MULTIVARIABLE OPERATOR THEORY (S5)

SANNE TER HORST (NWU), TIRTHA BHATTACHARYYA (INDIAN INSTITUTE OF SCIENCE), NICHOLAS YOUNG (NEWCASTLE & LEEDS)

Tuesday 16:30-19:00 (SIBLT1) chair: Nicholas Young

16:30-16:55 Hugo Woerdeman
Optimal interpolation in Hardy and Bergman spaces: a reproducing kernel Banach space approach

17:00-17:25 Lijia Ding
The biholomorphic invariance of essential normality

Wednesday 11:40-12:40 (SIBLT1) chair: Nicholas Young

11:40-12:05 Piotr Pikul
The Szász inequality for matrix polynomials and functional calculus

12:10-12:35 Jeet Sampat
Jointly cyclic polynomials and maximal domains

Wednesday 14:00–15:00 (SIBLT1) chair: Sanne ter Horst

14:00-14:25 Raul Curto
Slantification of Hankel Operators on the Hardy Space of the n–torus

14:30-14:55 Dariusz Bugajewski
On composition of formal power series of multiple variables with applications

Thursday 14:00-16:00 (SIBSR7) chair: Sanne ter Horst

14:00-14:25 Greg Knese
Boundary local Integrability of rational functions in two variables

14:30-14:55 Connor Evans
Types of singularity on the boundary for Schur-Agler class functions on $\mathbb{D}^d$

15:00-15:25 Kenta Kojin
Some relations between Schwarz-Pick inequality and von Neumann's inequality

15:30-15:55 Robert T.M. Martin
A Fejér–Riesz theorem for non-commutative rational functions

Thursday 16:30–18:30 (SIBSR7) chair: Sanne ter Horst

16:30-16:55 Baruch Solel
Isometric dilation for representations of product systems

17:00-17:25 Amit Maji
Wold decomposition for isometries with equal range

17:30-17:55 Victor Bailey
Frames Generated by Unilateral Iterations of Bounded Commuting Operators

Abstracts.

Victor Bailey, University of Oklahoma

Frames Generated by Unilateral Iterations of Bounded Commuting Operators

Abstract. Recent work in Dynamical Sampling has been centered on characterizing frames obtained from the orbit of a vector under a bounded operator [1]. In this talk, we will provide a necessary and sufficient condition for a frame in a separable infinite-dimensional Hilbert space to be generated by unilateral iterations of a pair of bounded commuting operators on a vector. Applying the theory of shift-invariant subspaces of the Hardy Space on the bidimensional torus, we characterize these frames and provide several properties of frames of this form. This is joint work with Carlos Cabrelli.

References

[1] O. Christensen, M. Hasannasab, A Survey on Frame Representations via Dynamical Sampling, *arXiv:2201.00038* (2021)

Dariusz Bugajewski, Adam Mickiewicz University, Poznań

On composition of formal power series of multiple variables with applications

Abstract. Formal power series as well as formal Laurent series play an important role in various branches of mathematics. In particular, they are frequently used in solving various type of equations, like ordinary differential equations or partial differential equations. Other applications of formal power series can be found in combinatorics, in Riordan groups and also in the proofs of some classical results, like the Cayley-Hamilton Theorem.

During the talk I am going to focus mainly on a necessary and sufficient condition for the existence of the composition of formal power series in the case when the outer series is a series of one variable while the inner one is a series of multiple variables. I am planning to discuss some ambiguities connected with the Right Distributive Law for formal power series of one variable, and next to present analogues of that law in the multivariable case. In the second part of the talk (depending on time), I am going to discuss the famous J.C.P. Miller formula which provides a recurrence algorithm for the composition of the formal binomial series and a formal power series f being a nonunit. I will present the general J.C.P. Miller formula which eliminates the requirement of nonunitness of f, formulating a necessary and sufficient condition for the existence of such composition.
The results presented in this talk come mainly from the papers [1] and [2].

