QUANTUM INFORMATION (S8)

IGOR KLEP (LJUBLJANA), DAVID GROSS (COLOGNE), WILLIAM SLOFSTRA (WATERLOO)

Monday 14:00-16:00 (PSR4) chair: Igor Klep

14:00-14:25 Reinhard Werner
Von Neumann algebraic quantum information
14:30-14:55 Jurij Volcic
Post-hoc self-testing of quantum measurements
15:00-15:25 Arthur Mehta
New Approaches to Complexity via Quantum Graphs
15:30-15:55 Anna Skripka
Statistical inference for fermionic quantum time series

Monday 16:30-19:00 (PSR4) chair: Anand Natarajan

16:30-16:55 Bruno Nachtergaele
Ground state gap stability in the GNS representation
17:00-17:25 Hamza Fawzi
Certified algorithms for equilibrium states of local quantum Hamiltonians
17:30-17:55 Angela Capel
Quantum Markov Semigroups and Modified Logarithmic Sobolev Inequalities
18:00-18:25 Omar Fawzi
Capacities of quantum Markovian noise for large times

Tuesday 14:00-16:00 (PSR4) chair: Anna Skripka

14:00-14:25 Anand Natarajan
Bounding the quantum value of compiled nonlocal games: from CHSH to BQP verification
14:30-14:55 Yantian Zhang
Succinct arguments for QMA from standard assumptions via compiled nonlocal games
15:00-15:25 Simon Schmidt
On the quantum value of compiled nonlocal games
15:30-15:55 Tobias Fritz
An approach to homological algebra up to ε

Tuesday 16:30-19:00 (PSR4) chair: Tobias Fritz

16:30-16:55 Tim Netzer
Beyond Operator Systems
17:00-17:25 James Pascoe
Noncommutative change of variables
17:30-17:55 David E. Roberson
Quantum isomorphism and the NPA hierarchy
18:00-18:25 Josse van Dobben de Bruyn
Asymmetric graphs with quantum symmetry

Wednesday 11:40-12:40 (SIBSR2) chair: David Gross

11:40-12:05 Bill Helton
3XOR Games: Critical Thresholds for Solvability and Quantum Solvability

12:10-12:35 Seyed Sajjad Nezhadi
Quantum Perfect Matching Games

Wednesday 14:00–15:00 (SIBSR2) chair: David Gross

14:00-14:25 Miguel Navascués
First-order optimality conditions for non-commutative optimization problems

14:30-14:55 Sander Gribling
Bounding the separable rank via polynomial optimization

Abstracts.

Angela Capel, University of Cambridge

Quantum Markov Semigroups and Modified Logarithmic Sobolev Inequalities

Abstract. A dissipative evolution of an open quantum many-body system weakly coupled to an environment can be modelled by a quantum Markov semigroup, and its mixing time can be bounded using optimal constants of certain quantum functional inequalities, such as the modified logarithmic Sobolev constant. In this talk, we will review the mathematical formalism of dissipative evolutions governed by Lindbladians, and we will summarize the current state of the art on mixing times when the system has an associated commuting Hamiltonian.

References

[1] J. Kochanowski, Á. M. Alhambra, Á. Capel and C. Rouzé, *Rapid thermalization of dissipative many-body dynamics of commuting Hamiltonians*, **arXiv: 2404.16780**, 2024,
[2] I. Bardet, Á. Capel, L. Gao, A. Lucia, D. Pérez-García and C. Rouzé, *Entropy decay for Davies semigroups of a one dimensional quantum lattice*, **Communications in Mathematical Physics**, 405, 42, 2024 .

Omar Fawzi, Inria, ENS Lyon

Capacities of quantum Markovian noise for large times

Abstract. Given a quantum Markovian noise model, we study the maximum dimension of a classical or quantum system that can be stored for arbitrarily large time. We show that, unlike the fixed time setting, in the limit of infinite time, the classical and quantum capacities are characterized by efficiently computable properties of the peripheral spectrum of the quantum channel. In addition, the capacities are additive under tensor product. Based on joint work with Mizanur Rahaman and Mostafa Taheri.

Tobias Fritz, University of Innsbruck

An approach to homological algebra up to ε

Abstract. A theorem of Kazhdan on approximate representations of groups is based on a proof which seems to use cohomological methods "up to ε". This means that being a cocycle or a coboundary is not a yes/no-property of a cochain C, but rather a quantitative statement where one measures how strongly C deviates from being either. Based on Grandis's work on nonabelian homological algebra, I will present a framework for such quantitative homological algebra and sketch the intuition behind the resulting definitions of kernel and cokernel.

