DIFFERENTIAL OPERATORS AND MATHEMATICAL PHYSICS (S4)

SABINE BÖGLI (DURHAM), JEAN-CLAUDE CUENIN (LOUGHBOROUGH), PETR SIEGL (GRAZ)

Monday 14:00-16:00 (SIBLT1) chair: SB/JCC

14:00-14:25 Ivica Nakic
Spectrum of operators on equilateral metric graphs
14:30-14:55 Marzieh Baradaran
Spectrum of quantum graphs with time-reversal non-invariant vertex couplings
15:00-15:25 Jon Harrison
A Discrete Analog of Quantum Unique Ergodicity on Circulant Graphs
15:30-15:55 Bryn Davies
Subwavelength spectra for highly contrast coefficients: an asymptotic framework for metamaterial design

Monday 16:30-19:00 (SIBLT1) chair: SB/JCC

16:30-16:55 Yuri Latushkin
The Duistermaat index and eigenvalue interlacing for self-adjoint extensions of a symmetric operator
17:00-17:25 Oliver Fürst
Regularized Index of non-Fredholm Callias Operators
17:30-17:55 Sukrid Petpradittha
Lieb-Thirring type inequalities for multidimensional Schrödinger operators with complex-valued potentials
18:00-18:25 Rongwei Yang
Yang-Mills equations in C^*-algebras

Tuesday 14:00-16:00 (SIBLT1) chair: SB/JCC

14:00-14:25 Piero D'Ancona
Dispersion estimates for Dirac equations with Aharonov–Bohm magnetic fields
14:30-14:55 Łukasz Rzepnicki
Dirac system with an integrable potential and asymptotic behavior of its solutions on the plane
15:00-15:25 Jakob Reiffenstein
Eigenvalue density of limit circle Jacobi operators
15:30-15:55 Giovanni Bracchi
The propagator for the operator curl

Thursday 14:00-16:00 (SIBLT1) chair: SB/JCC

14:00-14:25 Bernhard Aigner
Well-posedness for a generalisation of a model for cell migration

14:30-14:55 Andreas Buchinger
Strong Operator Convergence in Homogenization of PDEs with Nonlocal Coefficients

15:00-15:25 Davide Macera
Disordered Dyson chains and disordered dimer models

15:30-15:55 Esmanur Yıldız Akil
Transition Dynamics of Reaction-Diffusion Equations at the kc-th Eigenvalue

Thursday 16:30–18:30 (SIBLT1) chair: SB/JCC

16:30-16:55 Filippo Santi
Galilei covariance of the theory of Thouless pumps

17:00-17:25 Elmira Nabizadeh Morsalfard
Continuity properties and Bargmann mappings of quasi-Banach Orlicz modulation spaces

17:30-17:55 Mohammed Sanduk
Is the complex harmonic oscillator a transformation due to a problem of partial observation? Rolling Circles Theory

Abstracts.

Bernhard Aigner, TU Bergakademie Freiberg

Well-posedness in H^1 of a model for stem cell growth

Abstract. Delay differential equations are notoriously difficult to tackle with standard approaches. Recent results from [1] for ordinary state-dependent delay differential equations, that are based on the theory of evolutionary equations (cf. [2,3]), show that a traditional approach using exponentially weighted Sobolev-spaces is possible though. I will apply these results, in particular a generalisation of the Picard-Lindelöf theorem for H^1, to the setting of ordinary state-dependent delay differential equations to give an easier proof for well-posedness of a generalized population model from cell-biology. This approach requires less assumptions than previous well-posedness results (cf. [4,5]), that relied upon classical $\mathcal{C}$- or $\mathcal{C}^1$-theory such as [6,7]. The talk is based upon joint research with Marcus Waurick in [8].

References

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[3] R. Picard, A structural observation for linear material laws in classical mathematical physics, *Mathematical Methods in the Applied Sciences* **32**(14), (2009), 1768–1803.
[4] P. Getto and M. Waurick, A differential equation with state-dependent delay from cell population biology, *J. Differential Equations* **260**(7), (2016), 6176–6200.
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Esmanur Yıldız Akıl, Yeditepe University

Transition Dynamics of Reaction-Diffusion Equations at the k_c-th Eigenvalue

Abstract. Dynamic transition theory has been developed to understand transitions in nonlinear sciences, examining transitions between stable states of a system and classifying these dynamics [1]. The applications of dynamic transition theory in nonlinear sciences span across various physics, biology, and chemistry models [2, 3]. In this study, under certain assumptions, the dynamic transitions of the basic solution of a partial differential reaction-diffusion equation with high-order nonlinearity, given by $u_t = L_\lambda u + g(u, u_x)$, will be classified. Here, our first fundamental assumption is that the eigenvectors of the linear operator L form a basis of sine functions. The second fundamental assumption is that the critical value of the first eigenvalue where the transition occurs is taken at $\lambda_c = k_c$. In this study, the dynamics at the k_c-th eigenvalue of the one spatial dimensional reaction-diffusion equation with higher order nonlinearity will be examined.