References

[1] Dariusz Bugajewski, Dawid Bugajewski, X.-X. Gan and P. J. Maćkowiak, On the recursive and the explicit form of the general J.P.C. Miller formula with applications, *Adv. Appl. Math.* **156**, (2024), 102688, 1–21.
[2] Dariusz Bugajewski, A. Galimberti and P. Maćkowiak, *On composition and Right Distributive Law for formal power series of multiple variables*, arXiv:2211.06879 (2022).

Raúl E. Curto, The University of Iowa

Slantification of Hankel Operators on the Hardy Space of the n–torus

Abstract. In the case of Toeplitz operators on the Hardy space $H^2(\mathbb{T})$ of the unit circle, the notion of slant Toeplitz operator was introduced by Mark Ho in 1996. Subsequently, slantification has been extended to Hankel operators on $H^2(\mathbb{T})$ and to Toeplitz operators acting on $H^2(\mathbb{T}^n)$, the Hardy space of the n–torus. In this talk, we will first introduce the notion of slantification of a Hankel operator acting on the space $H^2(\mathbb{T}^n)$. We will then study various properties of these operators, including hyponormality, isometric behavior, co-isometric behavior, and compactness.

For our purposes, the appropriate definition of Hankel operator is as follows. Given a symbol $\phi \in L^\infty(\mathbb{T}^n)$, the Hankel operator on $H^2(\mathbb{T}^n)$ induced by ϕ is given as

$$H_{\phi,n} := PJ_nM_\phi|_{H^2(\mathbb{T}^n)};$$

here J_n is the flip operator defined on $L^2(\mathbb{T}^n)$ by

$$(J_nf)(z_1,\ldots,z_n) := f(\bar{z}_1,\ldots,\bar{z}_n) \qquad (f \in L^2(\mathbb{T}^n))$$

and P is the orthogonal projection of $L^2(\mathbb{T}^n)$ onto $H^2(\mathbb{T}^n)$.

References

[1] R.E. Curto, G. Datt and B.B. Gupta, Slantification of Hankel Operators on the Hardy Space of the n–torus, *Complex Anal. Oper. Theory* **17**, 48(2023); 16 pp.

Lijia Ding, Zhengzhou University

The biholomorphic invariance of essential normality

Abstract. The study of p-essentially normal Hilbert modules originated from Arveson's seminal works circa 2000, where he applied the results to investigate the dilation theory and the geometric invariant theory of the commuting operator tuple. Recently, there has been an increasing focus on the p-essential normality of Hilbert modules determined by subvarieties. In this talk, I shall discuss the biholomorphic invariance of p-essential normality of analytic Hilbert modules and give some intrinsic characterizations of p-essential normality. Furthermore, I will provide an application that extends the recent results on the equivalence between essential normality and hyperrigidity.

Connor Evans, Newcastle University, UK

Types of singularity on the boundary for Schur-Agler class functions on $\mathbb{D}^d$

Abstract. Classical results for analytic functions on $\mathbb{D}$ often have non-trivial generalizations, so how can we find analogous results for the polydisc $\mathbb{D}^d$? Recent success in the case of the bidisk $\mathbb{D}^2$ was found by Agler, McCarthy and Young, [1], through the use of Hilbert space models on $\mathbb{D}^2$, specifically, let φ be a function on $\mathbb{D}^2$, then $(\mathcal{H}, u)$ is said to be a model for φ if $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$ is a Hilbert space, $\mathcal{H}_1$ and $\mathcal{H}_2$ are orthogonally complementary subspaces of $\mathcal{H}$ and $u = (u_1, u_2)$ is a pair of analytic maps from $\mathbb{D}^2$ to $\mathcal{H}_1, \mathcal{H}_2$ respectively such that, for all $\lambda, \mu \in \mathbb{D}^2$

$$1 - \overline{\varphi(\mu)}\varphi(\lambda) = \left\langle (1 - \overline{\mu_1}\lambda_1)u_1(\lambda), u_1(\mu) \right\rangle_{\mathcal{H}_1} + \left\langle (1 - \overline{\mu_2}\lambda_2)u_2(\lambda), u_2(\mu) \right\rangle_{\mathcal{H}_2}.$$

