \{KAK^\dagger \,|\, K \in \mathbf{K} \subsetneq \mathbf{U}(n)\}$, one arrives at the so-called restricted C-numerical range $W_{\mathbf{K}}(C, A)$ [1].

New observability results in quantum systems theory [2] are taken over to establish symmetry conditions for A and C in relation to $\mathbf{K}$ under which the restricted C-numerical range of A exhausts the full C-numerical range, i.e. $W_{\mathbf{K}}(C, A) = W(C, A)$. In turn, similar symmetry arguments classify tomographiable pairs (A, C) as well as scenarios when $W_{\mathbf{K}}(C, A)$ collapses to a singleton *beyond* scalar A or C—the latter case being necessary and sufficient for the full $W(C, A)$ to be a singleton [3,4,5].

These results are placed into a symmetry-based Lie-theoretical framework of quantum systems theory.

Relies on joint work with Markus Wiener.

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Helena Soares, ISCTE

Quaternionic essential numerical range of complex operators

Abstract. We study the essential numerical range of complex operators on a quaternionic Hilbert space and its relation with the essential S-spectrum. We give a new characterization of the essential numerical range relating it to the complex essential numerical range. Moreover, we show that the quaternionic essential numerical range of a normal operator is the convex hull of the essential S-spectrum.

Tin-Yau Tam, University of Nevada, Reno

Generalized numerical ranges in Lie framework

Abstract. Let G be a complex semisimple Lie group with Lie algebra $\mathfrak{g}$, K connected subgroup of G with Lie algebra $\mathfrak{k}$, $B_\theta(\cdot,\cdot)$ the inner product on $\mathfrak{g}$ induced by the Killing form $B(\cdot,\cdot)$, where θ is the Cartan involution of $\mathfrak{g}$ associated with $\mathfrak{k}$. For $X, C \in \mathfrak{g}$, the C-numerical range of X is

$$W_C(X) := \{B_\theta(C, \mathrm{Ad}(k)X) : k \in K\}, \quad C, X \in \mathfrak{g}.$$

Geometric properties of $W_C(X)$ and related results will be discussed.

Stephan Weis, Czech Technical University in Prague

Restricting states to a matrix algebra is an open map

Abstract. Two topological problems appeared in matrix theory about ten years ago. One of them [1] concerns the map $f : x \mapsto \langle x|Ax\rangle$ from the unit sphere of $\mathbb{C}^n$ to the numerical range, where $A \in M_n$ is a complex $n \times n$-matrix. The other one [2] involves the linear map $g : D_n \to \mathbb{R}^k, \rho \mapsto (\mathrm{Tr}(\rho A_1), \ldots, \mathrm{Tr}(\rho A_k))$ on the convex set of density matrices $D_n = \{\rho \in M_n : \mathrm{Tr}(\rho) = 1, \rho \text{ is psd}\}$, where $A_1, \ldots, A_k \in M_n$ are hermitian matrices. It turns out [3] that the openness of f is equivalent to that of g in case $k = 2$ (taking $A = A_1 + \mathrm{i}A_2$).

The openness of g is relevant in physics, as it governs the continuity of the maximum-entropy inference map, and of other inference maps [2]. For example, the phenomenon that a smooth change of local observations can lead to a discontinuous change of global inference states is discussed as a signal of a quantum phase transition [4].

In the theory of operator systems [5], the openness of g is equivalent to the openness of the orthogonal projection (with respect to the Frobenius inner product) of D_n onto the operator system L defined as the complex span of the identity matrix and the matrices $A_1, \ldots, A_k$. This, in turn, is equivalent to the openness of the linear map that restricts states on M_n to L (by virtue of the Riesz representation theorem).

This talk presents a novel theorem that asserts that the map restricting states to an operator system L is an open map if L is a *-algebra. This result extends a theorem by Vesterstrøm [6] into a non-commutative setting. It simplifies the topological analysis of the map that restricts states to an operator system included in a proper *-subalgebra.

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Hugo J. Woerdeman, Drexel University

Partial isospectrality of a matrix pencil and circularity of the c-numerical range

Abstract. We study when functions of the eigenvalues of the pencil

$$\mathrm{Re}(e^{-it}A) = \cos(t)\mathrm{Re}A + \sin(t)\mathrm{Im}A \tag{10}$$

are constant functions of t. The results are then applied to questions regarding the numerical range, the higher rank numerical range and the c-numerical range, and we derive trace type conditions for when these numerical ranges are disks centered at 0. The theory of symmetric polynomials plays an important part in the proofs. This talk is based on joint work with Alma van der Merwe and Madelein Thiersen.

Takeaki Yamazaki, Toyo University

Generalizations of Aluthge transform, numerical ranges and spectral radii

Abstract. In this talk, we shall introduce two types of generalizations of the Aluthge transformation, the one is called the induced Aluthge transform and another is called the spherical Aluthge transform. In this talk, we shall introduce (i) inclusion relations of numerical ranges among induced Aluthge transforms and (ii) a characterization of the Taylor spectrum radius via spherical Aluthge transforms. This is partially joint work with Professor Kais Feki.

Nicholas Young, Newcastle University (emeritus)

Operators with numerical range in an elliptical region

Abstract. We give new necessary and sufficient conditions for the numerical range $W(T)$ of a bounded linear operator T on a Hilbert space $\mathcal{H}$ to be a subset of the closed elliptical set $K_\delta \subseteq \mathbb{C}$ given by

$$K_\delta \stackrel{\text{def}}{=} \left\{ x+iy : \frac{x^2}{(1+\delta)^2} + \frac{y^2}{(1-\delta)^2} \le 1 \right\},$$

where $0 < \delta < 1$. We start by generalizing Berger's well-known criterion for an operator to have numerical radius at most one, his so-called *strange dilation theorem*. Specifically, we show that, for $\delta \in (0,1)$ and for an operator $T \in \mathcal{B}(\mathcal{H})$, $W(T) \subseteq K_\delta$ if and only if there exist a Hilbert space $\mathcal{K}$, an isometry $I : \mathcal{H} \to \mathcal{K}$ and a unitary operator U on $\mathcal{K}$ such that

$$\frac{1-\frac{1}{2}zT}{1-zT+\delta z^2} = I^* \frac{1}{1-zU} I$$

for all $z \in \mathbb{D}$.

We next generalize the lemma of Sarason that describes power dilations in terms of semi-invariant subspaces to operators T that satisfy $W(T) \subseteq K_\delta$. This generalization yields a characterization of the operators $T \in \mathcal{B}(\mathcal{H})$ such that $W(T)$ is contained in K_δ in terms of certain structured contractions that act on $\mathcal{H} \oplus \mathcal{H}$. As a corollary of our results we extend Ando's parametrization of operators having numerical range in a disc to those T such that $W(T) \subseteq K_\delta$. Indeed, if $\dim \mathcal{H} < \infty$, then $W(T) \subseteq K_\delta$ if and only if there exist contractions A, B on $\mathcal{H}$ with $A = A^*$ such that

$$T = 2\sqrt{\delta}A + (1-\delta)\sqrt{1+A}B\sqrt{1-A}.$$

The talk is based on joint work with Jim Agler and Zinaida Lykova.

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4.16. **Spectral Problems and Computation.**
Organizers: Christiane Tretter (Bern), Marco Marletta (Cardiff)

Wednesday 11:40-12:40 (SIBLT3) chair: Marco Marletta

11:40-12:05 Matteo Capoferri
Curl and asymmetric pseudodifferential projections
12:10-12:35 Karl Michael Schmidt
On the continuum limit for discrete Dirac operators on square lattices

Wednesday 14:00-14:25 (SIBLT3) chair: Karl Michael Schmidt

14:00-14:25 Siavash Sadeghi
Wavenumber-explicit bounds for first kind integral equations in wave scattering
14:30-14:55 Anna Rozanova-Pierrat
Spectral stability under domain convergence in a class of non-Lipschitz uniform domains

Thursday 14:00-16:00 (SIBLT3) chair: Matteo Capoferri

14:00-14:25 Christiane Tretter
Spectral bounds for damped systems
14:30-14:55 Iveta Semorádová
$\mathcal{PT}$-symmetric oscillators with one-center point interactions
15:00-15:25 Marko Lindner
Localisation of pseudospectra on discrete groups
15:30-15:55 Christian Wyss
Computing the quadratic numerical range

Thursday 16:30-19:00 (SIBLT3) chair: Christian Wyss

16:30-16:55 Matt Colbrook
Barriers and Classifications of Robust Koopman Learning
17:00-17:25 Sugirtha Gayathri
A study on the exponential spectrum in Banach algebras
17:30-17:55 Lyonell Boulton
Spectral analysis of Dirac operators with a purely imaginary dislocation

4.16.1. *Abstracts.*

Lyonell Boulton, Heriot-Watt University

Spectral analysis of Dirac operators with a purely imaginary dislocation

Abstract. In this talk we present a complete spectral analysis of Dirac operators with non-Hermitian matrix potentials of the form $i\,\mathrm{sgn}+V$ where $V \in L^1$. For $V = 0$ we compute explicitly the matrix Green function. This allows us to determine the spectrum, which is purely essential, and its different types. It also allows us to find sharp enclosures for the pseudospectrum and its complement, in all parts of the complex plane. Notably, this includes the instability region, corresponding to the interior of the band that forms the numerical range. Then, with the help of a Birman-Schwinger principle, we establish in precise manner how the spectrum and pseudospectrum change when $V \neq 0$, assuming the hypotheses $\|V\|_{L^1} < 1$ or $V \in L^1 \cap L^p$ where $p > 1$. We show that the essential spectra remain unchanged and that the ε-pseudospectrum stays close to the instability region for small ε. We determine sharp asymptotics for the discrete spectrum, whenever V satisfies further conditions of decay at infinity. Finally, in one of our main findings, we give a complete description of the weakly-coupled model.

The research has been conducted jointly with Tho Nguyen Duc and David Krejčiřík.

Matteo Capoferri, Heriot-Watt University

Curl and asymmetric pseudodifferential projections

Abstract. In my talk I will present a new approach to the spectral theory of systems of PDEs on closed manifolds, developed in a series of recent papers by Dmitri Vassiliev (UCL) and myself, based on the use of pseudodifferential projections. After discussing the general theory, I will turn to the (non-elliptic) operator curl, and explain how our techniques offer a new pathway to the study of spectral asymmetry.

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This is joint work with D. Vassilev (UCL) which was partially supported by EPSRC Fellowship EP/X01021X/1.

Matthew J. Colbrook, University of Cambridge

Barriers and Classifications of Robust Koopman Learning

Abstract. Many modern dynamical systems are too complicated to analyze directly or we do not have access to models, driving significant interest in learning methods. Koopman operators, though classical, have recently emerged as a dominant approach because they allow the study of nonlinear dynamics using linear techniques by solving an infinite-dimensional spectral problem. However, current algorithms face challenges such as lack of convergence (e.g., spectral pollution), hindering practical progress. In this talk, I will explore the fundamental question: *When can we robustly learn spectral properties of Koopman operators from trajectory data of dynamical systems, and when can we not?* Understanding these boundaries is crucial for analysis, applications, and designing algorithms. We establish a foundational approach combining computational analysis and ergodic theory, revealing the first fundamental barriers – universal for any algorithm – associated with system geometry and complexity, regardless of data quality and quantity. For instance, we demonstrate well-behaved smooth systems on tori where non-trivial

eigenfunctions of Koopman operators cannot be determined by any sequence of (even randomized) algorithms, even with unlimited training data. Additionally, we identify when learning is possible and introduce optimal algorithms with verification that overcome issues in standard methods. These results pave the way for a sharp classification theory of data-driven dynamical systems, including beyond Koopman operators, based on how many limits are needed to solve a problem (the SCI hierarchy). These limits also characterize all previous methods. The talk is based on joint work with Igor Mezić and Alexei Stepanenko.

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Sugirtha Gayathri, Indian Institute of Technology Hyderabad

A study on the exponential spectrum in Banach algebras

Abstract. In this talk, we give an outline of the literature about the notion of generalized exponentials in a Banach algebra and the corresponding spectrum. Towards a question of commutativity of the exponential spectrum, we investigate an associated set.

Marko Lindner, Hamburg University of Technology

Localisation of pseudospectra on discrete groups

Abstract. Given a bounded and possibly non-normal band operator A on $\ell^2(G)$ with a discrete group G and an integer $n \in \mathbb{N}$, we show that every nonzero vector $x \in \ell^2(G)$ has a finite subvector $x_{n,k}$ of size n at some (typically unknown) position $k \in G$ such that

$$\frac{\|Ax_{n,k}\|}{\|x_{n,k}\|} \;\leq\; \frac{\|Ax\|}{\|x\|} \;+\; \varepsilon_n,$$

where $x_{n,k}$ was extended to G by zero, $\varepsilon_n \sim 1/\sqrt{n}$ and we know the proportionality constant.

Evaluating this inequality for an x that (almost) minimises $\frac{\|Ax\|}{\|x\|}$, one can draw conclusions about the norm of the inverse, resolvent norms, and pseudospectra of A versus those of its restrictions to subspaces $X_{n,k}$ of vectors with n-sized support.

As a result, we cover the pseudospectrum of A by a union of pseudospectra of its restrictions to spaces $X_{n,k}$, proving the absence of (pseudo)spectrum outside that union.

We show how to improve the result to $\varepsilon_n \sim 1/n$ and we explain the connection of ε_n to Dirichlet eigenvalues of an associated graph Laplacian.

This is joint work with Simon Chandler-Wilde (Reading) and Christian Seifert (Hamburg).

Siavash Sadeghi, University of Reading

Wavenumber-explicit bounds for first kind integral equations in wave scattering

Abstract. There has been significant interest in the derivation of wavenumber-explicit bounds for the inverses of operators arising in the boundary integral equation formulation of time-harmonic scattering problems, when the scatterer is a bounded Lipschitz domain (see e.g. [1, 2]). In the first part of this talk, we will obtain such bounds for the interior Dirichlet to Neumann map. Building on results from [2], we prove that the norm of the inverse of the boundary single-layer potential operator grows at worst as a polynomial function of the wavenumber, provided that a set of positive wavenumbers of arbitrarily small Lebesgue measure is excluded. This result holds even in cases where the exterior of the obstacle is strongly trapping, and although the integral equation fails to be uniquely solvable at every Dirichlet eigenvalue of the domain.

