FREE ANALYSIS AND CONVEXITY (S7)

ROBERT MARTIN (MANITOBA), JAMES PASCOE (DREXEL, PHILADELPHIA), ELI SHAMOVICH (BEN GURION)

Monday 16:30-19:00 (SIBLT3) chair: Eli Shamovich

16:30-16:55 Bill Helton
Solving Quantum Max Cut via Swap Operators, I
17:00-17:25 Igor Klep
Solving Quantum Max Cut via Swap Operators, II
17:30-17:55 William Slofstra
Positivity is undecidable in free product algebras
18:00-18:25 Tea Štrekelj
Duality and extreme points for Γ*-convex sets*
18:30-18:55 Jiří Spurnỳ
Boundary integral representation of multipliers of fragmented affine functions

Tuesday 14:00-16:00 (SIBLT3) chair: Rob Martin

14:00-14:25 Jurij Volčič
Waring problems for noncommutative rational functions
14:30-14:55 Matt Kennedy
Noncommutative Majorization
15:00-15:25 Jeet Sampat
Biholomorphisms between subvarieties of noncommutative operator balls
15:30-15:55 Hridoyananda Saikia
A non-commutative boundary for the dilation order

Abstracts.

Bill Helton, University of California San Diego

Solving Quantum Max Cut via Swap Operators, I

Abstract. The Quantum Max Cut (QMC) problem has emerged as a test-problem for designing approximation algorithms for local Hamiltonian problems in quantum physics. In this talk we attack this problem using the algebraic structure of QMC; we will explore the relationship between QMC and the representation theory of the symmetric group.

The first major contribution is an extension of noncommutative Sum of Squares optimization techniques championed by Helton and McCullough to give a new hierarchy of relaxations to Quantum Max Cut. The hierarchy we present is based on polynomials in the swap operators. To prove completeness of this hierarchy, we give a finite presentation of the algebra generated by the swap operators. We find that level-2 of this new hierarchy is exact (up to tolerance 10^7) on all QMC instances with uniform edge weights on small graphs.

The second major contribution of this talk is a polynomial-time algorithm that exactly computes the maximum eigenvalue of the QMC Hamiltonian for certain graphs, including graphs that can be "decomposed" as a signed combination of cliques. A special case of the latter are complete bipartite graphs with uniform edge-weights, for which exact solutions are known from the work of Lieb and Mattis (1962).

The talk is based on joint work with Adam Bene Watts, Anirban Chowdhury, Aidan Epperly, and Igor Klep.

Matthew Kennedy, University of Waterloo

Noncommutative Majorization

Abstract. I will introduce a notion of noncommutative majorization and discuss some application to operator algebras, including a multivariate generalization of the Schur-Horn theorem for finite von Neumann algebras. I will also briefly discuss some applications to quantum information theory. This is joint work with Laurent Marcoux and Paul Skoufranis.

Igor Klep, University of Ljubljana

Solving Quantum Max Cut via Swap Operators, II

Abstract. The Quantum Max Cut (QMC) problem has emerged as a test-problem for designing approximation algorithms for local Hamiltonian problems in quantum physics. In this talk we attack this problem using the algebraic structure of QMC; we will explore the relationship between QMC and the representation theory of the symmetric group.

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The talk is based on joint work with Adam Bene Watts, Anirban Chowdhury, Aidan Epperly, Bill Helton, and work in progress with Tea Štrekelj and Jurij Volčič.

Hridoyananda Saikia, University of Manitoba

A non-commutative boundary for the dilation order

Abstract. Arveson's hyperrigidity conjecture focuses on the unique extension property (UEP) of representations of C^*-algebras with respect to a generating operator system. The states that are maximal in the dilation order fully encapsulate the cyclic representations of a C^*-algebra with the UEP. The set of all maximal states form a norm-closed set which remains stable under absolute continuity. In this talk, we will discuss an equivalent characterization of the dilation maximal states in terms of a boundary projection. Subsequently, we will state a reformulation of Arveson's hyperrigidity conjecture in terms of the non-commutative topological properties of this boundary projection. This is a joint work with Raphaël Clouâtre.

Jeet Sampat, Technion

Biholomorphisms between subvarieties of noncommutative operator balls

Abstract. This talk is based on recent joint work with O. M. Shalit.