Every Schur class function φ on $\mathbb{D}$ and $\mathbb{D}^2$ has such a model, but for $d \geq 3$ this is not true. We instead remedy this by considering the Schur-Agler class, $\mathcal{SA}_d$, this is the set of holomorphic functions $\varphi : \mathbb{D}^d \to \mathbb{C}$ such that, for all commuting d-tuples $(T_1, \ldots, T_d)$ of contractions on a Hilbert space $\mathcal{H}$,

$$\|\varphi\|_{(d,\infty)} = \sup_{r<1} \|\varphi(rT_1, \dots, rT_d)\|_\infty \leq 1.$$

Every function $\varphi \in \mathcal{SA}_d$ does have a Hilbert space model.

In this talk, we consider singularities $\tau \in \partial\mathbb{D}^d$ for functions $\varphi \in \mathcal{SA}_d$ and give specific criteria for the study of such singularities by means of Hilbert space models.

This talk is based on joint work with Dr. Zinaida Lykova and Prof. Nicholas Young.

References

[1] J. Agler, J. E. McCarthy and N. J. Young. A Carathéodory Theorem for the bidisc via Hilbert space methods. *Math. Annalen*, **352**(2), (2012), 581–624.

Greg Knese, Washington University in St. Louis

Boundary local Integrability of rational functions in two variables

Abstract. Motivated by studying boundary singularities of rational functions in two variables that are analytic on a domain, we investigate local integrability on $\mathbb{R}^2$ near $(0,0)$ of rational functions with denominator non-vanishing in the bi-upper half-plane but with an isolated zero (with respect to $\mathbb{R}^2$) at the origin. Building on work of Bickel-Pascoe-Sola, we give a necessary and sufficient test for membership in a local $L^p(\mathbb{R}^2)$ space and we give a complete description of all numerators Q such that Q/P is locally in a given L^p space. As applications, we prove that every bounded rational function on the bidisk has partial derivatives belonging to L^1 on the two-torus. In addition, we give a new proof of a conjecture started in work of Bickel-Knese-Pascoe-Sola and completed by Kollár characterizing the ideal of Q such that Q/P is locally bounded. A larger takeaway from this work is that a local model for stable polynomials we employ is a flexible tool and may be of use for other local questions about stable polynomials.

Kenta Kojin, Nagoya University

Some relations between Schwarz-Pick inequality and von Neumann's inequality

Abstract. I will talk about a Schwarz-Pick type inequality for the Schur-Agler class $SA(B_\delta)$ on a polynomial polyhedron B_δ, where δ is a matrix of polynomials in d-variables. We define a pseudo-distance on B_δ by

$$d_\delta(z,w) := \Big\| (I - \delta(w)\delta(w)^*)^{-\frac{1}{2}}(\delta(z) - \delta(w)) \\ \times (I - \delta(w)^*\delta(z))^{-1}(I - \delta(w)^*\delta(w))^{\frac{1}{2}} \Big\| \quad (z, w \in B_\delta).$$

This is a generalization of the pseudo-hyperbolic distance on the open unit disk $\mathbb{D}$ defined by

$$d_{\mathbb{D}}(z,w) = \left| \frac{z-w}{1-\overline{w}z} \right| \quad (z, w \in \mathbb{D}).$$

Then, we can prove that

$$\sup_{f \in SA(B_\delta)} d_{\mathbb{D}}(f(z), f(w)) = d_\delta(z,w)$$

holds for any pair $z, w \in B_\delta$. As an application of this result and Jim Agler's deep observations on relations between operator theory and complex geometry [1], we can give a quantitative sufficient condition on a diagonalizable commuting tuple T acting on $\mathbb{C}^2$ for B_δ to be a complete spectral domain for T. We apply this sufficient condition to generalizing von Neumann's inequalities studied by Drury and by Hartz-Richter-Shalit.