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This is joint work with my supervisor Simon N. Chandler-Wilde.

Karl Michael Schmidt, Cardiff University

On the continuum limit for discrete Dirac operators on square lattices

Abstract. The talk discusses the continuum limit of discrete Dirac operators on the two-dimensional square lattice as the mesh size tends to zero. We use the most natural and simplest embedding of the discrete Hilbert space into the continuum Hilbert space, and the question arises naturally when discretising the Dirac operator in two-dimensional Euclidean space, e.g. for numerical analysis. The discrete Dirac operator converges to the continuum Dirac operator in the strong resolvent sense, but not in the norm resolvent sense. The latter result is closely related to the observation that the Liouville theorem does not hold in discrete complex analysis. These results extend to the three-dimensional Dirac operator. This is joint work with Tomio Umeda.

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Anna Rozanova-Pierrat, Supélec

Spectral stability under domain convergence in a class of non-Lipschitz uniform domains

Abstract. I will present recent results from **[1]** and **[2]** (see also **[3]** for the independant on the boundary measure trace theory), focusing on the stability questions under domain convergence. Firstly, I will introduce the functional framework of the trace operator allowing to work with boundaries as supports of the upper regular Borel measures. Hence, these supports can define non-Lipschitz, and possibly fractal/multi-fractal, boundaries. In this framework, I introduce generalized Dirichlet, Neumann, and Robin problems for Poisson-type equations, for which we

proved the Mosco convergence of the associated energy functionals along sequences of suitably converging domains. Generally, the Mosco convergence does not imply the operator norm convergence of resolvents. I will present the sufficient conditions on the domain convergence which imply a stability result for weak solutions, the norm convergence of the associated resolvents, and the convergence of the corresponding eigenvalues and eigenfunctions.

In the end of my talk, if I have some time, I will finish with the existence of optimal shapes for the Robin boundary problems (not known before [**1**]) in the parametrized classes of admissible domains in the sense that they minimize the initially given energy functionals. The keys for this result are the uniform on the shape of the domains the Poincaré inequality and the compactness of the introduced parametrized classes of admissible domains.

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I would like to thank the CNRS IEA ("International Emerging Actions 2022") project Functional and applied analysis with fractal or non-Lipschitz boundaries for the financial support in attending the conference.

Iveta Semorádová, Cardiff University & Czech Technical university

$\mathcal{PT}$-symmetric oscillators with one-center point interactions

Abstract. We investigate the spectrum of Schrödinger operators with imaginary polynomial potentials in $L^2(R)$, perturbed with δ, or δ' interaction, centered at the origin

$$-\partial_x^2 + ix^{2k-1} + \alpha\delta, \quad -\partial_x^2 + ix^{2k-1} + \beta\delta', \tag{11}$$

where $\alpha \in R$, $\beta \in R$, $k \in N$.

It is well established that the spectrum of the unperturbed operators consists of countable many real, isolated and simple eigenvalues for $k \geq 2$, and it is empty for $k = 1$.

When $\alpha \neq 0$ or $\beta \neq 0$, for $k \geq 1$, we observe countable many non-real eigenvalues appearing in complex conjugate pairs, and at maximum finitely many real eigenvalues. The non-real eigenvalues asymptotically converge to the eigenvalues of the unperturbed problems defined on $L^2(R_+)$ and $L^2(R_-)$ with Dirichlet, resp. with Neumann boundary conditions for δ, resp. δ' interaction.

Moreover, for $\alpha \leq C_k < 0$, we show the existence of negative real eigenvalue, diverging to $-\infty$ as $\alpha \to -\infty$.

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Christiane Tretter, University of Bern

Spectral bounds for damped systems

Abstract. In this talk we present enclosures for the spectra of operators associated with second order Cauchy problems for the case of non-selfadjoint damping. These new results yield much better bounds than the numerical range for both uniformly accretive and sectorial damping, and even in the case of selfadjoint damping. Applications e.g. to wave equations illustrate the results.
(joint work with B. Jacob, C. Trunk and H. Vogt as well as with N. Hefti)

Christian Wyss, University of Wuppertal

Computing the quadratic numerical range

Abstract. We present a new algorithm for the computation of the quadratic numerical range of a matrix. So far the canonical approach has been random vector sampling. While the random vector method works well for matrices of small dimension, it fails to compute the full quadratic numerical range of moderately sized matrices already. This is due to a concentration phenomenon, which makes it increasingly unlikely for the randomly computed points to lie close to the boundary of the quadratic numerical range. In our new algorithm we overcome this difficulty by using a steepest ascent procedure to generate sequences of points that converge to the boundary. We combine this with additional techniques to handle non-convex parts of the boundary and to correctly fill the interior. In a side-by-side comparison we illustrate that the resulting algorithm performs significantly better than random vector sampling.

4.17. **Positive Operators and Their Dynamics.**
Organizers: Jochen Glueck (Wuppertal), Anke Kalauch (Dresden)

Monday 14:00-16:00 (CHLT) chair: Jochen Glück

14:00-14:25 Florian Boisen
A generalization of Riesz homomorphisms on order unit spaces*
14:30-14:55 Janko Stennder
Markov operators on order unit spaces
15:00-15:25 Onno van Gaans
The order center and the algebraic center of a JB-algebra
15:30-15:55 Melchior Wirth
Symmetric quantum Markov semigroups and their generators

Monday 16:30-19:00 (CHLT) chair: Anke Kalauch

16:30-16:55 Christian Budde
Positive Desch-Schappacher perturbations of bi-continuous semigroups on AM-*spaces*
17:00-17:25 Marianna Porfido
Kernel estimates for parabolic systems of PDEs with unbounded coefficients
17:30-17:55 Sahiba Arora
Asymptotics of eventually positive semigroups
18:00-18:25 Jonathan Mui
Positivity as a stability condition

Tuesday 14:00-16:00 (SIBSR2) chair: Anke Kalauch

14:00-14:25 Alexander Dobrick
On buffered flows in infinite networks
14:30-14:55 Marianne Akian
Escape rate games and competitive spectral radii
15:00-15:25 Painos Chitanga
Hausdorff dimension of continued fractions and Perron-Frobenius operators
15:30-15:55 Julian Hölz
Uniform ergodicity of Banach lattice homomorphisms

4.17.1. *Abstracts.*

Marianne Akian, Inria and CMAP, École polytechnique, CNRS, IP Paris

Escape Rate Games and competitive spectral radii

Abstract. We consider a new class of repeated zero-sum games in which the payoff of one player is the escape rate of a dynamical system which evolves according to a nonexpansive nonlinear operator depending on the actions of both players. Considering order preserving finite dimensional linear operators over the positive cone endowed with Hilbert's projective (semi-)metric, we recover the matrix multiplication games introduced by Asarin et al. [1], which generalize the joint spectral radius of sets of nonnegative matrices. We establish a two-player version of Mañe's Lemma characterizing the value of the game in terms of a nonlinear eigenproblem. We deduce the existence of optimal strategies of both players. This is motivated by applications to population dynamics (growth maximization and minimization). This also provides a vector valued generalization of mean-payoff games.

This is a joint work with Stéphane Gaubert (Inria and CMAP, École polytechnique, CNRS, IP Paris), and Loic Marchesini (CMAP, École polytechnique, CNRS, IP Paris and Inria).

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Sahiba Arora, University of Twente

Asymptotics of eventually positive semigroups

Abstract. In many concrete applications, operator semigroups exhibit positivity, ensuring that a positive initial datum leads to a positive solution for all time $t \geq 0$. Consequently, positive semigroups have been extensively studied. However, recent years have witnessed a surge in the exploration of *eventually positive* semigroups, where a positive initial datum results in a solution that becomes and remains positive (only) for sufficiently large times.

This talk delves into the question of whether the favourable asymptotic properties observed in positive semigroups extend to eventual positivity. We explore the challenges encountered in adapting proofs and discuss strategies to overcome these obstacles.

Florian Boisen, Dresden University of Technology

A generalization of Riesz homomorphisms on order unit spaces*

Abstract. Riesz homomorphisms between vector lattices are generalized by van Haandel to Riesz* homomorphisms between pre-Riesz spaces. Riesz* homomorphisms are characterized, intrinsically, via a condition on finite sets. Originally, van Haandel claimed that sets with at most two elements are sufficient. In this talk, I illustrate that, even in the setting of finite-dimensional order unit spaces, this is not true, in general.

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Christian Budde, University of the Free State

Positive Desch-Schappacher perturbations of bi-continuous semigroups on AM*-spaces*

Abstract. In this talk, we consider positive Desch-Schappacher perturbations of bi-continuous semigroups on AM-spaces with an additional property concerning the additional locally convex topology [2]. As an example, we discuss perturbations of the left-translation semigroup on the space of bounded continuous functions on the real line and on the space of bounded linear operators. As main reference serves the work of A. Batkái, B. Jacob, J. Wintermayr and J. Voigt [1] on positive Desch-Schappacher perturbations of strongly continuous operator semigroups.

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Painos Chitanga, University of Kent

Hausdorff Dimension of continued fractions and Perron-Frobenius operators

Abstract. Given a subset E of $\mathbb{N}$ the set of continued fraction expansions is given by

$$J_E = \{x \in (0,1) \colon x = [a_1, a_2, a_3 \cdots] \text{ with } a_i \in E \text{ for all } i\},$$

where

$$[a_1, a_2, a_3, \cdots] = \cfrac{1}{a_1 + \cfrac{1}{a_2 + \frac{1}{a_3 + \cdots}}}.$$

These sets typically have a fractal nature and their Hausdorff dimension, denoted $\dim_{\mathcal{H}}(J_E)$, has been studied extensively. In this talk we will discuss the *dimension spectrum of* E,

$$\mathrm{DS}(E) = \{\dim_{\mathcal{H}}(J_F) \colon F \subseteq E\},$$

for different sets E. The structure of the dimension spectrum can be analysed using Perron-Frobenius operators, which are positive operators. Among other results we will see how the spectral theory of these operators can be used to answer a question raised by Chousionis, Leykekhman and Urbanski [1] concerning the dimension spectrum of sets of powers $E_k = \{n^k \colon k = 1, 2, 3, \ldots\}$.

Based on joint work with Bas Lemmens and Roger Nussbaum.

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Alexander Dobrick, CAU Kiel

On buffered flows in infinite networks

Abstract. We consider a transport problem on an infinite metric graph, focusing on its well-posedness and long-term behaviour, given that the mass flow is buffered in at least one vertex. In this context, we address the well-posedness of the flow by leveraging a recent boundary perturbation result on AL-spaces. Furthermore, we discuss the long-term behaviour of the flow using recent results about the convergence of stochastic semigroups that dominate a kernel operator. This is joint work with Florian G. Martin.

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Julian Hölz, University of Wuppertal

Uniform ergodicity of Banach lattice homomorphisms

Abstract. Measure preserving and topological dynamical systems can be associated with a homomorphism of a Banach lattice of functions by means of so-called composition operators. By studying properties of lattice homomorphisms, we thus show that the uniform ergodicity for topological dynamical systems implies eventual periodicity and for measure preserving system it implies periodicity.

Jonathan Mui, University of Wuppertal

Positivity as a stability condition

Abstract. This talk concerns C_0-semigroups and resolvent operators on Banach lattices which are not only positive, but even 'positivity improving' in a precise sense. This stronger property is often encountered in PDE problems, especially in the context of diffusion equations (e.g. [1]). Loosely speaking, the main goal is to demonstrate how positivity improvement is stable under bounded perturbations in an abstract setting which is motivated by the study of Schrödinger operators and applications to semilinear evolution equations [2].

This is joint work in progress with Daniel Daners and Jochen Glück.

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Marianna Porfido, TU Bergakademie Freiberg

Kernel estimates for parabolic systems of PDEs with unbounded coefficients

Abstract. In this talk we consider a class of systems of nondegenerate elliptic partial differential equations with unbounded coefficients with possibly unbounded diffusion coefficients which may vary equation by equation. In particular, we deal with a vector-valued elliptic operator $\boldsymbol{\mathcal{A}}$ in divergence form defined on smooth functions $\boldsymbol{f} : \mathbb{R}^d \to \mathbb{R}^m$ by

$$(\boldsymbol{\mathcal{A}}\boldsymbol{f})_h = \operatorname{div}(Q^h \nabla f_h) + \langle b^h, \nabla f_h \rangle - (V\boldsymbol{f})_h$$

for $h = 1, \dots, m$, where $Q^h : \mathbb{R}^d \to \mathbb{R}^{d\times d}$, $b^h : \mathbb{R}^d \to \mathbb{R}^d$ for every $h = 1, \dots, m$ and $V : \mathbb{R}^d \to \mathbb{R}^{m\times m}$. Under suitable assumptions, we prove pointwise upper bounds for the transition kernels of the semigroup associated in $C_b(\mathbb{R}^d; \mathbb{R}^m)$ with the operator $\boldsymbol{\mathcal{A}}$. The idea is to adapt and generalize to our setting the techniques exploited in the scalar case based on time-dependent

Lyapunov functions for the parabolic operator $D_t + \boldsymbol{\mathcal{A}}$. We finally illustrate our results in case of polynomially and exponentially growing coefficients.

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Janko Stennder, TU Dresden

Markov operators on order unit spaces

Abstract. Let Ω be a topological space. An operator T on $\mathrm{C}(\Omega)$ is called a Markov operator if T is positive and maps the constant 1 function to itself. It is well-known that T is an extreme point in the set of Markov operators if and only if T is a Riesz homomorphism. This was also shown in the more general context of unital f-algebras. We investigate Markov operators in the setting of order unit spaces. By means of the functional representation order unit spaces can be represented as order dense subspaces of $\mathrm{C}(\Omega)$ for some compact Hausdorff space Ω. We present a similar result as above in the case that the order unit space is a subalgebra of its functional representation.

Onno van Gaans, Leiden University

The order center and the algebraic center of a JB-algebra

Abstract. The vector space of all self-adjoint operators on a Hilbert space with the Jordan product $A \circ B = (AB + BA)/2$ is a typical example of a JB-algebra. A JB-algebra is both a Jordan algebra and a Banach space with with a suitably compatible norm. A JB-algebra is commutative but typically not associative. The set of all squares in a JB-algebra is a closed cone and, with the induced order, a JB-algebra is an Archimedean directed partially ordered vector space.