References

[1] Ma, Tian, and Shouhong Wang, Phase transition dynamics, New York: Springer, 2014.
[2]Junyan Li and Ruili Wu. Dynamic transition analysis for activator-substrate system. Journal of Nonlinear Mathematical Physics, 30(3):956-979, 2023.
[3]Mickaël D Chekroun, Henk Dijkstra, Taylan Şengül, and Shouhong Wang. Transitions of zonal flows in a two-layer quasi-geostrophic ocean model. Nonlinear dynamics, 109(3):1887–1904, 2022.

Marzieh Baradaran, University of Hradec Králové

Quantum graphs with time-reversal non-invariant vertex coupling

Abstract. Motivated by the application of quantum graphs to model the anomalous Hall effect, we discuss spectral properties of magnetic and non-magnetic quantum graphs assuming a preferred-orientation coupling at the graph vertices. The used vertex coupling violates the time reversal invariance, and its high-energy behavior depends on the vertex degree parity. Special attention is paid to the asymptotic behavior of the spectral bands in the high-energy regime. We see that the Band-Berkolaiko universality holds as long as the graph edge lengths are incommensurate. The talk is based on joint works with Pavel Exner and Jiří Lipovský [1-3].

References

[1] M. Baradaran and P. Exner, Cairo lattice with time-reversal non-invariant vertex couplings, , *J. Phys. A: Math. Theor.* **57**, (2024), 265202.
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[3] M. Baradaran, P. Exner and J. Lipovský, Magnetic ring chains with vertex coupling of a preferred orientation, *J. Phys. A: Math. Theor.* **55**, (2022),375203.

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Giovanni Bracchi, UCL

Hyperbolic propagator for the operator curl

Abstract. In this talk, I will describe an algorithm for computing Weyl coefficients of the operator curl = $*$d on a connected oriented closed Riemannian 3-manifold. This approach hinges on careful examination of the propagator of curl, *i.e.* the solution of the initial value problem $(-i\partial_t + \mathrm{curl})U(t) = 0$, $U(0) = \mathrm{Id}$, as a Fourier integral operator in the limit as $t \to 0^+$. In the end, I will provide formulae for the first three Weyl coefficients of curl.

Andreas Buchinger, TU Bergakademie Freiberg

Strong Operator Convergence in Homogenization of PDEs with Nonlocal Coefficients

Abstract. In this talk, we will revisit the classical notion of homogenization of div-grad-systems (H-convergence) and its operator-theoretic description that allows for more general systems with possibly nonlocal coefficients (nonlocal H-convergence provided by M. Waurick). We will introduce a convergence theorem for the corresponding solution operators, and we will discuss its sharpness in the sense of weak vs. strong operator convergence. This is joint work with S. Franz, N. Skrepek and M. Waurick.

Piero D'Ancona, Sapienza University of Rome

Dispersion estimates for Dirac equations with Aharonov–Bohm magnetic fields

Abstract. We examine the dispersive properties of a two dimensional Dirac operator perturbed by a critical Aharonov–Bohm potential. The flow can be split into a dispersive part which decays like in the unperturbed case, plus a singular component with weaker decay. For a partial range of indices, we deduce sharp Strichartz estimates for the flow. This is a joint work with Federico Cacciafesta, Zhiqing Yin and Junyong Zhang.

References

[1] F. Cacciafesta, P. D'Ancona, Z. Yin, J. Zhang, Dispersive estimates for Dirac equations in Aharonov–Bohm magnetic fields: massless case, `https://arxiv.org/abs/2407.12369`

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Bryn Davies, Imperial College London

Subwavelength spectra for high-contrast coefficients: an asymptotic framework for metamaterial design

Abstract. High-contrast heterogeneous media are an attractive platform for microscopic wave control, thanks to their ability to support resonance at subwavelength scales. We have developed an asymptotic method for characterising these high-contrast scattering problems, which uses boundary integral representations to derive a characterisation of subwavelength resonance in terms of eigenstates of the generalised capacitance matrix (GCM). The GCM provides a concise framework to explore some of the important applications and exotic phenomena related to these materials. In this talk, we will introduce this approximation strategy and survey some of its applications to metamaterial design problems. These results are collaborations with Habib Ammari and Erik Orvehed Hiltunen.