Let $\mathbb{M}^d$ be the graded union of d-tuples of complex matrices of arbitrary size. Given any d-dimensional operator space $\mathcal{E}$ with basis $\{Q_1, \ldots, Q_d\}$, we define the corresponding noncommutative (nc) operator ball as

$$\mathbb{D}_Q := \left\{ X \in \mathbb{M}^d : \left\| \sum_{j=1}^{d} Q_j \otimes X_j \right\| < 1 \right\}.$$

We shall discuss the problem of extending a nc map between subvarieties $f : \mathfrak{V}_1 \to \mathfrak{V}_2$ to a map $F : \mathbb{D}_{Q_1} \to \mathbb{D}_{Q_2}$. We show that such an extension cannot be guaranteed unless $\mathcal{E}_2$ is an injective operator space. In particular, this holds for the nc unit polydisk $\mathfrak{D}_d$ and the nc unit row-ball $\mathfrak{B}_d$. We shall use this extension property to identify nc biholomorphisms between subvarieties $\mathfrak{V}_j \subseteq \mathbb{D}_{Q_j}$ and show that when $\mathfrak{V}_j$ is 'nice enough,' such a biholomorphism can be modified to be the restriction of a linear isomorphism between $\mathbb{D}_{Q_j}$.

William Slofstra, University of Waterloo

Positivity is undecidable for products of free algebras

Abstract. For free *-algebras, free group algebras, and related algebras, it is possible to decide if an element is positive (in all representations) using results of Helton, Bakonyi-Timotin, Helton-McCullough, and others. In this talk, I'll discuss joint work with Arthur Mehta and Yuming Zhao showing that this problem becomes undecidable for a product of free *-algebras. I'll also discuss how results of this type could be aided by having a Higman embedding theorem for algebras with states, as well as work in progress on this question.

Jičí Spurný, Charles University

Boundary integral representation of multipliers of affine functions

Abstract. We develop a theory of abstract intermediate function spaces on a compact convex set X and study the behaviour of multipliers and centers of these spaces. In particular, we provide some criteria for coincidence of the center with the space of multipliers and a general theorem on boundary integral representation of multipliers. We apply the general theory in several concrete cases, among others to strongly affine Baire functions, to the space $A_f(X)$ of fragmented affine functions, to the space $(A_f(X))^\mu$, the monotone sequential closure of $A_f(X)$, to their natural subspaces formed by Borel functions, or, in some special cases, to the space of all strongly affine functions. In addition, we prove that the space $(A_f(X))^\mu$ is determined by extreme points and provide a large number of illustrating examples and counterexamples.

References

[1] J. Rondoš, O.F.K. Kalenda, J. Spurný, Boundary integral representation of multipliers of fragmented affine functions and other intermediate function spaces, *preprint.*

Tea Štrekelj, Institute of Mathematics, Physics and Mechanics

Duality and extreme points for Γ*-convex sets*

Abstract. In this talk we discuss several generalizations of (matrix) convexity, e.g., partial convexity or biconvexity, which are summed up in the term Γ-convexity. Here Γ is a tuple of free symmetric polynomials determining the geometry of a Γ-convex set. We introduce the notions of Γ-operator systems and Γ-ucp maps and establish a Webster-Winkler type categorical duality between Γ-operator systems and Γ-convex sets. Next, we introduce a notion of an extreme point of a Γ-convex set extending the concept of a free extreme point. To ensure existence of such points, matrix convex sets are extended to include an operator level. The so called Γ-extreme points of an operator Γ-convex set K are in bijective correspondence with the free extreme points of the operator convex hull of $\Gamma(K)$. From this result, a Krein-Milman theorem for Γ-convex sets follows.

Jurij Volčič, Drexel University

Waring problems for noncommutative rational functions

Abstract. Noncommutative polynomials and noncommutative rational functions are elements of the free associative algebra and free skew field, respectively. One may view them as multivariate functions in matrix arguments; this perspective is common in noncommutative function theory and free real algebraic geometry. This talk concerns the images of noncommutative rational functions on large matrices. Firstly, every nonconstant noncommutative rational function attains values with pairwise distinct eigenvalues on sufficiently large matrix tuples. Secondly, one can then apply this to noncommutative variants of the Waring problem. In particular, given a nonconstant noncommutative rational function, every large enough trace-zero matrix is a difference of its values, and every large enough nonscalar determinant-one matrix is a quotient of its values. Based on joint work with Matej Brešar.