There are two natural notions of a center in a JB-algebra. The algebraic center of a JB-algebra is defined to be the set of those elements whose corresponding left multiplication operator commutes with all other left multiplication operators. The order center is the subspace of all linear operators on the JB-algebra consisting of the operators that are below and above a multiple of the identity operator, where the space of operators is ordered by the cone of positive operators. We will show that the order center and the algebraic center of a unital JB-algebra are isomorphic.

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Melchior Wirth, Institute of Science and Technology

Symmetric Quantum Markov Semigroups and Their Generators

Abstract. A quantum Markov semigroup is a point-weak* continuous semigroup of unit-preserving, completely positive maps on a von Neumann algebra. For such semigroups there are several non-equivalent notions of symmetry, including GNS and KMS symmetry. In this talk I will report on recent progress in the project to characterize the generators of GNS- and KMS-symmetric quantum Markov semigroups and discuss some open questions. (This is partly joint work with Matthijs Vernooij.)

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The author was funded by the Austrian Science Fund (FWF) under the Esprit Programme [ESP 156].

4.18. **Operator Semigroups and Evolution Equations.**
Organizers: Lyonell Boulton (Heriot-Watt), Christian Budde (Free State)

Monday 14:00-16:00 (SIBLT3) chair: Lyonell Boulton

14:00-14:25 Alexander Dobrick
Towards a general framework for queueing and reliability theory
14:30-14:55 David Seifert
A Katznelson-Tzafriri theorem for analytic Besov functions
15:00-15:25 Felix Schwenninger
On the solvability of the radiative transfer equations with polarization
15:30-15:55 Christian Budde
Well-posedness of non-autonomous transport equation on metric graphs

Tuesday 16:30-19:00 (SIBLT3) chair: Christian Budde

16:30-16:55 Hannes Gernandt
On a class of dissipative boundary control systems and networks
17:00-17:25 François Genoud
Finite time blow-up for the nonlinear Schrödinger equation on a star graph
17:30-17:55 Lyonell Boulton
Weak revivals in time-evolution models

4.18.1. *Abstracts.*

Lyonell Boulton *Weak revivals in time-evolution models*

Abstract. The mysterious phenomenon of *revivals* in linear dispersive periodic equations, was discovered first experimentally in optics in around 1834, then rediscovered several times by theoretical investigations. While the term has been used systematically and consistently by many authors, there is no consensus on a rigorous definition. Several have described it by stating that a given periodic time-dependent boundary value problem exhibits *revivals at rational times*, if the solution evaluated at a certain dense subset of times is given by finite superposition of translated copies of the initial conditions. When this initial condition has jump discontinuities at time zero, these discontinuities propagate and remain present in the solution at each rational time but disappear completely at irrational times.

In this talk, I will report on the presence of revivals in three distinct models of parabolic differential equations: (1) non-local equations that arise in water wave theory and are defined by convolution kernels [1]; (2) Schrödinger equations with different types of boundary conditions [2] or with complex potentials [3]; (3) dislocated Laplacian time-evolution equations [4]. As we shall see, in all cases the solution is given explicitly by finite combination of translations, dilations and scaling of the initial datum, plus additional regular terms. When present, these extra terms can be interpreted as a weak manifestation of the classical revivals phenomenon.

The research has been conducted jointly with George Farmakis (London South Bank University), Peter Olver (University of Minnesota), Beatrice Pelloni (Heriot-Watt University) and David Smith (Yale NUS).

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Christian Budde

Well-posedness of non-autonomous transport equation on metric graphs

Abstract. We consider transport processes on metric graphs with time-dependent velocities and show that, under continuity assumption of the velocity coefficients, the corresponding non-autonomous abstract Cauchy problem is well-posed by means of evolution families and evolution semigroups.

Consider a finite network (i.e., of pipelines) where some material is transported along its branches (i.e., pipes). The velocity of the transport depends on a given branch but may also change in time. We would like to know under which condition such a system can be modelled in a way that for any given initial distribution we are able to predict the state of the system in any time. We would also like to obtain stable solutions that continuously depend on the initial state. In this case we will call our problem well-posed.

Such transport problems on networks have already been studied by several authors. The operator theoretical approach by means of abstract Cauchy problems on Banach spaces was initiated by the second author and Sikolya [3]. However, the majority of the publications concentrates on time-independent transport and hence autonomous abstract Cauchy problems. A first attempt to non-autonomous problems of this kind was performed by Bayazit et al. [1]. They considered transport on networks with boundary conditions changing in time. The advantage of such an

approach is that the corresponding operator does not change its action on the Banach space, only its domain changes in time. Our aim is to consider also the non-autonomous operator, that is, we study transport problems on finite metric graphs with time-dependent velocities along the edges. We use evolution families and evolution semigroups as studied by Nickel [4] and show that the abstract Cauchy problem, which can be associated to the transport equation on these graphs, is well-posed.

This talk is based on joint work with M. Kramar Fijavž [2].

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Alexander Dobrick, CAU Kiel

Towards a general framework for queueing and reliability theory

Abstract. Building on Greiner's result on boundary perturbations, we develop an abstract framework within the context of AL-spaces. This framework allows proving a generation result for C_0-semigroups arising from various examples from the queueing and reliability theory. Leveraging recent results of Glück, Gerlach and Martin, it further allows investigating the long-term behaviour of these semigroups without requiring an in-depth analysis of the spectrum of their generators. Finally, we sketch an extension of the framework that can be used to discuss buffered transport problems on infinite networks.

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François Genoud, EPFL

Finite time blow-up for the nonlinear Schrödinger equation on a star graph

Abstract. The construction of a finite time blow-up solution for a nonlinear Schrödinger equation (NLS) on a star graph will be presented. The simplest configuration of a graph with two branches corresponds to the NLS on the line with a delta potential at the origin. The general case involves a one-dimensional Laplace operator on the graph with Robin boundary conditions at the vertex. The blow-up analysis relies on the resolution of the nonlinear Cauchy problem within the domain of the corresponding linear operator. This is joint work with Stefan Le Coz and Julien Royer.

Hannes Gernandt, University of Wuppertal

On a class of dissipative boundary control systems and networks

Abstract. In this talk, we consider a class of dissipative boundary control systems whose dynamics is generated by a 2x2 block operator in a Hilbert space that has a bounded dissipative diagonal and a possibly unbounded skew-adjoint off-diagonal. Sufficient conditions for the strong and exponential stability of the underlying semigroup generators are provided along with the derivation of a power balance equation for classical solutions of the boundary control system. Furthermore, we consider interconnections of several such dissipative boundary control systems and show that Kirchhoff-type interconnections preserve the underlying structure of the considered block operators, and thus, of the aforementioned stability and passivity properties. The results are illustrated for a power network connecting several prosumers via distributed transmission lines that are modeled based on the telegraph equations.

This talk is based on the preprint [1] with Dorothea Hinsen (TU Berlin)

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Felix L. Schwenninger, University of Twente

On the solvability of the radiative transfer equations with polarization

Abstract. The well-posedness of the radiative transfer equation with polarization and varying refractive index is investigated using semigroup techniques. This includes non-homogeneous boundary value problems on bounded spatial domains, which requires the analysis of suitable trace spaces. Additionally, we discuss positivity, Hermiticity, and norm-preservation of the matrix-valued solution. This is joint work with M. Schlottbom and V. Bosboom (Twente).

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David Seifert, Newcastle University

A Katznelson–Tzafriri theorem for analytic Besov functions

Abstract. Let $-A$ be the generator of a bounded C_0-semigroup $(T(t))_{t\geq 0}$, and suppose that A admits a bounded functional calculus with respect to an algebra $\mathcal{A}$ of holomorphic functions on the the open right half-plane $\mathbb{C}_+$. Theorems of Katznelson–Tzafriri type provide sufficient conditions under which

$$(*) \qquad \lim_{t\to\infty} \|T(t)f(A)\| = 0$$

for suitable functions $f \in \mathcal{A}$. Such theorems play an important role in the asymptotic theory of C_0-semigroups, and in particular have been used to give an alternative proof of the famous countable spectrum theorem. The main result to be presented in this talk is a new Katznelson–Tzafriri theorem for operators A admitting a bounded functional calculus with respect to a certain algebra $\mathcal{B}$ of analytic Besov functions. The theorem states that $(*)$ holds for all $f \in \mathcal{B}$ such that f vanishes on the boundary spectrum $\sigma(A) \cap i\mathbb{R}$ of A and $|f(z)| \to 0$ as $|z| \to \infty$ with $z \in \mathbb{C}_+$. The talk is based on joint work with Charles Batty.

4.19. **Fractional Calculus Operators and Their Applications.**
Organizers: Arran Fernandez (Eastern Mediterranean University), Milton Ferreira (Polytechnic of Leiria), Manuela Rodrigues (University of Aveiro), Nelson Vieira (University of Aveiro)

Tuesday 14:00-16:00 (SIBSR5) chair: Nelson Vieira

14:00-14:25 Jaan Janno
Inverse problems to determine sources of fractional diffusion equations in a form of separated variables
14:30-14:55 Hafiz Muhammad Fahad
Generalised operational calculus approach for fractional differential equations
15:00-15:25 Haniyyah Ul Irshad
Transmuted fractional operators with general analytic kernels
15:30-15:55 M. Manuela Rodrigues
Generalized fractional gradient

Thursday 14:00-16:00 (SIBSR4) chair: Milton Ferreira

14:00-14:25 Arran Fernandez
Extending operational calculus and Sonine kernels to higher dimensions
14:30-14:55 Sunday Simon Isah
Bivariate fractional calculus with general bivariate analytic kernels
15:00-15:25 Katarzyna Gorska
Operator solutions for fractional Fokker-Planck equation and diffusion-wave equation
15:30-15:55 Tobiasz Pietrzak
The wave aspect of the solution to the generalized telegraph equation

Thursday 16:30–18:30 (SIBSR4) chair: Arran Fernandez

16:30-16:55 Katarzyna Ryszewska
Harnack estimates for parabolic-type problems with time nonlocalities
17:00-17:25 Nelson Vieira
Dirac's method applied to the time-fractional telegraph equation

4.19.1. *Abstracts.*

Hafiz Muhammad Fahad, National University of Sciences and Technology, Islamabad, Pakistan

Generalised Operational Calculus Approach for Fractional Differential Equations

Abstract. Mikusiński's operational calculus is a method for interpreting and solving fractional differential equations, formally similar to Laplace transforms but more rigorously justified. This formalism was established for Riemann–Liouville and Caputo fractional calculi in the 1990s, and more recently for other types of fractional calculus. In this talk, we consider the operators of Riemann–Liouville and Caputo fractional differentiation of a function with respect to another function, and discover that the approach of Luchko can be followed, with small modifications, in the more general settings too. We establish all the function spaces, formalisms, and identities required to build the versions of Mikusiński's operational calculus which cover Riemann–Liouville and Caputo derivatives with respect to functions. The mathematical structure established here is used to solve fractional differential equations using Riemann–Liouville and Caputo derivatives with respect to functions, the solutions being written using multivariate Mittag-Leffler functions, in agreement with the results found in other recent work.

It is useful to understand how the various operators of fractional calculus relate to each other, especially relations between newly defined operators and classical well-studied ones. If time allows, we will also focus on an important type of such relationship, namely conjugation relations, also called transmutation relations. We define a general abstract setting in which such relations are relevant, and indicate how they can be used to prove many results easily in general settings such as fractional calculus with respect to functions and weighted fractional calculus.

Arran Fernandez, Eastern Mediterranean University

Extending operational calculus and Sonine kernels to higher dimensions

Abstract. Mikusiński's operational calculus is an algebraic method for interpreting integro-differential operators and solving equations that involve these operators. Starting from the 1990s, it was used to solve, for the first time, multi-term incommensurate linear fractional differential equations [1,2]. These solutions were constructed in the Dimovski space

$$C_\alpha = \left\{ f : (0,\infty) \to \mathbb{C} \;\middle|\; f(t) = t^p f_1(t),\, p > \alpha,\, f_1 \in C[0,\infty) \right\},$$

which has also been used recently as a setting for the theory of Sonine kernels [3,4].

In the current work, we discuss how the method of Mikusiński's operational calculus, as well as the concept and spaces of Sonine kernels, can be extended to higher dimensions. Instead of considering functions f of a single variable, we study functions of several variables, and solve some simple fractional PDEs for such functions. Part of this work is soon to appear in [5].

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Katarzyna Górska, Institute of Nuclear Physics, Polish Academy of Sciences

Operator solutions for fractional Fokker-Planck equation and diffusion-wave equation

Abstract. The evolution operator method will be applied to solve the fractional Fokker-Planck equation and diffusion wave equation in a $(1+1)$-dimensional setting involving time derivatives which are smeared using a power-law memory function. Two types of evolution operators with kernels determined by functions proportional to the stable distribution will be presented. It will be demonstrated that distinguishing which equations govern the spreading particles evolution, if based solely on the mean square displacement knowledge, becomes challenging for longer periods of time. The examination of evolution operators' evolution and their self-reproducing properties will also be conducted.

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I would like to thank the National Science Center, Poland, Programe Preludium Bis for their generous support.

Haniyyah Ul Irshad, National University of Sciences and Technology

Transmuted Fractional Operators with General Analytic Kernels

Abstract. Many different types of fractional calculus are defined by various kernel functions, and one example of a very general class is fractional calculus with analytic kernels. Others arise from transmuting the usual fractional calculus with invertible linear operators, for example, composition and multiplication operators. In this talk, we combine these two ideas to create a new and very general model of fractional calculus using analytic kernels with transmutations. We prove fundamental theorems of calculus and other results on function spaces and compositions in the framework of these general operators. As special cases, we obtain left-sided and right-sided operators with analytic kernels on arbitrary intervals, as well as operators with analytic kernels with respect to functions, weighted operators with analytic kernels, and more. We also briefly discuss the utility of a generalised Laplace transform for solving fractional differential equations in the setting of generalised transmuted fractional operators with analytic kernels.