I would like to thank the EPSRC for their support under grant number EP/X027422/1.

Oliver Fürst, University of Bonn

Trace and generalized index of Callias operators

Abstract. A Callias operator D is a Dirac-Schrödinger operator over a non-compact manifold M with an operator valued potential A admitting decay of its derivative in some sense. Usually it is required that the potential is invertible outside some compact region, allowing the operator D to be Fredholm. In this talk, this assumption is suspended, leading to a new trace formula for the heat-semigroups associated to D for $M = \mathbb{R}^d$. We will also see that the trace formula leads to a new index formula, where the Fredholm index needs to be replaced by the Witten index. We will calculate the Witten index for $(d+1)$-massless Dirac-Schrödinger operators in $\mathbb{R}^{d+1}$, and show that it may attain any real number on that class of operators, in contrast to the Fredholm index.

References

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[2] O. Fürst, *The Witten Index of massless* $(d{+}1)$*-Dirac-Schrödinger Operators*, arXiv:2405.17123

Jon Harrison, Baylor University

Discrete Quantum Unique Ergodicity on Circulant Graphs

Abstract. A discrete analog of quantum unique ergodicity was proved for Cayley graphs of quasirandom groups by Magee, Thomas and Zhao [1]. They show that for large graphs there exists an orthonormal basis of Eigenfunctions of the adjacency matrix such that quantum probability measures of the eigenfunctions put approximately the correct proportion of their mass on subsets of the vertices that are not too small. We investigate this property for families of circulant graphs with prime order. We see that the property holds for an orthonormal Eigenfunction basis but fails if the basis is also required to be real. The equivalent result for a real basis holds for the Cayley graphs of quasirandom groups. This is work with Clare Pruss at Baylor University.

References

[1] M. Magee, J. Thomas, Y. Zhao, Quantum unique ergodicity for Cayley graphs of quasirandom groups, *Commun. Math. Phys.* **402**, (2023), 3021–3044.

Yuri Latushkin, University of Missouri

The Duistermaat index and eigenvalue interlacing for self-adjoint extensions of a symmetric operator

Abstract. Eigenvalue interlacing is a useful tool in linear algebra and spectral analysis. In its simplest form, the interlacing inequality states that a rank-one positive perturbation shifts each eigenvalue up, but not further than the next unperturbed eigenvalue. We prove a sharp version of the interlacing inequalities for "finite-dimensional perturbations in boundary conditions," expressed as bounds on the spectral shift between two self-adjoint extensions of a fixed symmetric operator with finite and equal defect numbers. The bounds are given in terms of the Duistermaat index, a topological invariant describing the relative position of three Lagrangian planes in a symplectic space. Two of the Lagrangian planes describe the self-adjoint extensions being compared, while the third corresponds to the Friedrichs extension, which acts as a reference point.

This is a joint work with G. Berkolaiko, G. Cox and S. Sukhtaiev.

Davide Macera, Durham University

On a dimer model with random weights

Abstract. After a brief outline on dimer models and their relevance in statistical physics, I will introduce a dimer model on the half-lattice whose weights are random variables which are independent in the horizontal direction and equal in the vertical one. I will then explain how to relate the rate of convergence of dimer-dimer correlations in such a model to the spectral properties of a disordered linear chain. This is a joint work (in progress) with Sunil Chhita.

Elmira Nabizadeh Morsalfard, Linnaeus University

Continuity properties and Bargmann mappings of quasi-Banach Orlicz modulation spaces

Abstract. We deduce continuity, compactness and invariance properties for quasi-Banach Orlicz modulation spaces on $\mathbf{R}^d$. We characterize such spaces in terms of Gabor expansions and by their images under the Bargmann transform.

Ivica Nakić, University of Zagreb

Spectrum of operators on equilateral metric graphs

Abstract. It is well known that the spectrum of the Laplacian on an equilateral metric graph is essentially determined by the spectrum of the corresponding Laplacian matrix. Similar results of this type are also known for the transport equations on metric graphs and in some other specific cases. In this talk we consider general operators on equilateral metric graphs with the property that their actions on all edges coincide. I will show that the spectrum of such operators can be elegantly described in terms of the spectrum of the underlying discrete graph and the spectrum of the restriction of the operator on one edge.