References

[1] J. Agler, Operator theory and the Carathéodory metric. *Invent. Math.*, **101**, (1990), 483–500.
[2] K. Kojin, Some relations between Schwarz–Pick inequality and von Neumann's inequality, *Complex Analysis and Operator Theory*, **18**, (2024).

Amit Maji, Indian Institute of Technology Roorkee, India

Wold decomposition for isometries with equal range

Abstract. The main aim of this talk is the *Wold decomposition* of a large class of tuples of isometries on Hilbert spaces. More specifically, let $n \geq 2$, and $V = (V_1, \ldots, V_n)$ be an n-tuple of isometries acting on a Hilbert space $\mathcal{H}$. We say that V is an n-tuple of *isometries with equal range* if $V_i^{m_i} V_j^{m_j} \mathcal{H} = V_j^{m_j} V_i^{m_i} \mathcal{H}$ and $V_i^{*m_i} V_j^{m_j} \mathcal{H} = V_j^{m_j} V_i^{*m_i} \mathcal{H}$ for $m_i, m_j \in \mathbb{Z}_+$, where $1 \leq i < j \leq n$.

We prove that each n-tuple of *isometries with equal range* admits a unique *Wold decomposition*. We further obtain analytic models of the above class, and as a consequence, we show that the wandering data are complete unitary invariants for n-tuples of *isometries with equal range*. Our results unify all prior findings on the decomposition for tuples of isometries in the existing literature.

References

[1] S. Majee, A. Maji, Wold decomposition for isometries with equal range, accepted in Journal of Operator Theory, 2024, 33 pp. arXiv:2309.04445.

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Robert T. W. Martin, University of Manitoba

A Fejér–Riesz theorem for non-commutative rational functions

Abstract. We prove a Fejér–Riesz theorem for bounded, positive semi-definite Toeplitz operators on the full Fock space with 'non-commutative (NC) rational symbols'. Here, the full Fock space can be identified with the 'free Hardy space' of all square–summable power series in several non-commuting variables $\{\mathfrak{z}_1, \cdots, \mathfrak{z}_d\}$. A bounded linear operator, T, on this space is left-Toeplitz, if $L_j^* T L_k = \delta_{j,k} T$, where $L_j = M^L_{\mathfrak{z}_j}$ are the left free shifts, the isometries of left multiplication by any of the formal NC variables, $\mathfrak{z}_j$. A non-commutative rational function is (essentially) any valid expression that can be obtained by applying the arithmetic operations of summation, multiplication and inversion to the free algebra of all free (non-commutative) polynomials with complex coefficients. Any element of the free Hardy space can be viewed as a non-commutative analytic function on the NC unit row-ball consisting of all row-contractive $d-$tuples of $n \times n$ complex matrices of any fixed size $n \in \mathbb{N}$, and an NC rational function defines a bounded left and right multiplier of this space, if and only if its domain includes a row-ball of radius greater than one.

Our NC rational Fejér–Riesz theorem then states that if $\mathfrak{h}$ is any bounded NC rational multiplier of the free Hardy space, and if $T = \operatorname{Re} M^R_{\mathfrak{h}}$ is positive semi-definite, where $M^R_{\mathfrak{h}}$ denotes right multiplication by $\mathfrak{h}$, then it factors as $T = (M^R_{\mathfrak{r}})^* M^R_{\mathfrak{r}}$, for some bounded NC rational multiplier, $\mathfrak{r}$. The Fejér–Riesz theorem for free polynomials of G. Popescu is recovered as a special case.

This is joint work with Michael T. Jury (U. Florida).