Sunday Simon Isah, Eastern Mediterranean University

Bivariate Fractional Calculus with General Bivariate Analytic Kernels

Abstract. We use a general bivariate analytic function with fractional power substitutions to define a bivariate fractional integral operator which can be expressed as a double infinite sum of classical Riemann—Liouville operators, using analyticity, which allows many interesting and useful fundamental properties. We further consider inversion properties of our proposed model, which in turn motivate the definition of a bivariate fractional derivative operator based on the same bivariate analytic kernels modified by fractional powers. We then prove the analogues of the fundamental theorems of calculus, Leibniz rule, and consider the functional maps and bounds, Laplace, Fourier, and Mellin transforms in this model of bivariate fractional calculus. As an application, we consider some illustrative examples which have already found applications in the literature using our new model.

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Jaan Janno, Tallinn University of Technology

Inverse problems to determine sources of fractional diffusion equations in a form of separated variables

Abstract. We consider inverse problems to determine source terms of the form

$$F(t,x) = g(t)f(x) \tag{$*$}$$

of a time-fractional diffusion equation of the order $\alpha \in (0,1)$ in a bounded domain Ω. In a first class of problems a normal derivative of u over a portion of a boundary of Ω in a time interval $(T-\epsilon, T)$ is given. Uniqueness of f is proved. Under the additional condition that α is irrational, the uniqueness of full F of the form ($*$) is shown. The proofs use a branch line of a kernel of a solution operator of a direct problem. In a second class of problems an instant over-determination condition for u at $t = T$ is given. Uniqueness of g is shown. The proof uses asymptotics at ∞ of Mittag-Leffler functions involved in the solution operator of the direct problem.

Tobiasz Pietrzak, Institute of Nuclear Physics Polish Academy of Sciences

The wave aspect of the solution to the generalized telegraph equation

Abstract. In order to overcome the problem of the long transmission time of information in the transatlantic telegraph cable, scientists such as Maxwell, Lord Kelvin, and Heaviside began working on a profound understanding of the physics of electrical impulse propagation in long cables. An important achievement from these efforts was the so-called telegraphers' equation introduced by Heaviside. The telegraph equation originally used for transmission line analysis, also has other no less important applications, such as the description of heat transport in liquid helium or the description of the so-called bioheat transfer occurring in biological tissues. The telegraph equation has also been generalized using the fractional derivative concept. An equation of this type (generalized version) has been used to describe anomalous diffusion and chemical reactions, as well as biological applications such as heat conduction in muscles and blood. During the presentation, the generalized telegraph equation with the power-law memory function, its solution, and the frequency shift effect for the obtained solution will be presented.

M. Manuela Rodrigues, Nelson Vieira, CIDMA & University of Aveiro

Generalized fractional gradient

Abstract. Motivated by the increasing practical applications in fractional calculus, we study the classical gradient method under the perspective of the ψ-Hilfer derivative. This allows us to cover in our study several definitions of fractional derivatives that are found in the literature. We develop an algorithm for the ψ-Hilfer fractional order gradient method using a series representation of the target function. Using benchmark functions, the numerical method obtained by truncating higher-order terms was tested and analyzed. Considering variable order differentiation and optimizing the step size, the ψ-Hilfer fractional gradient method shows better results in terms of speed and accuracy. Our results generalize previous works in the literature.

This is a joint work with M. Ferreira (Polytechnic University of Leiria & CIDMA) and N. Vieira (CIDMA & University of Aveiro).

References

[1] N. Vieira, M.M. Rodrigues, and M. Ferreira, Fractional gradient methods via ψ-Hilfer derivative, *Fractal and Fractional*, **7**-No.3, (2023), Article No. 275 (30pp.).

Katarzyna Ryszewska, Warsaw University of Technology

Harnack estimates for parabolic-type problems with time nonlocalities

Abstract. The theory of Harnack inequalities is a wide and important topic in the analysis of elliptic and parabolic equations. It provides several properties of weak solutions to respective problems, the most significant of which is the Hölder continuity. Thus, developing this theory for nonlocal problems is very desirable. In the talk I will describe the main ideas regarding the application of de Giorgi-Nash -Moser theory for nonlocal problems. Firstly, to give an intuition, I will present the results concerning the Harnack estimates for parabolic-type problem with time fractional derivative of order $\alpha \in (0, 1)$. Then, I will comment on recent results for problems with more general time nonlocality.

This is a joint work with Prof. Rico Zacher and Adam Kubica.

Nelson Vieira, CIDMA & University of Aveiro

Dirac's method applied to the time-fractional telegraph equation

Abstract. The free Dirac equation arises from the factorization of the Klein-Gordon equation using matrix coefficients satisfying anticommutation relations. In this talk, we focus on the factorization of the multidimensional time-fractional telegraph equation, applying Dirac's factorization method.

Explicit representations for solutions in the Fourier domain will be presented in terms of bivariate Mittag-Leffler functions and some results will be presented regarding their asymptotic behaviour at the origin and infinity. To obtain explicit representations of our solutions in the space-time domain, new results were obtained involving the inverse Fourier transform, bivariate Mittag-Leffler functions and two-variable Fox-H functions. Finally, some graphical representations of our solutions in the Fourier domain will be presented.

This is a joint work with M. Ferreira (Polytechnic University of Leiria & CIDMA) and M.M. Rodrigues (CIDMA & University of Aveiro).

4.20. **Operators on Banach Spaces and Lattices.**
Organizers: Niels Laustsen (Lancaster), Kevin Beanland (Washington & Lee University)

Monday 14:00-16:00 (SIBSR2) chair: Niels Laustsen

14:00-14:25 Mitchell Taylor
Free Banach lattices
14:30-14:55 Krystian Kazaniecki
Martingale Type, the Gamlen–Gaudet Construction and a Greedy Algorithm

Tuesday 16:30-19:00 (SIBSR2) chair: Niels Laustsen

16:30-16:55 Jacek Chmieliński
Approximate smoothness of operators
17:00-17:25 Syamantak Das
Various generalizations of generalized centers and related geometric properties in Banach spaces
17:30-17:55 Tanmoy Paul
U-embedding of Banach spaces
18:00-18:25 Elroy Zeekoei
Some Dunford–Pettis-like properties in Banach lattices

Thursday 14:00-16:00 (SIBSR2) chair: Niels Laustsen

14:00-14:25 Richard Lechner
Haar multipliers on $L^p(L^1)$
14:30-14:55 Thomas Speckhofer
Factorization in Haar system Hardy spaces
15:00-15:25 James Smith
Closed ideals of operators on Baernstein spaces
15:30-15:55 Sukumar Daniel
Exponential spectrum commutes in $B(\ell_p \oplus \ell_q)$

Friday 14:00-16:00 (SIBSR2) chair: Niels Laustsen

14:00-14:25 Richard Smith
L-embeddability in Lipschitz-free spaces
14:30-14:55 Tomasz Kania
Polish spaces of Banach lattices
15:00-15:25 Jan Bíma
Nagata Dimension and Lipschitz Extensions Into Quasi-Banach Spaces
15:30-15:55 Natalia Maślany
On isometries and Tingley's problem for combinatorial Tsirelson spaces

4.20.1. *Abstracts.*

Jan Bíma, Charles University

Nagata Dimension and Lipschitz Extensions Into Quasi-Banach Spaces

Abstract. Given two metric spaces $\mathcal{N} \subseteq \mathcal{M}$ in inclusion and $0 < p \leq 1$, we wish to determine the smallest constant $\mathfrak{t}_p(\mathcal{N}, \mathcal{M})$ such that any Lipschitz map $f : \mathcal{N} \to Z$ into any p-Banach space Z can be extended to a Lipschitz map $f' : \mathcal{M} \to Z$ satisfying $\operatorname{Lip} f' \leq \mathfrak{t}_p(\mathcal{N}, \mathcal{M}) \cdot \operatorname{Lip} f$. In my talk, I will present a recent result showing that if $\mathcal{N}$ has finite Nagata dimension at most d with constant γ, then $\mathfrak{t}_p(\mathcal{N}, \mathcal{M}) \lesssim_p \gamma \cdot (d+1)^{1/p-1} \cdot \log(d+2)$ for all $0 < p \leq 1$. I will show that examples of spaces with finite Nagata dimension include doubling spaces, as well as minor-excluded metric graphs. Interestingly, I will also establish that the constant $\mathfrak{t}_p(\mathcal{N}, \mathcal{M})$ generally increases as p approaches zero.

References

[1] J. Bíma, "Nagata Dimension and Lipschitz Extensions Into Quasi-Banach Spaces," *arXiv preprint arXiv:2402.03189*, 2024. URL: https://arxiv.org/abs/2402.03189.

Jacek Chmieliński, UKEN Krakow

Approximate smoothness of operators

Abstract. We develop the notion of approximate smoothness introduced in [1] and apply it for spaces of operators acting on Banach spaces. Some results from [1] will be presented as well as new ones.

References

[1] J. Chmieliński, D. Khurana and D. Sain, *Approximate smoothness in normed linear spaces*, Banach J. Math. Anal. (2023), 17:41.

Syamantak Das, Indian Institute of Technology Hyderabad

Various generalizations of Generalized centers and related geometric properties in Banach spaces

Abstract. Veselý (1997) developed the idea of generalized centers for finite sets in Banach spaces. In this talk, we explore the concept of *restricted $\mathcal{F}$-center property* for a triplet $(X, Y, \mathcal{F}(X))$, where Y is a subspace of a Banach space X and $\mathcal{F}(X)$ is the family of finite subsets of X. In addition, we generalize the analysis to include all closed, bounded subsets of X. We show that Y has $n.X.I.P.$ in X for all natural numbers n if and only if $\operatorname{rad}_Y(F) = \operatorname{rad}_X(F)$ for all finite subsets F of Y. It then turns out that, for all continuous, monotone functions f, the f-radii viz. $\operatorname{rad}_Y^f(F), \operatorname{rad}_X^f(F)$ are the same whenever the generalized radii viz. $\operatorname{rad}_Y(F), \operatorname{rad}_X(F)$ are also the same, for all finite subsets F of Y. We establish a variety of characterizations of central subspaces of Banach spaces. With reference to an appropriate subfamily of closed and bounded subsets, we will see that a number of function spaces and subspaces exhibit the restricted weighted Chebyshev center property.

References

[1] Syamantak Das, Tanmoy Paul, *A study on various generalizations of Generalized centers* (**GC**) *in Banach spaces*, arXiv:2311.15818.

[2] Libor Veselý, *Generalized centers of finite sets in Banach spaces*, Acta Math. Univ. Comenian. (N.S.) **66**(1) (1997), 83–115.

I would like to thank IIT Hyderabad for their generous support.

Jorge González Camus, Universidad Tecnológica Metropolitana

Representation of Solution for Fractional Damped Heat and Wave Equation on an Infinite Lattice Via Subordination Techniques and Banach Algebras.

Abstract. In this talk, we present a study of the non-local in-time damped wave and heat equations on an infinite lattice in the linear case. Under suitable assumptions, we establish a representation of the solutions in terms of subordinators, as well as an explicit formula via discrete Fourier transform, on the Banach Algebras framework. Moreover, we establish sufficient conditions to guarantee the solution as a probability distribution, and we point out the differences to its continuous counterpart. The discrete maximal regularity of the non-homogeneous cases on $\ell^p(\mathbb{Z})$ also are presented.

References

[1] L. Abadias, M. de León-Contreras, J.L. Torrea. *Non-local fractional derivatives. Discrete and continuous.* J. Math. Anal. Appl., **449**(1) (2017) 734–755.

[2] F. Alegría, V. Poblete and J. C. Pozo. *Nonlocal in-time telegraph equation and telegraph processes with random time.* J. Differential Equations, Vol 347,(2023),310–347.

[3] H. Bateman. *Some simple differential difference equations and the related functions.* Bull. Amer. Math. Soc. 49 (1943), 494–512.

[4] O. Ciaurri, T.A. Gillespie, L. Roncal, J.L. Torrea and J.L. Varona. Harmonic analysis associated with a discrete Laplacian, J. d'Analyse Mathématique, 132 (2017), 109–131.

[5] J. González-Camus, C. Lizama and P.J. Miana. Fundamental solutions for semi discrete evolution equations via Banach algebras. Adv Differ Equ, 35 (2021).

[6] E. Orsingher and L. Beghin. *Time-fractional telegraph equations and telegraph processes with Brownian time* Probab. Theory Relat. Fields 128(1) (2004) 141–160.

[7] J.C. Pozo, V. Vergara, A non-local in time telegraph equation, Nonlinear Anal. 193 (2020) 111411.

I would like to thank to ANID, Fondecyt Iniciación 2023 Folio 11230182 for their support.

Tomasz Kania, Jagiellonian University

Polish spaces of Banach lattices

Abstract. Using admissible topologies on spaces of closed subspaces of the universal Banach space C[0,1], introduced by Godefroy and Saint-Raymond in 2018, we develop the descriptive set theory of Polish spaces whose points are separable Banach spaces with extra structure such as Banach lattices. To achieve this, we exploit the recent construction of a free Banach lattice. Within this framework we will show how to bound the Borel complexity of various Banach lattice properties.

Krystian Kazaniecki, Johannes Kepler University Linz

Martingale Type, the Gamlen-Gaudet Construction and a Greedy Algorithm

Abstract. A Banach space X satisfies Martingale Type p if there exists $C = C_p$ such that any filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$ gives rise to the upper ℓ^p estimates,

$$(*) \qquad \|f\|^p_{L^p(\Omega,X)} \leq C^p \left(\|\mathbb{E}(f|\mathcal{F}_0)\|^p_{L^p(\Omega,X)} + \sum_{n=1}^{\infty} \|\mathbf{\Delta}_n f\|^p_{L^p(\Omega,X)} \right)$$

where $\mathbf{\Delta}_n f = \mathbb{E}(f\big|\mathcal{F}_n) - \mathbb{E}(f\big|\mathcal{F}_{n-1})$.

Assuming that $\mathcal{F}_n$ is a sequence of increasing purely atomic sub-σ-algebras of $\mathcal{F}$ and $\mathcal{F} = \sigma(\bigcup \mathcal{F}_n)$ we will identify precisely all filtered probability spaces $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$, that are able to determine the martingale type of Banach space X. We associate explicit intrinsic conditions on the filtration $(\mathcal{F}_n)$ which determine that the upper ℓ^p estimates $(*)$ imply the martingale type p.