Unfortunately, the resulting category does not satisfy the axioms required of homological categories in Grandis's sense. Our main result solves this problem by showing that an arrow category is a homological category already under very weak assumptions. It follows that versions of derived functors and long exact sequences can be constructed for arrow categories quite generally, and this applies to quantitative homological algebra in particular.

References

[1] T. Fritz, *Non-abelian and ε-curved homological algebra with arrow categories*, J. Pure Appl. Algebra **227**, (2023), 107314.
[2] M. Grandis, *Homological Algebra: In Strongly Non-Abelian Settings*, World Scientific, Singapore, 2013.
[3] D. Kazhdan, *On ε-representations*, Israel J. Math. **43**, (1982), 315–323.

Sander Gribling, Tilburg University

Bounding the separable rank via polynomial optimization

Abstract. In this talk we consider the set $\mathcal{SEP}_d$ consisting of the linear maps ρ acting on $\mathbb{C}^d \otimes \mathbb{C}^d$ that can be written as a convex combination of rank-one matrices of the form $xx^* \otimes yy^*$, i.e., the set of separable states. We first revisit the well-known Doherty-Parrilo-Spedalieri hierarchy from the perspective of moment techniques, giving a new proof of convergence. We then introduce a new hierarchy of outer approximations based on lower bounding the *separable rank* of the state ρ.

References

[1] S. Gribling, M. Laurent, A. Steenkamp, Bounding the separable rank via polynomial optimization, *Lin. Alg. Appl.* **648**, (2022), 1–55.

Hamza Fawzi, University of Cambridge

Certified algorithms for equilibrium states of local quantum Hamiltonians

Abstract. Predicting observables in equilibrium states is a central yet notoriously hard question in quantum many-body systems. In the physically relevant thermodynamic limit, certain mathematical formulations of this task have even been shown to result in undecidable problems. Using a finite-size scaling of algorithms devised for finite systems often fails due to the lack of certified convergence bounds for this limit. In this work, we design certified algorithms for computing expectation values of observables in the equilibrium states of local quantum Hamiltonians, both at zero and positive temperature. Importantly, our algorithms output rigorous lower and upper bounds on these values. This allows us to show that expectation values of local observables can be approximated in finite time, contrasting related undecidability results. When the Hamiltonian is commuting on a 2-dimensional lattice, we prove fast convergence of the hierarchy at high temperature and as a result for a desired precision ϵ, local observables can be approximated by a convex optimization program of quasi-polynomial size in $1/\epsilon$.

Based on joint work with Omar Fawzi and Samuel Scalet (arXiv:2311.18706).

Bill Helton, UC San Diego

3XOR Games: Critical Thresholds for Solvability and Quantum Solvability

Abstract. Most of the talk will be devoted to proving that a particular function h of six variables is nonpositive. The approach envolves many cases and calculations which reduce the problem to analysis of one dimensional functions. These can be successfully bounded above by interval arithmetic.

The motivation for this is to determine if a 3 XOR game (asymptotically) has a sharp phase transition between solvability and nonsolvability as the number of constraints vs unknowns increase. Also what is the critical threshold for the phase transition? Our work parallels the approach by Dubois-Mandler 2003 which solved the classical 3 SAT problem (and has some connection to spin glass models). However, the formulas are much more complicated.

To back up further, while the talk is on purely classical mathematics, it arose from ways to find perfect quantum strategies to games, in work with Adam Bene Watts, Igor Klep and Zehong Zhang.

Arthur Mehta

New Approaches to Complexity via Quantum Graphs

Abstract. Problems based on the structure of graphs — for example finding cliques, independent sets, or colourings — are of fundamental importance in classical complexity. In this work, we introduce and study the clique problem for quantum graphs presented as quantum channels. We show that, by varying the collection of channels in the language, these give rise to complete problems for the classes NP, MA, QMA, and QMA(2). In this way, we exhibit a classical complexity problem whose natural quantisation is QMA(2), rather than QMA, which is commonly assumed. This talk is based on work with Eric Culf.

Bruno Nachtergaele, University of California, Davis

Ground state gap stability in the GNS representation

Abstract. Quantum many-body systems describe a wide variety of fascinating phenomena in the physical world. The last decade has seen impressive progress in our understanding of these systems. Notable topics are the classification of symmetry protected and symmetry enhanced topological phases, the fractional quantum Hall effect, and the stability of spectral gaps for many-body quantum systems. As a rule, these topics call for the analysis of arbitrarily large systems or infinite systems. While many works restrict themselves to obtain results for finite systems that are uniform in the system size, studies of infinite systems are increasingly common and are often preferred. Analysis of infinite many-body systems leads one to transition from the standard algebraic setting to the Hilbert space description provided by the GNS representation. In this talk I will explain a recent result proving stability of the ground state gap of infinite quantum spin systems (joint work with Robert Sims and Amanda Young [1]).