This is a joint work with Marjeta Kramar Fijavž.

Sukrid Petpradittha, Durham University

Lieb-Thirring type inequalities for multidimensional Schrödinger operators with complex-valued potentials

Abstract. The purpose of this research is to investigate a conjecture that was stated by Demuth, Hansmann and Katriel in 2013. We study a possible generalization of Lieb-Thirring type inequalities for eigenvalues of non-selfadjoint Schrödinger operators, with complex-valued potentials, acting on $L^2(\mathbb{R}^d)$ where $d \geq 2$. In particular, we find the asymptotic behavior for the discrete spectra of Schrödinger operators with a one-parameter family of rapidly decaying complex-valued potentials and present a disproof of this conjecture. This is a joint work with Sabine Bögli (Durham) and František Štampach (Prague).

I would like to thank the Isaac Newton Institute for Mathematical Sciences (INI) and the Engineering and Physical Sciences Research Council (EPSRC) grant (Ref. EP/V521929/1) via the UK Spectral Theory Network for their support.

Jakob Reiffenstein, Stockholm University

Eigenvalue density of limit circle Jacobi operators

Abstract. The eigenvalues of a limit circle Jacobi operator can be described on a quantitative level in terms of the growth of its Nevanlinna matrix. The most prominent result in this direction is a theorem of Berezanskii which gives a sufficient condition for limit circle case to take place and states that the exponential order of the Nevanlinna matrix is equal to the convergence exponent of the off-diagonal Jacobi sequence. However, its assumptions are very restrictive.
It turns out that the same assertion holds in significantly more general settings, which may lead to the intuition that the order of the Nevanlinna matrix should generically be equal to the above-mentioned convergence exponent. We will show that, for *any* limit circle Jacobi operator, the order is not less than the convergence exponent, and present a situation where it is actually larger.

Łukasz Rzepnicki, Nicolaus Copernicus University, Toruń

Dirac system with an integrable potential and asymptotic behavior of its solutions on the plane

Abstract. The main focus of this talk is a Dirac-type system considered on the interval $[0, 1]$ with a potential from L_p space, where $1 \leq p < 2$. We propose a new approach to study asymptotic behaviour of its solutions with respect to spectral parameter $\mu \in \mathbb{C}$ and $\mu \to \infty$. Our method gives results not only for μ in a horizontal stripe but for $\mathrm{Im}\mu \geq -c$, where $c > 0$. This allows us to obtain asymptotic identities valid in the whole complex plane which generalize those for the stripe. Moreover, using our approach one can get asymptotic formulas for spectral problems with eigenvalues lying outside a horizontal stripe.

Presented results are joint work with Alexander Gomilko.

Mohammed Sanduk, University of Surrey

Is the complex harmonic oscillator a transformation due to a problem of partial observation? Rolling Circles Theory

Abstract. The present project aims to establish a physical foundation for the complex harmonic oscillator. It is based on two main steps:

First: In the Bohr model, the particle was considered a classical particle. However, a classical particle does not exhibit the electron's spin. In the present project, the particle is considered an extended body with an internal clock, as hypothesized by Dirac and de Broglie. The resulting system is termed the Modified Bohr Model (MBM). This model depicts a system of two rolling circles (the orbit and the extended object). The MBM is a classical hypothetical model.

Second: In 1925, Heisenberg noted that some quantities in the Bohr model are observable, while others are not. In the present project, that Heisenberg's observation is interpreted as a partial observation process, where some quantities are observable and others are unobservable due to the resolving power. For mathematical application, this observation can be considered a measurement process. Substituting the quantities from the partial observation (as those of the quantum mechanics) into the mathematical forms MBM produces the mathematical forms of the observable MBM (OMBM).

These mathematical forms of the OMBM show similarities with those of quantum mechanics. In this context, the complex harmonic oscillator can be attributed to a classical model (rolling circles) and a physical process (partial observation).

Rongwei Yang, University at Albany, the State University of New York

Pluri-harmonic solutions to the Maxwell's equations and Yang-Mills equations

Abstract. Abstract: This talk provides a view of Maxwell's equations and Yang-Mills equations from the perspective of complex analysis, operator theory, and C^*-algebras. Based on pluri-harmonic differential forms, we will present a new class of instanton (self-dual or anti-self-dual) solutions to the equations. It is a joint work with Marius Beceanu and Sachin Munshi.