Theorem 1. For each fixed $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$ the following dichotomy holds true: Either, there exists $C > 0$ such that for any Banach space X and any $f \in L^p(\Omega, \mathcal{F}, \mathbb{P}, X)$

$$(**) \qquad \|f\|^p_{L^p(\Omega,X)} \leq C \left(\|\mathbb{E}(f|\mathcal{F}_0)\|^p_{L^p(\Omega,X)} + \sum_{n=1}^{\infty} \left\|\mathbb{E}(f\big|\mathcal{F}_n) - (f\big|\mathcal{F}_{n-1})\right\|^p_{L^p(\Omega,X)} \right)$$

or the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$ and upper ℓ^p estimates $(**)$ already determine that the Banach space X is of martingale type p.

Talk is based on a joint work with Paul F.X. Müller [1].

References

[1] K. Kazaniecki, P. F. X. Müller, *Martingale type, the Gamlen-Gaudet construction and a greedy algorithm.* J. Funct. Anal., 286(9):Paper No. 110369, 43, 2024.

I would like to thank the Austrian Science Fund FWF (grant I 5231) for their generous support.

Richard Lechner, Johannes Kepler University Linz

Haar multipliers on $L^p(L^1)$

Abstract. Let (h_I) denote the standard Haar system on $[0,1]$ and let $h_I \otimes h_J$ denote the tensor product $(s,t) \mapsto h_I(s)h_J(t)$. Given $1 \leq p, q < \infty$, we define the *biparameter Lebesgue space* $L^p(L^q)$ as the completion of $\operatorname{span}\{h_I \otimes h_J\}$ under the norm

$$\left\| \sum_{I,J} a_{I,J} h_I(s) h_J(t) \right\|_{L^p(L^q)} = \left\| s \mapsto \left\| t \mapsto \sum_{I,J} a_{I,J} h_I(s) h_J(t) \right\|_{L^q} \right\|_{L^p}.$$

We say that $D\colon L^p(L^q) \to L^p(L^q)$ is a Haar multiplier if $Dh_I \otimes h_J = d_{I,J} h_I \otimes h_J$, where $d_{I,J} \in \mathbb{R}$. The decisive representative of Haar multipliers is the Capon projection $\mathcal{C}\colon L^p(L^q) \to L^p(L^q)$ given by $\mathcal{C}h_I \otimes h_J = h_I \otimes h_J$ if $|I| \leq |J|$, and $\mathcal{C}h_I \otimes h_J = 0$ if $|I| > |J|$, as our main result highlights:

For any bounded Haar multiplier $D\colon L^p(L^q) \to L^p(L^q)$, there exist $\lambda, \mu \in \mathbb{R}$ such that

$$\lambda\mathcal{C} + \mu(\mathrm{Id} - \mathcal{C}) \text{ factors through } D,$$

i.e., there exist bounded operators A, B so that $\lambda\mathcal{C} + \mu(\mathrm{Id} - \mathcal{C}) = ADB$. Additionally, if $\mathcal{C}$ is unbounded on $L^p(L^q)$ (which is true if $p = 1$ or $q = 1$), then $\lambda = \mu$ and the identity operator Id either factors through D or $\mathrm{Id} - D$.

This constitutes an important step towards establishing the primarity of $L^p(L^1)$—which is among the most prominent examples of classical Banach spaces for which primarity is still open.

This talk is based on joint work Motakis, Müller and Schlumprecht.

References

[1] R. Lechner, P. Motakis, P. F. X. Müller, Th. Schlumprecht, Multipliers on bi-parameter Haar system Hardy spaces, *Mathematische Annalen*, https://doi.org/10.1007/s00208-024-02887-9.

Tanmoy Paul, IIT Hyderabad

U-embedding of Banach spaces

Abstract. Given two Banach spaces X, Y, I will discuss some instances when there exists an into isometry $\Phi : Y \to X$ such that $\Phi(Y)$ is a U-subspace of X. We study the cases when X is of the form $C(K)$, the continuous function spaces on a compact Hausdorff space. I will discuss some necessary and sufficient conditions Y must satisfy for this case. I will discuss that even if we consider a particular subclass of functionals on Y have unique norm preserving extensions over $C(K)$ then Y must satisfy some geometric properties similar to $C(K)$.

References

[1] T. S. S. R. K. Rao, *A geometric Jordan decomposition theorem*, Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser.,(2024), 118(68) 1–8.
[2] Francis Sullivan, *Geometric properties determined by the higher duals of Banach spaces*, Illinois J. Math.,(1977) 21(2), 315–331.

James Smith, Lancaster University

Closed ideals of operators on Baernstein spaces

Abstract. In the presence of a basis for a Banach space X, $\mathcal{L}(X)$ contains at most $2^{|\mathbb{R}|}$ closed ideals. In this talk, we discuss how the Baernstein spaces B_p (where $1 < p < \infty$) can be added to the growing list of spaces in which this upper bound is attained. Along the way, we explain how subsequences of the unit vectors in B_p are equivalent precisely when they are equivalent in the Schreier space, a somewhat surprising outcome. Finally, we deduce that $\mathcal{L}(B_p)$ contains at least $|\mathbb{R}|$ maximal closed ideals, and whether $2^{|\mathbb{R}|}$ many could exist. This is joint work with Niels Laustsen.

Richard Smith, University College Dublin, Ireland

L-embeddability in Lipschitz-free spaces

Abstract. Let $\mathrm{Lip}_0(M)$ denote the Banach space of real-valued Lipschitz functions on a complete metric space (M, d) that vanish at a point $0 \in M$. It has a natural isometric predual which is sometimes called the Lipschitz-free space over M, and denoted $\mathcal{F}(M)$. We consider the question of when $\mathcal{F}(M)$ is L-embedded in its bidual $\mathrm{Lip}_0(M)^*$, that is, when there exists a projection $P : \mathrm{Lip}_0(M)^* \to \mathcal{F}(M)$ such that $\|\psi\| = \|P\psi\| + \|(I - P)\psi\|$, $\psi \in \mathrm{Lip}_0(M)^*$. The question of L-embeddability, or more generally complementability of $\mathcal{F}(M)$ in $\mathrm{Lip}_0(M)^*$, has implications for the non-linear geometry of Banach spaces, which we discuss. We give an explicit and natural description of a projection P as above when M is proper and purely 1-unrectifiable. This is done using the De Leeuw transform, which is an important tool for representing elements of $\mathrm{Lip}_0(M)^*$ by signed Radon measures on a certain compact space.

This talk is based on joint work with Ramón Aliaga (Universitat Politècnica de València) and Eva Pernecká (Czech Technical University, Prague).

Thomas Speckhofer, Johannes Kepler University Linz

Factorization in Haar system Hardy spaces

Abstract. A Haar system Hardy space is the completion of the linear span of the Haar system $(h_I)_I$, either under a rearrangement-invariant norm $\|\cdot\|$ or under the associated square function norm given by

$$\Big\|\sum_I a_I h_I\Big\|_* = \Big\|\Big(\sum_I a_I^2 h_I^2\Big)^{1/2}\Big\|.$$

Apart from L^p, $1 \le p < \infty$, the class of these spaces includes all separable rearrangement-invariant function spaces on $[0,1]$ and also the dyadic Hardy space H^1.

Using a unified and systematic approach, we prove that every Haar system Hardy space X with $X \neq C(\Delta)$ (where $C(\Delta)$ denotes the continuous functions on the Cantor set) has the following property: For every bounded linear operator T on X, the identity I_X factors either through T or through $I_X - T$. Moreover, if T has large diagonal with respect to the Haar system, then the identity factors through T. In particular, we obtain that

$$\mathcal{M}_X = \{T \in \mathcal{B}(X) : I_X \text{ does not factor through } T\}$$

is the unique maximal ideal of the algebra $\mathcal{B}(X)$ of bounded linear operators on X. Finally, we establish analogous factorization results for the spaces $\ell^p(X)$, $1 \le p < \infty$, and we use Pełczyński's decomposition method to show that these spaces are primary.

Based on joint work with Richard Lechner [1].

References

[1] R. Lechner, T. Speckhofer, Factorization in Haar system Hardy spaces. Preprint available on arXiv:2212.06723, October 2023.

Sukumar Daniel, Indian Institute of Technology Hyderabad

Exponential spectrum commutes in $B(\ell_p \oplus \ell_q)$

Abstract. The exponential spectrum does not commute, as shown by Klaja and Ransford. In literature, the operator algebra $\mathcal{B}(\ell^p \oplus \ell^q)$ was also anticipated to be a potential algebra of the same type. But, we show that the exponential spectrum commutes in this algebra.

References

[1] Hubert Klaja and Thomas Ransford, Non-commutativity of the exponential spectrum, *J. Funct. Anal.* **272**, (2017), no. 10, 4158–4164. MR3626037.

This is a joint work with Dr. Arindam Ghosh.

Mitchell Taylor, ETH Zurich

Free Banach lattices

Abstract. Given a Banach space E, one may construct a Banach lattice FBL$[E]$ with the property that every bounded linear operator from E into a Banach lattice X extends uniquely to a lattice homomorphism from FBL$[E]$ into X. We will discuss the structure of FBL$[E]$ and its larger role in the theory of Banach spaces and lattices.

Elroy Zeekoei, North-West University

Some Dunford–Pettis-like operators on Banach lattices

Abstract. The notion of a p-convergent operator on a Banach space was originally introduced in 1993 by Castillo and Sánchez in the paper entitled "Dunford-Pettis-like properties of continuous vector function spaces". In this talk we consider the p-convergent operators on Banach lattices as well as the notion of a weak p-convergent operator. We also discuss the notion of a disjoint p-convergent operator on Banach lattices and apply it to a study of the positive Schur property of order p are considered.

4.21. **Operator Theory on Analytic Function Spaces 1.**
Organizers: Maribel Loaiza-Leyva (Centro de investigacion y de estudios avanzados del IPN, Mexico), Raul Quiroga Barranco (CIMAT, Guanajuato), Armando Sanchez-Nungaray (Universidad Veracruzana), Kehe Zhu (State University of New York at Albany)

Monday 14:00-16:00 (PLT1) chair: Maribel Loaiza-Leyva

14:00-14:25 Nina Zorboska
Measure induced Hankel and Toeplitz type operators on weighted Dirichlet spaces

14:30-14:55 Ghazaleh Asghari
Schatten class Hankel operators on doubling Fock spaces and the Berger-Coburn phenomenon

15:00-15:25 Cezhong Tong
A new class of Carleson embeddings

15:30-15:55 Zengjian Lou
Carleson embedding on Bergman spaces with applications

Monday 16:30-19:00 (PLT1) chair: Raul Quiroga-Barranco

16:30-16:55 Wolfram Bauer
Operators in the Fock-Toeplitz Algebra

17:00-17:25 Robert Fulsche
A Wiener algebra on the Fock space

17:30-17:55 Egor Maximenko
Horizontal Fourier transform of the polyanalytic Fock kernel

18:00-18:25 Stephen Sontz
Non-commutative Toeplitz Quantization of Euclidean Planes

18:30-18:55 Pindoli Mohan
Von Neumann algebras of analytic functions on the unit ball

Tuesday 14:00-16:00 (PLT1) chair: Armando Sanchez-Nungaray

14:00-14:25 Jari Taskinen
Bergman projection induced by radial weight acting on growth spaces

14:30-14:55 Maribel Loaiza-Leyva
On C^*-algebras generated by Toeplitz operators and projections

15:00-15:25 Shubham Rameshsingh Bais
Integral representation of angular operators on the Bergman space over the upper half-plane

15:30-15:55 Miron Bekker
On Generators of the Hardy and the Bergman Spaces

Tuesday 16:30-19:00 (PLT1) chair: Maribel Loaiza-Leyva

16:30-16:55 Jani Virtanen
Asymptotics of determinants for structured matrices

17:00-17:25 Željko Čučković
A geometric condition for the invertibility of Toeplitz operators on the Bergman space

17:30-17:55 Hyungwoon Koo
Local Hopf lemma for degenerate elliptic operator

18:00-18:25 Hicham Arroussi
Generalized Volterra type integral operators on large Bergman spaces

Wednesday 11:40-12:40 (PLT1) chair: Raul Quiroga-Barranco

11:40-12:05 David Norrbo
Asymptotic Toeplitzness of weighted composition operators on Abstract Hardy spaces

12:10-12:35 Armando Sanchez-Nungaray
Toeplitz Operators with symbols invariant under the action of a subgroup of the nilpotent group on the Siegel Domain D_2

Thursday 14:00-16:00 (PLT1) chair: Armando Sanchez-Nungaray

14:00-14:25 Ching-on Lo
Complex Symmetric Weighted Composition-Differentiation Operators on Weighted Hardy Spaces

14:30-14:55 Marek Ptak
Invariant Subspaces for conjugations with special behavior with respect to given unitary operator

15:00-15:25 Anusree Sreedharan
Multiresolution Analysis on the Weighted Bergman spaces

4.21.1. *Abstracts.*

Hicham Arroussi, University of Reading and Helsinki

Generalized Volterra type integral operators on large Bergman spaces

Abstract. : Let ϕ be an analytic self-map of the open unit disk $\mathbb{D}$ and g analytic in $\mathbb{D}$. We characterize boundedness and compactness of generalized Volterra type integral operators

$$GI_{(\phi,g)}f(z) = \int_0^z f'(\phi(\xi))\, g(\xi)\, d\xi$$

and

$$GV_{(\phi,g)}f(z) = \int_0^z f(\phi(\xi))\, g(\xi)\, d\xi,$$

acting between large Bergman spaces A^p_ω and A^q_ω for $0 < p, q \leq \infty$. To prove our characterizations, which involve Berezin type integral transforms, we use the Littlewood-Paley formula of Constantin and Peláez and establish corresponding embedding theorems, which are also of independent interest. When $\phi(z) = z$, our results for $GV_{(\phi,g)}$ complement the descriptions of Pau and Peláez.

Ghazaleh Asghari, University of Reading

Schatten class Hankel operators on doubling Fock spaces and the Berger-Coburn phenomenon

Abstract. Using the notion of integral distance to analytic functions, we give a characterization of Schatten class Hankel operators acting on doubling Fock spaces on the complex plane and use it to show that for $f \in L^\infty$, if H_f is Hilbert-Schmidt, then so is $H_{\bar{f}}$. This property is known as the Berger-Coburn phenomenon. When $0 < p \leq 1$, we show that the Berger-Coburn phenomenon fails for a large class of doubling Fock spaces. Along the way, we illustrate our results for the canonical weights $|z|^m$ when $m > 0$.