References

[1] M. T. Jury and R. T. W. Martin, Sub-Hardy Hilbert spaces in the non-commutative unit row-ball, *Fields Institute Communications: Function Spaces, Theory and Applications* **87**, (2023), 349–398.

Piotr Pikul, Jagiellonian University in Kraków

The Szász inequality for matrix polynomials and functional calculus

Abstract. Szász inequality is a classical result providing a bound for polynomials with zeros in the upper half of the complex plane in terms of its low-order coefficients. Some generalisations of this result to multivariable polynomials were done by Borcea, Brändén and Knese. In the talk there will be presented inequalities of this kind for matrix polynomials in scalar variable and scalar polynomials in one and multiple matrix variables.

Joint work with Michał Wojtylak nad Oskar J. Szymański.

References

[1] P. Pikul, O. J. Szymański, M. Wojtylak, The Szász inequality for matrix polynomials and functional calculus, preprint arXiv:2406.08965 (2024)

[2] G. Knese, Global bounds on stable polynomials, *Complex Anal. Oper. Theory*, **13**, (2019), 1895–1915.

Jeet Sampat, Technion

Jointly cyclic polynomials and maximal domains

Abstract. This talk is based on recent joint work with M. Mironov.

Let $\mathcal{X}$ be a topological vector space of holomorphic functions on an open set $\Omega \subset \mathbb{C}^d$ for which the polynomials $\mathcal{P}_d$ are dense. A family $\mathcal{F} \subset \mathcal{P}_d$ is called *jointly cyclic* if the shift invariant subspace they generate is $\mathcal{X}$. The *maximal domain* Ω_{max} is the set of all $w \in \mathbb{C}^d$ at which the evaluation functional $\Lambda_w : P \mapsto P(w)$ on $\mathcal{P}_d$ extends continuously to $\mathcal{X}$.

For $d = 1$, we completely determine when a family $\mathcal{F} \subset \mathcal{P}_d$ is jointly cyclic using Ω_{max}. For $d = 2$, we show that this can be reduced to determining the cyclicity of $GCD(\mathcal{F})$. We also examine the topology of Ω_{max} and show that if $\mathcal{X}$ is metrizable then Ω_{max} is an F_σ set. Lastly, we construct Hilbert function spaces on the unit disk $\mathbb{D}$ with $\Omega_{max} = \mathbb{D} \cup \Gamma$, where $\Gamma \subseteq \partial\mathbb{D}$ is both F_σ and G_δ.

Baruch Solel, Technion, Haifa, Israel

Isometric dilation for representations of product systems

Abstract. We discuss representations of product systems (of W^*-correspondences) over the semigroup $\mathbf{Z}_+^n$ and show that, under certain pureness and Szegö positivity conditions, a completely contractive representation can be dilated to an isometric representation. For $n = 1, 2$ this is known to hold in general (without assuming the conditions) but, for $n \geq 3$, it does not hold in general (as is known for the special case of isometric dilations of a tuple of commuting contractions). Restricting to the case of tuples of commuting contractions, our result reduces to a result of Barik, Das, Haria and Sarkar. Our dilation is explicitly constructed and we present some applications.

This is a joint work with S. Barik and M. Bhattacharjee.

Hugo J. Woerdeman, Drexel University

Optimal interpolation in Hardy and Bergman spaces: a reproducing kernel Banach space approach

Abstract. After a review of the reproducing kernel Banach space framework and semi-inner products, we apply the techniques to the setting of Hardy spaces H^p and Bergman spaces A^p, $1 < p < \infty$, on the unit ball in $\mathbb{C}^n$, as well as the Hardy space on the polydisk and half-space. In particular, we show how the framework leads to a procedure to find a minimal norm element f satisfying interpolation conditions $f(z_j) = w_j$, $j = 1, \ldots, n$. We also explain the techniques in the setting of ℓ^p spaces where the norm is defined via a change of variables and provide numerical examples. This talk is based on joint work with Gilbert Groenewald and Sanne ter Horst.