References

[1] B. Nachtergaele, Robert Sims, and Amanda Young, *Stability of the bulk gap for frustration-free topologically ordered quantum lattice systems*, Lett. Math. Phys. **114**, 24 (2024).

Anand Natarajan, Massachusetts Institute of Technology

Bounding the quantum value of compiled nonlocal games: from CHSH to BQP verification

Abstract. In the classical world, an extremely fruitful technique for constructing interactive protocols is “compiling” a multiprover game, using cryptography to simulate the separation between the provers. In the quantum world, the study of compiled nonlocal games was introduced by Kalai et al. (STOC’23), who defined a compilation procedure that applies to any nonlocal game and preserves the *classical* value; however, they did not show any bounds on the *quantum* value of their protocols. In this work, we make progress towards a full understanding of the quantum value of compiled nonlocal games. For the special case of the CHSH game, we show that the Tsirelson bound holds for the compiled game in two ways: by extending the “macroscopic locality” argument of Rohrlich, and by showing that strategies for the compiled game yield feasible solutions to the Tsirelson SDP. We conjecture that the latter argument can be extended to all XOR games. Using our SDP argument, we are able to recover a strong version of the “rigidity” property that makes CHSH so useful in applications; specifically, we show that compiled CHSH is a “computational self-test” in the sense of Metger and Vidick. As an application, we give a classical verification protocol for BQP based on a compiled nonlocal game and prove soundness. Our protocol replicates the functionality of Mahadev ’18 but with two advantages: (1) the soundness analysis is much simpler, and directly follows the analysis of the nonlocal case, and (2) the soundness does not “explicitly” use the assumption of a TCF or an adaptive hardcore bit, and only requires QFHE as a black box (though currently the only known constructions of QFHE use TCFs).

Miguel Navascués

First-order optimality conditions for non-commutative optimization problems

Abstract. We consider the problem of optimizing the state average of a polynomial of non-commuting variables, over all states and operators satisfying a number of polynomial constraints, and over all Hilbert spaces where such states and operators are defined. Such non-commutative polynomial optimization (NPO) problems are routinely solved through hierarchies of semidefinite programming (SDP) relaxations. By phrasing the general NPO problem in Lagrangian form, we heuristically derive, via small variations on the problem variables, state and operator optimality conditions, both of which can be enforced by adding new positive semidefinite constraints to the SDP hierarchies. State optimality conditions are satisfied by all Archimedean (that is, bounded) NPO problems, and allow enforcing a new type of constraints: namely, restricting the optimization over states to the set of common ground states of an arbitrary number of operators. Operator optimality conditions are the non-commutative analogs of the Karush–Kuhn–Tucker (KKT) conditions, which are known to hold in many classical optimization problems. In this regard, we prove that a weak form of non-commutative operator optimality holds for all Archimedean NPO problems; stronger versions require the problem constraints to satisfy some qualification criterion, just like in the classical case. We test the power of the new optimality conditions by computing local properties of ground states of many-body spin systems and the maximum quantum violation of Bell inequalities.

References

[1] Mateus Araújo, Igor Klep, Andrew J. P. Garner, Tamás Vértesi, Miguel Navascués, First-order optimality conditions for non-commutative optimization problems, *arXiv:2311.18707*

Tim Netzer, University of Innsbruck

Beyond Operator Systems

Abstract. We generalize the setup of finite-dimensional operator systems to so-called *cone systems*, and prove the most important results in this general setup. This has several interesting consequences. We obtain a vector-valued Krein extension theorem, prove that the free mapping cones of positive maps and completely positive maps are not finitely generated, and deduce that there exists a self-dual functorial tensor products for finite-dimensional cones and for finite-dimensional operator systems.

This is joint work with Gemma De les Coves and Mirte van der Eyden.

James Eldred Pascoe, Drexel University

Noncommutative change of variables

Abstract. We give an overview of the change of variables theory for natural functions of several operator variables with emphasis on contributions from a variety of authors. Specializing, Augat's Ax-Grothendieck theorem for free polynomials as functions on tuples of matrices of arbitrary size gives that an invertible free polynomial map has a free polynomial inverse. If one instead evaluates over all linear maps on complex vector spaces, Augat's theorem is grossly facilitated and extensions to rational maps are available.