References

[1] G. Asghari, Z. Hu, J. A. Virtanen, Schatten class Hankel operators on doubling Fock spaces and the Berger-Coburn phenomenon, *J. Math. Anal. Appl.* **540**, (2024).

Shubham R. Bais, The Institute of Mathematical Sciences

Integral representation of angular operators on the Bergman space over the upper half-plane

Abstract. Let Π denote the upper half-plane and $\mathcal{A}^2(\Pi)$ be the Bergman space over the upper half-plane. In this talk, we define a class of integral operators on the space $\mathcal{A}^2(\Pi)$. We characterize the integral kernels so that the operators are bounded. We show that this class coincides with the class of angular operators on $\mathcal{A}^2(\Pi)$. As a consequence, we discuss various operator theoretic properties of angular operators, and a C^*-subalgebra generated by Toeplitz operators with special symbols. This is based on the joint work with D. Venku Naidu [1].

References

[1] S. R. Bais and D. Venku Naidu, Integral representation of angular operators on the Bergman space over the upper half-plane, *New York J. Math.* **30** (2024), paper no. 4, 42–57 (2024).

Wolfram Bauer, Leibniz Universität Hannover

Operators in the Fock-Toeplitz Algebra

Abstract. We consider various classes of bounded operators on the Fock space of Gaussian square integrable entire functions over the complex plane. These include Toeplitz (type) operators, weighted composition operators, singular integral operators, Volterra-type operators and Hausdorff operators. As a leading problem and closely linked to well-known compactness or boundedness characterizations we pursue the question of when these operators are contained in the Toeplitz algebra. Some new proofs from the perspective of quantum harmonic analysis will be explained. This is joint work with Robert Fulsche and Miguel A. Rodriguez Rodriguez.

Miron Bekker, University of Pittsburgh at Johnstown

On Generators of the Hardy and the Bergman Spaces

Abstract. A function φ which is analytic and bounded in the unit disk $\mathbb{D}$ is called a generator for the Hardy space $H^2(\mathbb{D})$ or the Bergman space $A^2(\mathbb{D})$ if polynomials in φ are dense in the corresponding space. We characterize generators in terms of $\varphi-$invariant subspaces which are also $z-$invariant and study wandering properties of such subspaces. Density of bounded analytic functions in the $\varphi-$invariant subspaces is also investigated.

References

[1] John Akeroyd, *Density of the polynomials in the Hardy space of certain slit domains.* Proceeding of the AMS, **15** (1992), No 4, 1013-1021.
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Željko Čučković, University of Toledo, USA

A geometric condition for the invertibility of Toeplitz operators on the Bergman space

Abstract. Invertibility of Toeplitz operators on the Bergman space and the related Douglas problem are long standing open problems. In this paper we study the invertibility problem under the novel geometric condition on the image of the symbols, which relaxes the standard positivity condition. We show that under our geometric assumption, the Toeplitz operator T_φ is invertible if and only if the Berezin transform of $|\varphi|$ is invertible in L^∞. It is well known that the Douglas problem is still open for harmonic functions. We study a class of rather general

harmonic polynomials and characterize the invertibility of the corresponding Toeplitz operators. We also give a number of related results and examples. (Joint work with Jari Taskinen)

Robert Fulsche, Leibniz Universität Hannover

A Wiener algebra on the Fock space

Abstract. In the operator theory of Hardy spaces H^p, it is well known that the spectrum of the Toeplitz operator T_f can depend on the precise Hardy space on which the operator is considered, i.e., the spectrum can depend on the choice of the parameter p. In the present talk, we will demonstrate that for Toeplitz operators on the Fock space such behaviour is impossible. More precisely, we will introduce an algebra of integral operators $\mathcal{W}_t$ (called the *Wiener algebra*), which act boundedly on each of the Fock spaces F^p_t, $1 \leq p \leq \infty$, and contains all Toeplitz operators with bounded symbols. As the main results, we show that the spectrum, the essential spectrum, and the Fredholm index of an operator from $\mathcal{W}_t$ do not depend on the choice of the parameter p.

Hyungwoon Koo, Korea University

Local Hopf lemma for degenerate elliptic operator

Abstract. We prove that the local Hopf lemma of Baouendi-Rothschild for harmonic functions continues to hold for the degenerate elliptic operator, $\mathcal{L}_\alpha = x^\alpha \partial_x^2 + \sum_{j=1}^n \partial_{y_j}^2$, on the half-space when the degeneracy exponent α is less than 2. We provide examples of degenerate elliptic operators with the degeneracy exponent greater or equals to 2 for which the local Hopf lemma fail.

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Ching-on LO, College of Professional and Continuing Education, The Hong Kong Polytechnic University

Complex Symmetric Weighted Composition-Differentiation Operators on Weighted Hardy Spaces

Abstract. Complex symmetric weighted composition-differentiation operators on the weighted Hardy spaces $H^2(\beta)$ with respect to the standard conjugation are investigated. Our results partially answer a question from a paper of Lim and Khoi (2018) [1] and subsume several existing ones in the literature. We also study the classes of self-adjoint, normal, co-isometric, unitary weighted composition-differentiation operators on $H^2(\beta)$ and their relations with the property of complex symmetry. The talk is based on joint work with Anthony Wai-keung LOH.

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Maribel Loaiza-Leyva, Centro de investigación y de estudios avanzados del IPN

On C^*-algebras generated by Toeplitz operators and projections

Abstract. The C*-algebra generated by Toeplitz operators, acting on the poly-Bergman space of order n, with bounded vertical symbols (and with finite limits at the points 0 and ∞), is isomorphic and isometric to the algebra $\{M(x) \in M_n(\mathbb{C}) \otimes C[0,+\infty] : M(0), M(\infty) \in \mathbb{C}I\}$. Then this algebra looks close to the C^*-algebra generated by n orthogonal projections. This implies that, inside the algebra of all bounded operators acting on the Poly-Bergman space of order n, there are n orthogonal projections that generate a C^*-algebra close related to the C^*-algebra generated by all Toeplitz operators with vertical symbols. In this talk we construct a family of projections with these characteristics. One of them is in terms of Toeplitz operators. Besides, we study the C*-algebra $\mathcal{A}$ generated by a single Toeplitz operator and the orthogonal projections $P_1, \ldots, P_n$, where P_k is the orthogonal projection from the $n-$poli-Bergman space onto the true poli-Bergman space of order k. It turns out that, if the vertical symbol of the Toeplitz operator is a characteristic function, the C*-algebra $\mathcal{A}$ is isomorphic and isometric to the algebra

$$\mathcal{D}_n := \{M \in M_n(\mathbb{C}) \otimes C[0,\infty] : M(0), M(\infty) \text{ are diagonal matrices}\}.$$

In particular, this C*-algebra contains (isometrically) the C*-algebra generated by all Toeplitz operators with vertical symbols under the conditions given above.

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Zengjian Lou, Shantou University

Carleson embedding on Bergman spaces with applications

Abstract. In this talk, we consider Carleson embedding and its applications. We will introduce the recent development of absolutely summing Carleson embedding on Bergman spaces with applications on composition operators (Based on joint work with B. He (FDU), J. Jreis and P. Lefèvre (Université d'Artois, France))

I would like to thank NNSF of China and Li Ka Shing Foundation for their support.

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Egor Maximenko, Instituto Politécnico Nacional (Mexico)

Horizontal Fourier transform of the polyanalytic Fock kernel

Abstract. This is a joint work with Erick Lee-Guzmán, Gerardo Ramos-Vazquez, and Armando Sánchez-Nungaray.

Let $\mathcal{F} = \mathcal{F}_m(\mathbb{C}^n)$ be the space of m-analytic functions on $\mathbb{C}^n$ square integrable with the Gaussian weight. Its reproducing kernel was recently computed by Youssfi [1]:

$$K_{\mathcal{F}}(w,z) = e^{\langle w,z\rangle}\, L^{(n)}_{m-1}(|w-z|^2).$$

We construct a new RKHS $\mathcal{H}$ multiplying all elements of $\mathcal{F}$ by the weight

$$2^{\frac{n}{2}}\, e^{-\frac{1}{2}|w|^2 - i\langle \mathrm{Re}(w), \mathrm{Im}(w)\rangle}.$$

$\mathcal{H}$ is invariant under the usual translations in the horizontal direction. The reproducing kernel of $\mathcal{H}$ is

$$K_{\mathcal{H}}(w,z) = 2^n\, e^{-\frac{1}{2}|w-z|^2 - i\langle \mathrm{Re}(w-z), \mathrm{Im}(w+z)\rangle}\, L^{(n)}_{m-1}(|w-z|^2).$$

Then, we compute the Fourier transform of $K_{\mathcal{H}}$ in the horizontal direction. It decomposes into a sum of products of Hermite functions:

$$\frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^n} K_{\mathcal{H}}(u+iv, iy)\, e^{-i\langle u,\xi\rangle}\, \mathrm{d}u_1 \cdots \mathrm{d}u_n$$
$$= 2^n \pi^{\frac{n}{2}} \sum_{k_1+\ldots+k_n = m-1} \prod_{r=1}^{n} \psi_{k_r}\left(\frac{\xi_r + 2v_r}{\sqrt{2}}\right) \psi_{k_r}\left(\frac{\xi_r + 2y_r}{\sqrt{2}}\right).$$

The number of summands is $d = \binom{m+n-1}{n}$. As the main application, we construct an isometric isomorphism $R\colon \mathcal{F} \to L^2(\mathbb{R}^n)^d$ that intertwines Weyl translation operators with multiplication operators by characters of $\mathbb{R}^n$. Using [2] we show that the corresponding centralizer (i.e., the von Neumann algebra of "vertical" operators in $\mathcal{F}$) is isometrically isomorphic to $L^2(\mathbb{R})^{d\times d}$.

The speaker has been supported by CONAHCYT (Mexico) project "Ciencia de Frontera" FORDECYT-PRONACES/61517/2020 and by IPN-SIP projects.

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Pinlodi Mohan, Indian Institute of Technology Hyderabad,India

Von Neumann algebras of analytic functions on the unit ball

Abstract. This talk answers the challenge posed by Ma and Zhu in [1]. We provide examples of von Neumann algebras of analytic functions over the unit disc as well as over the unit ball. Also, we introduce a novel multiplication operation on the set of all analytic functions over the unit disc, ensuring that this collection forms a $*$-algebra of analytic functions over $\mathbb{D}$.

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David Norrbo, University of Reading

Asymptotic Toeplitzness of weighted composition operators on Abstract Hardy spaces

Abstract. How closely related are weighted composition operators to Toeplitz operators? This talk will provide an answer in terms of asymptotic Toeplitzness of weighted composition operators on abstract Hardy spaces on the disk. This class of spaces include the reflexive Hardy spaces and many Hardy-Lorentz and Orlicz spaces.

Marek Ptak, University of Agriculture in Kraków, Poland

Invariant Subspaces for conjugations with special behavior with respect to given unitary operator

Abstract. For a given unitary operator U we consider conjugation C (antilinear, isometric, involution) such that U is C–symmetric i.e. $CUC = U^*$. The subspaces which are invariant for all such conjugations are characterized. It turns out that it is all subspaces which are hyperinvariant for the given unitary operator U. Next, conjugations commuting with a given unitary operator are investigated. The necessary and sufficient condition when such conjugation exists is given. The description of subspaces which are invariant for all commuting conjugation is much complicated. The examples of specific unitary operators are pointed out.

Joint work with J. Mashreghi, W. Ross.

Armando Sánchez Nungaray, Universidad Veracruzana

Toeplitz Operators with symbols invariant under the action of a subgroup of the nilpotent group on the Siegel Domain D_2

Abstract. In this talk, we consider two types of subgroups of the Nilpotent group of the Siegel Domain of dimension two D_2. For each of these two subgroups we construct a Bargman-type transform adapted to the action of that subgroup over D_2, moreover using these Bargman-type transforme, we characterized the Toeplitz operators with invariant symbols under the action of these subgroups. I would like to thank the Harry Potter Foundation for their generous support.

This is a joint work with Nikolai Vasilevski (RIP).

Stephen Bruce Sontz, CIMAT, Guanajuato, Mexico

Non-commutative Toeplitz Quantization of Euclidean Planes

Abstract. We present a way of constructing Hilbert spaces from an algebraic representation of the Euclidean plane. If a certain Harmony condition is satisfied, this is a reproducing kernel Hilbert space of holomorphic functions. Then, together with a symbol space which is a non-commutative algebra and another Harmony condition, we can define Toeplitz operators and the corresponding quantization, which generalize greatly our previous research on this topic. In collaboration with Micho Durdevich.

References

Various eprints on arxiv.

Anusree Sreedharan, Cochin University of Science and Technology, India

Multiresolution Analysis on the Weighted Bergman spaces

Abstract. We examine rational Blaschke functions that are capable to formulate a Multiresolution on the weighted Bergman space of the open unit disc $A^2_\alpha(\mathbb{D})$. We construct a rational orthogonal wavelet system that generates the levels of the multiresolution. The levels of the multiresolution are finite dimensional, which makes it easier to find a basis on each level. We can approximate any $f \in A^2_\alpha(\mathbb{D})$ by the projection operator on the n^{th} resolution level. The projection will be an interpolation operator and whose coefficients can be computed through the evaluation of f on a given set of points in the unit disc. We extend the results to weighted Bergman space of upper half plane also.

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Jari Taskinen, University of Helsinki

Bergman projection induced by radial weight acting on growth spaces

Abstract. A radial weight ω on the unit disc $\mathbb{D}$ of the complex plane is said to belong to the class $\widehat{\mathcal{D}}$, if satisfies the upper doubling condition

$$\sup_{0<r<1} \frac{\widehat{\omega}(r)}{\widehat{\omega}\left(\frac{1+r}{2}\right)} < \infty,$$

where $\widehat{\omega}(r) = \int_r^1 \omega(s)\, ds$ denotes the tail integral. Given a weight ω, let P_ω be the Bergman projection in the space L^2_ω, endowed with the ω-weighted area measure. For a weight ν, denote $L^\infty_{\widehat{\nu}} = \{f : \|f\|_{\infty,v} = \operatorname{ess\,sup}_{z\in\mathbb{D}} |f(z)|\widehat{\nu}(z) < \infty\}$.

Under the general assumption that either ω or ν belongs to $\widehat{\mathcal{D}}$, we give several characterizations of pairs (ω, ν) such that P_ω is a bounded operator from $L^\infty_{\widehat{\nu}}$ onto its closed subspace $H^\infty_{\widehat{\nu}}$ consisting of analytic functions. The characterizations involve natural conditions on the moments and/or the tail integrals of ω and ν. Moreover, we solve the analogous problems for the boundedness of P_ω from $L^\infty_{\widehat{\nu}}$ onto the corresponding weighted Bloch type spaces and also study similar questions for exponentially decreasing radial weights.

This is a report on a joint work with Álvaro Miguel Moreno and José Ángel Peláez, Universidad de Málaga.

Cezhong Tong, Hebei University of Technology

A new class of Carleson embeddings

Abstract. In this talk I will introduce a new class of Carleson embeddings for Bergman-type spaces. This extends the embedding of a single derivative to the embedding of a combination of finite derivatives of different orders. As applications, we can use these new Carleson embeddings to characterize the generalized Volterra-type operators on Bergman-type spaces, and to characterize the boundedness and compactness of sums of weighted composition-differentiation operators of different orders.

Jani Virtanen, University of Reading

Asymptotics of determinants for structured matrices

Abstract. I discuss the determinant asymptotics for structured matrices with focus on Toeplitz and Toeplitz plus Hankel matrices, and also finite sections of Toeplitz operators, generated by matrix-valued symbols that may be smooth or possess singularities. Most of our results have been proved using operator theory but we also compare the operator-theoretic approach with the use of Riemann-Hilbert problems. Some applications to random matrix theory and mathematical physics are also mentioned.

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Nina Zorboska, University of Manitoba

Measure induced Hankel and Toeplitz type operators on weighted Dirichlet spaces

Abstract. I will talk about the boundedness of measure induced Hankel and Toeplitz type operators on weighted Dirichlet spaces, extending the known results for the case of the classical Hardy and Dirichlet spaces. The approach relies on recent results on weak products of complete Nevanlinna-Pick reproducing kernel Hilbert spaces.

4.22. **Special Matrices.**
Organizers: Natalia Bebiano (University of Coimbra), Mikhail Tyaglov (Shanghai Jiao Tong University)

Tuesday 16:30-19:00 (SIBSR3) chair: Sergei Grudsky

16:30-16:55 Jyoti Rani
Spectral properties of the Rhaly operator on weighted null sequence spaces and associated operator ideals

17:00-17:25 Sergei Grudsky
Asymptotics of eigenvalues and eigenvectors of Toeplitz matrices

17:30-17:55 Bibekananda Sitha
Generalized core-EP Inverse for Square Matrices

4.22.1. *Abstracts.*

Sergei Grudsky, CINVESTAV, Mexico

Asymptotics of eigenvalues and eigenvectors of Toeplitz matrices

Abstract. Analysis of the asymptotic behaviour of the spectral characteristics of Toeplitz matrices as the dimension of the matrix tends to infinity has a history of over 100 years. For instance, quite a number of versions of Szegö 's theorem on the asymptotic behaviour of eigenvalues and of the so-called strong Szegö theorem on the asymptotic behaviour of the determinants of Toeplitz matrices are known. Starting in the 1950s, the asymptotics of the maximum and minimum eigenvalues were actively investigated. However, investigation of the individual asymptotics of all the eigenvalues and eigenvectors of Toeplitz matrices started only quite recently: the first papers on this subject were published in 2009-2010. A survey of this new field is presented here.

Jyoti Rani, Indian Institute of Technology Bhilai

Spectral properties of the Rhaly operator on weighted null sequence spaces and associated operator ideals

Abstract. The research conducted in this work provides a thorough examination of the lower triangular terraced matrix, initially introduced by H. C. Rhaly, commonly referred to as the Rhaly matrix. In this work, we focused on continuity, compactness, and spectral properties of Rhaly matrix. For a sequence $a = \{a_n\}$ of real or complex numbers Rhaly [**?**] introduced the terraced matrix R_a also known as Rhaly matrix where

$$R_a = \begin{pmatrix} a_1 & 0 & 0 & 0 & \cdots \\ a_2 & a_2 & 0 & 0 & \cdots \\ a_3 & a_3 & a_3 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

If we consider the sequence $a_n = \frac{1}{n}$ for $n \in \mathbb{N}$, the operator R_a simplifies to the well-known Cesàro operator. In the specific case where $a_n = \frac{1}{n^p}$ with $p \in \mathbb{R}$, it reduces to the p-Cesàro operator. While numerous researchers have investigated the spectrum of Rhaly operators in various classical sequence spaces like c_0, ℓ_p, bv_0, etc. and over Hardy spaces. To the best of our knowledge, no investigation is carried out so far on the spectral properties of Rhaly operators over the weighted sequence spaces. This study aims to fill that gap. Within this article, we delve into the topics of boundedness, compactness, and various spectral properties of the Rhaly operator when it operates on the weighted sequence space $c_0(s)$, where $s = \{s_n\}$ represents the weight vector. We obtained that, for a bounded decreasing sequence $s = s_n$ of positive real numbers such that $R_a \in \mathcal{B}(c_0(s))$,(set of all bounded linear operators on $c_0(s)$), the spectrum $\sigma(R_a, c_0(s))$ satisfies the following relation

$$\sigma(R_a, c_0(s)) \subseteq \left\{\lambda \in \mathbb{C} \setminus \bar{S} : \left|\lambda - \frac{\mu}{2}\right| \leq \frac{\mu}{2}\right\} \cup \bar{S},$$

where $\mu = \lim_{n\to\infty} n a_n$ with $0 \leq \mu < \infty$ and $S = \{a_n : n \in \mathbb{N}\}$. Under similar assumptions, we have also obtained, the point spectrum $\sigma_p(R_a, c_0(s))$, continuous spectrum $\sigma_c(R_a, c_0(s))$ and residual spectrum $\sigma_r(R_a, c_0(s))$ of R_a as follows,

(i) $\sigma_p(R_a, c_0(s)) = A_1$,
(ii) $\sigma_r(R_a, c_0(s)) = (A_2 \cup S) \setminus A_1 = A_2 \cup (S \setminus A_1)$.

In addition, if the sequence $\{s_n\}$ is a decreasing sequence then

(iii) $\sigma(R_a, c_0(s)) \subseteq \left\{\lambda \in \mathbb{C} \setminus \bar{S} : \left|\lambda - \frac{\mu}{2}\right| \leq \frac{\mu}{2}\right\} \cup \bar{S}$,
(iv) $\{0\} \subseteq \sigma_c(R_a, c_0(s)) \subseteq (\left\{\lambda \in \mathbb{C} \setminus \bar{S} : \left|\lambda - \frac{\mu}{2}\right| \leq \frac{\mu}{2}\right\} \cup \bar{S}) \setminus (A_2 \cup S)$,

where

$$\begin{aligned} A_1 &= \left\{\lambda \in S : \lim_{n\to\infty} a_n s_n n^{\alpha\chi} = 0\right\}, \\ A_2 &= \left\{\lambda \in \mathbb{C} \setminus (S \cup \{0\}) : \sum_{n=1}^{\infty} \frac{1}{s_n n^{\alpha\chi}} < \infty\right\}, \end{aligned}$$

where $\alpha = \Re(\frac{1}{\lambda})$.

As as application, A novel class of operator ideals, denoted as $\chi^{(s)}_{c_0(r)}$, associated with the Rhaly operator acting on weighted c_0 spaces has been introduced. $\chi^{(s)}_{c_0(r)}$ is defined as follows:

$$\chi^{(s)}_{c_0(r)} = \left\{\phi \in \mathcal{B} : \lim_{i\to\infty} \left(a_i \sum_{j=1}^{i} s_j(\phi)\right) r_i = 0\right\},$$

where $\mathcal{B}$ denotes the class of all bounded linear operators between any pair of Banach spaces and $s_j(\phi)$ denotes the sequence of s-numbers. It has been demonstrated that, under certain assumptions on the sequence $\{a_n\}$, this class forms a closed quasi-Banach operator ideal.

Bibekananda Sitha, BITS Pilani Goa Campus

Generalized core-EP Inverse for Square Matrices

Abstract. In this paper, we introduce two new types of inverses for complex square matrices by using an inner inverse and core-EP inverse, called ICEP, and its dual called CEPI inverse. Further, we extend the notion of P-core inverse for square matrices with arbitrary index. A few equivalent characterizations of these inverses have been derived. In addition, the representations of these inverses are established via core-EP and HS decomposition. Moreover, we introduce a binary relation for these inverses and a few derived properties. An application of these inverses in solving linear systems also discussed.

4.23. **Operator Theory and Applications.**
Organizers: Bas Lemmens (Kent), Ana Loureiro (Kent), Marco Marletta (Cardiff), Ian Wood (Kent)

Monday 14:00-16:00 (SIBSR6) chair: TBD

14:00-14:25 Luis Manuel Tovar
Conjugate Complex Harmonic Functions (bicomplex case)
14:30-14:55 Kanat Tulenov
L^p-L^q boundedness of Fourier multipliers on quantum Euclidean spaces
15:00-15:25 Dimitri Bytchenkoff
Kernel theorems for operators on co-orbit spaces associated with localised frames
15:30-15:55 Peter Balazs
Representation of Operators Using Fusion Frames

Monday 16:30-18:30 (SIBSR6) chair: TBD

16:30-16:55 Jan Stochel
Similarity to conditionally positive definite unilateral weighted shifts
17:00-17:25 Zenon Jan Jabłoński
Bishop-type theorems for non-subnormal operators
17:30-17:55 Lav Kumar Singh
Geometry of Banach algebra A and the bidual of $L^1(G, A)$

Thursday 16:30-18:30 (SIBSR3) chair: TBD

16:30-16:55 Arvish Dabra
Arens regularity of $A_\Phi(G)$
17:00-17:25 Satyabrata Majee
On Decomposition for Pairs of Twisted Contractions
17:30-17:55 Andrew Pritchard
The Asymptotic Behaviour of the Cesàro Operator

Friday 14:00-16:00 (SIBSR3) chair: TBD

14:00-14:25 K.J. Vaishakh
TBC
14:30-14:55 A. Anju
TBC

4.23.1. *Abstracts.*

Peter Balazs, Acoustics Research Institute of the Austrian Academy of Sciences

Representation of Operators Using Fusion Frames

Abstract. To solve operator equations numerically matrix representations are needed, employing bases or more recently frames. This is done, e.g. in computational acoustics, by using the so called Galerkin approach. For finding the numerical solution of operator equations a decomposition in subspaces is needed in many applications. To combine those two approaches, it is necessary to extend the known methods of matrix representation to the utilization of fusion frames.

In this talk, we start with a motivation from computational acoustics (and domain decomposition). We give a short introduction to frames and fusion frame, that can be considered as a frame-like family of subspaces. We present the representation of operators on a Hilbert space $\mathcal{H}$ with fusion frames. Taking the particular property of the duality of fusion frames into account, we also define a matrix representation in an alternate way, the later being more efficient and well behaved in respect to inversion. We will show how this can be used for the solution of operator equations and link our approach to the well-known additive Schwarz algorithm.

This talk is based on the paper [1] - joint work with Mitra Shamsabadi, Ali Akbar Arefijamaal and Gilles Chardon.

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Dimitri Bytchenkoff, Acoustics Research Institute of the Austrian Academy of Sciences and Faculty of Mathematics of the University of Vienna

Kernel theorems for operators on co-orbit spaces associated with localised frames

Abstract. Kernel theorems provide a convenient representation of bounded linear operators on function spaces as an integral operator. In this speech I shall talk about kernel theorems for bounded linear operators acting on co-orbit spaces associated with localised frames. Two of our main results consist in characterising the spaces of the operators whose integral kernels belong to the co-orbit spaces of either test functions or distributions associated with the tensor product of the localised frames.

References

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I would like to thank Hans Georg Feichtinger, Karlheinz Gröchenig and Patrik Wahlberg for fruitful discussions and the Austrian Science Fund (FWF) for its fundings 10.55776/P34624 and 10.55776/Y1199 of this project.

Arvish Dabra, Indian Institute of Technology Delhi

Arens regularity of $A_\Phi(G)$

Abstract. For a locally compact group G, the L^p-analogue ($1 < p < \infty$) of the Fourier algebra $A(G)$ is called the Figà-Talamanca Herz algebra and is denoted by $A_p(G)$. It is well known that Orlicz spaces L^Φ are the natural generalisation of the classical L^p-spaces. Let $A_\Phi(G)$ denote the Orlicz version of the Figà-Talamanca Herz algebra of G associated with a Young function Φ. As Arens regularity is an important tool to study groups with the help of certain Banach algebras related to it, we show that if $A_\Phi(G)$ is Arens regular, then G is discrete. This generalises the result by Forrest about the Arens regularity of the $A_p(G)$ algebras. We also show that $A_\Phi(G)$ is finite-dimensional if and only if G is finite. Further, for amenable groups, we show that $A_\Phi(G)$ is reflexive if and only if G is finite, under the assumption that the associated Young function Φ satisfies the MA-condition.

This is a joint work with Dr. N. Shravan Kumar.

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Zenon Jan Jabłoński, Uniwersytet Jagielloński

Bishop-type theorems for non-subnormal operators

Abstract. The celebrated Bishop theorem states that an operator is subnormal if and only if it is the strong limit of a net (or a sequence) of normal operators. Since, by the Agler-Stankus theorem, 2-isometries behave in the sense similarly to subnormal operators (the role of normal extensions for 2-isometries is played by Brownian unitaries), we pose two problems: first, whether the set of all 2-isometries is equal to the strong closure of the set of all Brownian unitaries; second, whether the set of all 2-isometries is strongly closed. In this talk, we give partial solutions to both problems. The talk is based on a joint article [1] with I. B. Jung and J. Stochel.

References

[1] Z. J. Jabłońki, I. B. Jung, J. Stochel, Bishop-type theorems for non-subnormal operators, submitted.

Satyabrata Majee, Indian Institute of Technology Roorkee

On Decomposition for Pairs of Twisted Contractions

Abstract. In this talk, we present Wold-type decomposition for various pairs of twisted contractions on Hilbert spaces. As a consequence, we obtain a new and simple proof of Słoćinski's theorem for pairs of doubly commuting isometries and generalized that result for pairs of doubly twisted isometries. We also achieve an explicit decomposition for pairs of twisted contractions such that the c.n.u. parts of the contractions are in C_{00}. It is also shown that for a twisted pair (T, V^*) of operators with T as a contraction and V as an isometry, there exists a unique (upto unitary equivalence) pair of doubly twisted isometries on the minimal isometric dilation space of T. As an application, we provide a new proof for pairs of twisted operators consisting of an isometry and a co-isometry are doubly twisted. This is a joint work with Amit Maji.