Seyed Sajjad Nezhadi, University of Maryland

Quantum Perfect Matching Games

Abstract. I will talk about the notion of a quantum or no-signaling perfect matching (as well as fractional matching and bipartite matching) for graphs. These are defined via nonlocal games inspired by the homomorphism game which are classically winnable with probability 1 if and only if the graph has the matching properties. I will talk about when these games have quantum and no-signaling advantages. In particular, for bipartite and fractional matching I will provide an algorithm that checks if the game has a perfect no-signaling strategy, and show that while there can be a quantum advantage for these games there is no possibility for pseudo-telepathy. For perfect matching of general graphs I will show that all three models define distinct properties and provide some results that characterize when a game is quantum or no-signaling winnable.

Simon Schmidt, Ruhr University Bochum

On the quantum value of compiled nonlocal games

Abstract. Nonlocal games are a foundational tool for understanding entanglement and constructing quantum protocols in settings with spatially separated quantum devices. Recently, Kalai et al. (STOC '23) defined a cryptographic compilation procedure for nonlocal games. It translates a (two-player) nonlocal game into a single-player game, using cryptography to simulate the spatial separation between the players. This talk will be about the quantum value of such compiled nonlocal games. We will discuss techniques for bounding the quantum value and see when the value is preserved under the compilation procedure.

Anna Skripka, University of New Mexico

Statistical inference for fermionic quantum time series

Abstract. We will discuss asymptotic equivalence of quantum statistical experiments pertaining to certain gauge invariant quasifree states on a fermionic Fock space to certain classical experiments. The respective asymptotic equivalence is understood in the sense of the quantum Le Cam distance measuring the least trace-norm error incurred while mapping one model into another via quantum channels. The obtained result is applied to construct an asymptotically optimal estimator of an unknown state parameter. The talk is based on joint work with M. Nussbaum.

Josse van Dobben de Bruyn, Technical University of Denmark (DTU)

Asymmetric graphs with quantum symmetry

Abstract. The quantum symmetries of a graph form a C^*-algebraic compact quantum group, called the *quantum automorphism group* of the graph. In recent years, various authors have asked variations of the following question: how far can the classical and quantum automorphism group of a graph be apart? In this talk, after presenting some background on C^*-algebraic quantum groups, I will answer one of these questions by presenting a class of graphs which have trivial (classical) automorphism group and non-trivial quantum automorphism group. The construction is inspired by solution groups of binary linear systems, as defined by Cleve, Liu and Slofstra [1] in the context of linear constraint system games. This talk is based on joint work with David E. Roberson and Simon Schmidt [2].

References

[1] R. Cleve, L. Liu, W. Slofstra, Perfect commuting-operator strategies for linear system games, *J. Math. Phys.* **58** (2017), 012202.
[2] J. van Dobben de Bruyn, D. E. Roberson, S. Schmidt, *Asymmetric graphs with quantum symmetry*, Preprint, arXiv:2311.04889 (2023).

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Jurij Volčič, Drexel University

Post-hoc self-testing of quantum measurements

Abstract. Self-testing is the strongest form of quantum functionality verification, which allows one to deduce the quantum state and measurements of an entangled system from its classically observed statistics. From a mathematical perspective, self-testing is an intriguing uniqueness phenomenon, pertaining to functional analysis, moment problems, convexity and representation theory. This talk restricts itself to self-testing in bipartite systems. It focuses on a criterion for extending self-testing results when measurements are added to the system. This criterion in particular implies that every collection of real projective-valued measurements can be self-tested in a bipartite scenario. Based on joint work with Ranyiliu Chen and Laura Mančinska.

Reinhard F. Werner, Leibniz Universität Hannover, Germany

Von Neumann algebraic quantum information

Abstract. General von Neumann algebras appear as the observable algebras of systems with infinitely many degrees of freedom, as in quantum field theory, statistical mechanics in the thermodynamic limit, or as idealized resources in quantum information theory. Their structure has a direct relevance for the type of entanglement possible in such situations. I will report some old and some recent results in this regard.

References

[1] L. van Luijk, A. Stottmeister, R. F. Werner, and H. Wilming, Embezzlement of entanglement, quantum fields, and the classification of von Neumann algebras, arXiv:2401.07299
[2] L. van Luijk, A. Stottmeister, and H. Wilming, Critical Fermions are universal embezzlers, arXiv:2406.11747
[3] S.J. Summers and R.F. Werner, Maximal violation of Bell's inequalities for algebras of observables in tangent spacetime regions. *Ann. Inst. H. Poincaré A* **49** (1988) 215-243.