References

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Andrew Pritchard, Newcastle University

The Asymptotic Behaviour of the Cesàro Operator

Abstract. The (discrete) Cesàro operator T is defined for complex sequences as $Tx = (\phi_k(x))_{k\geq 0}$, where

$$\phi_k(x) = \frac{1}{k+1}\sum_{m=0}^{k} x_m.$$

In a 2010 paper, Adell and Lekuona proved that for sequences $x \in c_0$ satisfying certain additional conditions, it holds that $\|T^n x\| = O(n^{-1/2})$ as $n \to \infty$. Their approach was based on a probabilistic interpretation of the powers of the Cesàro operator.

In this talk, we present an operator-theoretic proof of this result as well as analogous results for the continuous Cesàro operator on certain function spaces. Our approach is based on a quantified version of the Katznelson-Tzafriri theorem.

This is joint work with David Seifert.

References

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Lav Kumar Singh

Geometry of Banach algebra A and the bidual of $L^1(G, A)$

Abstract. We shall start with the definition of two Arens product defined on the second conjugate algebra $\mathcal{A}^{**}$ of a Banach algebra $\mathcal{A}$. Few fundamental properties of these two products and some examples of Arens regular/irregular Banach algebras will be presented. Further, we will discuss about the topological center of the second dual of generalized group algebra $L^1(G, \mathcal{A})$, where G is any locally compact group and $\mathcal{A}$ is a Banach algebra. We will see that the topological center of $L^1(G, \mathcal{A})^{**}$ holds permanance property with respect to unitization of $\mathcal{A}$ and is a Banach $L^1(G)$-module in case G is abelian. Using these facts, we shall establish the result that the topological center of $L^1(G, \mathcal{A})^{**}$ is $L^1(G, \mathcal{A})$ itself when G is a compact abelian group and underlying Banach space of $\mathcal{A}$ is reflexive. Finally, we shall see a nice consequene of Cohen's factorization theorem regarding the elements in topological center of $L^1(G, \mathcal{A})^{**}$.

Jan Stochel, Jagiellonian University

Similarity to conditionally positive definite unilateral weighted shifts

Abstract. I will discuss the question of similarity of subnormal and CPD unilateral weighted shifts, where CPD is an abbreviation for "conditionally positive definite". These classes of operators emerged from the consideration of positive definite and conditionally positive definite functions on the discrete additive semigroup $(\mathbb{Z}_+, +)$, where $\mathbb{Z}_+$ is the set of all nonnegative integers. I will give several necessary conditions for similarity, which led us to distinguish CPD unilateral weighted shifts of types I, II and III. I will show that non-subnormal unilateral weighted shifts of types I and II are never similar to subnormal operators. This is a kind of dichotomy that excludes the case in which a non-subnormal CPD unilateral weighted shift is similar to a subnormal operator. We also give sufficient conditions for the similarity of CPD unilateral weighted shifts (necessarily of type III) to subnormal operators.

My talk is based on the content of the paper [3].

References

[1] J. B. Conway, *The theory of subnormal operators*, Mathematical Surveys and Monographs, **36**, American Mathematical Society, Providence, RI, 1991.

[2] Z. J. Jabłoński, I. B. Jung and J. Stochel, Conditional positive definiteness in operator theory, *Dissertationes Math.* **578**, (2022), pp. 64.

[3] Z. J. Jabłoński, I. B. Jung and J. Stochel, Similarity to conditionally positive definite unilateral weighted shifts, in progress.

Luis Manuel Tovar, Instituto Politecnico Nacional (MEXICO)

Conjugate Complex Harmonic Functions (bicomplex case)

Abstract. Abstract This paper presents several properties and relations that satisfy the components of a bicomplex holomorphic function. It also exhibits several analogies and differences with the case of analytic functions.

Kanat Tulenov, Ghent University

L^p -L^q boundedness of Fourier multipliers on quantum Euclidean spaces

Abstract. In this paper, we study Fourier multipliers on quantum Euclidean spaces and obtain results on their L^p -L^q boundedness.

4.24. **Operator Theory on Analytic Function Spaces 2.**
Organizers: Bas Lemmens, (Kent), Ana Loureiro (Kent), Marco Marletta (Cardiff), Ian Wood (Kent)

Wednesday 11:40-12:40 (SIBSR3) chair: TBD

11:40-12:05 Mohana Rahul Nandan
A study of multiplicative maps on Reproducing Kernel Hilbert spaces

12:10-12:35 Jaikishan
Multiplicativity of linear functionals on function spaces on an open disc

Wednesday 14:00-15:00 (SIBSR3) chair: TBD

14:00-14:25 Bharti Garg
J-contractive operator valued functions and vector valued de Branges spaces

14:30-14:55 Christian Emmel
A Generalization of Krein's extension formalism for simple symmetric operators with deficiency index (1, 1)

Thursday 16:30-18:30 (SIBSR2) chair: TBD

16:30-16:55 Chong Zhao
Essential normality of quotient modules vs. Hilbert-Schmidtness of submodules in $H^2(\mathbb{D}^2)$

17:00-17:25 Golla Ramesh
Denseness of a subclass of norm attaining operators

Friday 14:00-16:00 (SIBLT1) chair: TBD

14:00-14:25 Reid Johnson
Pullback Operators on Bargmann Spaces

14:30-14:55 Andrew Graven
On the Uniqueness of Generalized Quadrature Domains via the Faber Transform

15:00-15:25 Hui Dan
Gaussian Dirichlet series with periodic coefficients

15:30-15:55 Axel Renard
Criteria of contractivity for small size matrices and characterization of model operators

4.24.1. *Abstracts.*

Hui Dan, Sichuan University

Gaussian Dirichlet series with periodic coefficients

Abstract. In this talk, we consider Gaussian Dirichlet series $\sum_{n=1}^{\infty} X_n n^{-s}$ with periodic coefficients ($X_{n+q} = X_n$). These random series are Gaussian analytic functions on the half plane $\operatorname{Re} s > 1$. Some situations are discussed concering the probability of this Dirichlet series having zeros in $\operatorname{Re} s > 1$.

Christian Emmel, Stockholm University

A Generalization of Krein's extension formalism for simple symmetric operators with deficiency index $(1, 1)$

Abstract. Simple symmetric operators with deficiency index $(1, 1)$ can be realized as multiplication operators by the independent variable on suitable reproducing kernel Hilbert spaces. Their self-adjoint extensions are then characterized by Krein's extension formalism. In this talk, we extend this formalism to characterize all, not necessarily self-adjoint, extensions using comparably simple function-theoretic arguments. This is a special case of a result obtained for arbitrary symmetric operators with deficiency index $(1, 1)$.

Bharti Garg, Indian Institute of Technology Ropar

J-contractive operator valued functions and vector valued de Branges spaces

Abstract. This talk aims to give an overview of the reproducing kernel Hilbert spaces (RKHS) constructed from J-contractive operator valued analytic functions on the upper half-plane. First, we discuss the Potapov-Ginzburg transform of a certain class of bounded linear operators, investigate some of its properties, and outline sufficient conditions for a J-contractive operator to be J-bicontractive. Finally, a construction of the de Branges space of vector valued analytic functions denoted by $\mathcal{H}_{\mathfrak{S}_\infty}(U)$ is proposed with the help of J-contractive operator valued analytic functions.
This is part of an ongoing work with my Ph.D. supervisor Dr. Santanu Sarkar.

Andrew Graven, Caltech

On the Uniqueness of Generalized Quadrature Domains via the Faber Transform

Abstract. We study the theory of generalized quadrature domains (GQDs) and describe an approach to questions of uniqueness, involving the Faber transform and ideas from logarithmic potential theory. By GQDs, we mean quadrature domains with respect to weighted area measures and those with Abelian quadrature functions (i.e. arc integral terms in the quadrature identity). Connections to the integrability of the Hele-Shaw flow are also discussed. This is joint work with Nikolai Makarov.

I would like to thank the DOD National Defense Science and Engineering Graduate Fellowship Program (NDSEG) for their generous support.

Jaikishan, Shiv Nadar Institution of Eminence, Delhi NCR, India

Multiplicativity of linear functionals on function spaces on an open disc

Abstract. Gleason-Kahane-$\dot{Z}$elazko (GKZ) theorem characterizes all the multiplicative linear functionals on complex unital Banach algebras. Recently, the GKZ theorem has been extended by Javad Mashreghi and Thomas Ransford to function spaces that are not algebras.

In this talk, we present a general version of the GKZ theorem. First, we characterize a class of linear functionals as point evaluations on the vector space of all complex polynomials $\mathcal{P}$. We then apply this characterization to present a version of the GKZ theorem for a vast class of topological spaces of complex-valued functions, including the Hardy, Bergman, Dirichlet, and many other well-known function spaces. Also, we use the GKZ theorem for polynomials to obtain a version of the GKZ theorem for strictly cyclic weighted Hardy spaces.

References

[1] J. Mashreghi and T. Ransford. *A Gleason-Kahane-$\dot{Z}$elazko theorem for modules and applications to holomorphic function spaces.* Bull. Lond. Math. Soc., 47(6):1014-1020, 2015.
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I would like to thank the Shiv Nadar Institution of Eminence for providing me travel support to attend this conference.

Reid Johnson, UCLA

Pullback Operators on Bargmann Spaces

Abstract. We characterize boundedness and compactness of pullback operators under holomorphic maps between Bargmann spaces of entire holomorphic functions with quadratic strictly plurisubharmonic exponential weights, extending a result of Carswell-MacCluer-Schuster obtained in the case of the radial quadratic weight. We also show that the pullback operator between Bargmann spaces is compact precisely when it is of trace class, with sub-exponentially decaying singular values.

References

[1] B. J. Carswell, B. D. MacCluer, and A. Schuster, *Composition operators on the Fock Space*, Acta Sci. Math., **69**, (2003), 871–887.
[2] R Johnson, Pullback operators on Bargmann spaces, *Proc. Am. Math. Soc.*, to appear, arXiv:2403.13227.

Mohana Rahul Nandan, Indian Institute of Technology Hyderabad

A study of multiplicative maps on Reproducing Kernel Hilbert spaces

Abstract. The study of multiplicative linear maps in Banach algebras is a well-explored area in the mathematical literature. A key theorem in this context is the Gleason-Kahane-Żelazko (GKZ) theorem, which provides insights into the multiplicative properties of linear functionals. Similarly, the Kowalski-Słodkowski theorem addresses conditions under which a functional is both linear and multiplicative.

Recently, Cheng Chu, Michael Hartz, Javad Mashreghi, and Thomas Ransford have extended the GKZ theorem to reproducing kernel Hilbert spaces (RKHS) with normalized complete Pick kernels. In this talk, we will discuss about generalizing linearity in the hypothesis of the above

theorem. Also, we will discuss about generalizing the Kowalski-Słodkowski theorem for RKHS with normalized complete Pick kernel.

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[2] Sebastian, G., Daniel, S.: A weaker Gleason-Kahane-Żelazko theorem for modules and applications to Hardy spaces, Colloq. Math. **164(2)**, (2021), 273–282.
[3] Mashreghi, J., Ransford, T.: A Gleason-Kahane-Żelazko theorem for modules and applications to holomorphic function spaces, *Bull. Lond. Math. Soc.* **47(6)**, (2015), 1014–1020.

I would like to thank the Indian Institute of Technology Hyderabda for their generous support.

Golla Ramesh, Indian Institute of Technology

Denseness of a subclass of norm attaining operators

Abstract. Let H be a complex Hilbert space and $\mathcal{B}(H)$ denote the space of all bounded linear operators on H. We say $T \in \mathcal{B}(H)$ is norm attaining if there exists $x \in H$ with $\|x\| = 1$ such that $\|Tx\| = \|T\|$. We define a new class

$$\beta(H) := \{T \in \mathcal{B}(H) : \ T \text{ attains norm on every reducing subspace of } T\}.$$

In this talk we discuss the invariant subspace of operators in $\beta(H)$ and denseness of $\beta(H)$ in $\mathcal{B}(H)$ with respect to the operator norm.

This is a joint work with Hiroyuki Osaka and Shanola S. Sequeira.

References

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Axel Renard, University of Lille

Criteria of contractivity for small size matrices and characterization of model operators

Abstract. It is quite known in the literature that the Schwarz-Pick inequality can be obtained as a particular case of the von Neumann inequality, applied to a well chosen 2×2 matrix. Trying to generalize this observation to matrices of higher sizes leads us to the issue of estimating the (Euclidean) norm of a $n \times n$ upper-triangular matrix T_n, while the computations using the spectral radius of $T_n^* T_n$ become too intricate to give a useful criterion in practice. I will give an answer to this issue for 3×3 and 4×4 matrices. Then, this leads to a nice characterization of the matrix representation of the compressed shift acting on the model space $H^2(\mathbb{D}) \ominus uH^2(\mathbb{D})$, where u is a finite Blaschke product of degree $n \geq 2$.

References

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operators", hal-04565458f (2024), Preprint.

This talk is based on a joint work with Catalin Badea (University of Lille).

Chong Zhao, Shandong University

Essential normality of quotient modules vs. Hilbert-Schmidtness of submodules in $H^2(\mathbb{D}^2)$

Abstract. I would like to talk our latest work on the essential normality of quotient modules over the polydiscs, and the Hilbert-Schmidtness of submodules. We prove that all the quotient modules in $H^2(\mathbb{D}^2)$, associated to the finitely generated submodules containing a distinguished homogenous polynomial, are essentially normal, which is the first result on the essential normality of non-algebraic quotient modules in $H^2(\mathbb{D}^2)$. Moreover, we obtain the equivalence of the essential normality of a quotient module and the Hilbert-Schmidtness of its associated submodule in $H^2(\mathbb{D}^2)$, in the case that the submodule contains a distinguished homogenous polynomial. As an application, we prove that each finitely generated submodule containing a polynomial is Hilbert-Schmidt, which partially gives an affirmative answer to a conjecture of R. Yang.