JORDAN ALGEBRA STRUCTURES AND OPERATOR THEORY (S3)

MARIA CUETO AVELLANEDA (LISBON), BAS LEMMENS (KENT), AND ANTONIO M. PERALTA (GRANADA)

Tuesday 14:00-16:00 (SIBLT2) chair: Antonio Peralta

14:00-14:25 Harald Upmeier
Hilbert Modules and Jordan Algebras

14:30-14:55 Matthew Neal
Metric linear characterizations of algebraic properties in operator spaces

15:00-15:25 Shiho Oi
Isometries between groups of invertible elements in Fourier-Stieltjes algebras

15:30-15:55 María Cueto Avellaneda
Something old, something new, something borrowed, something...Jordan?

Tuesday 16:30-18:30 (SIBLT2) chair: Bas Lemmens

16:30-16:55 Gerardo Martin Escolano
Lie–Trotter formulae in Jordan–Banach algebras with applications to the study of spectral-valued multiplicative functionals

17:00-17:25 Kieran Power
The horofunction compactification of symmetric cones equipped with the Hilbert metric

17:30-17:55 Lina Oliveira
Matrix operator algebras of JB^*-triples

18:00-18:25 Jorge J. Garcés Pérez
Bilinear maps with product property

Wednesday 11:40-12:40 (SIBLT2) chair: María Cueto Avellaneda

11:40-12:05 Michael Mackey
Fixed points of holomorphic automorphisms in spin factors

12:10-12:35 Pauline Mellon
The Wolff hull of a compact holomorphic map

Wednesday 14:00–15:00 (SIBLT2) chair: María Cueto Avellaneda

14:00-14:25 Bas Lemmens
Horofunction compactifications of Hermitian symmetric spaces and JB^*-triples

14:30-15:00 Cormac Walsh
Isometric embeddings of Hermitian symmetric spaces

Thursday 14:00-16:00 (SIBLT2) chair: Bas Lemmens

14:00-14:25 Lajos Molnár
*Characterizations of Jordan *-isomorphisms between C^*-algebras by relative entropy preserving maps*

14:30-14:55 Jan Hamhalter
Maps preserving Jordan decompositions of functionals in preduals of JBW algebras

15:00-15:25 Antonio M. Peralta
Cartan factors as an ideal mathematical model for Wigner's theorem

15:30-15:55 Michiya Mori
Nonexpansive and noncontractive mappings on the set of quantum pure states

Thursday 16:30–18:30 (SIBLT2) chair: Anonio Peralta

16:30-16:55 Dániel Virosztek
Quantum Wasserstein distances: metric properties and isometries

17:00-17:25 Tamás Titkos
Lattice properties of strength functions

17:30-17:55 Curt Healey
Every Symmetric Kubo-Ando connection has the order determining property

Abstracts.

Gerardo Martín Escolano, University of Granada

Lie–Trotter formulae in Jordan–Banach algebras with applications to the study of spectral-valued multiplicative functionals

Abstract. We establish some Lie–Trotter formulae for unital complex Jordan–Banach algebras, showing that for each couple of elements a, b in a unital complex Jordan–Banach algebra $\mathfrak{A}$ the identities

$$\lim_{n\to\infty} \left(e^{\frac{a}{n}} \circ e^{\frac{b}{n}}\right)^n = e^{a+b}, \ \lim_{n\to\infty} \left(U_{e^{\frac{a}{n}}}\left(e^{\frac{b}{n}}\right)\right)^n = e^{2a+b}, \text{ and}$$

$$\lim_{n\to\infty} \left(U_{e^{\frac{a}{n}},e^{\frac{c}{n}}}\left(e^{\frac{b}{n}}\right)\right)^n = e^{a+b+c}$$

hold. These formulae are employed in the study of spectral-valued (non-necessarily linear) functionals $f : \mathfrak{A} \to \mathbb{C}$ satisfying $f(U_x(y)) = U_{f(x)}f(y)$, for all $x, y \in \mathfrak{A}$. We prove that for any such a functional f, there exists a unique continuous (Jordan-)multiplicative linear functional $\psi\colon \mathfrak{A} \to \mathbb{C}$ such that $f(x) = \psi(x)$, for every x in the connected component of set of all invertible elements of $\mathfrak{A}$ containing the unit element. If we additionally assume that $\mathfrak{A}$ is a JB*-algebra and f is continuous, then f is a linear multiplicative functional on $\mathfrak{A}$. The new conclusions are appropriate Jordan versions of results by Maouche, Brits, Mabrouk, Shulz, and Touré. This is a joint work with Antonio M. Peralta and Armando R. Villena.

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María Cueto Avellaneda, IST University of Lisbon

Something old, something new, something borrowed, something...Jordan?

Abstract. This talk is aimed to recap in a light way the description of linear isometries on Jordan structures. We shall do it by invoking celebrated results as well as applying well-known Jordan arguments to go a step further, highlighting new contributions on the topic.

Jorge J. Garcés Pérez, Universidad Politécnica de Madrid

Bilinear maps with product property

Abstract. In this talk we shall present some new trends in the study of linear preserves, such as the study of maps that behave like a homomorphism or a derivation at a fixed point z. In the case $z = 0$ these are the well-known zero-product preservers (or derivable maps at zero). More recently, there has been a lot of interest in studying the case when $z \neq 0$ for some special values of z (for instance for z invertible or a projection). In a recent work in collaboration with Mykola Khrypchenko we develop a unified approach to these type of problems by considering bilinear maps that "have product property at a fixed element". We shall present some of our results and their applications.

Jan Hamhalter, Czech Technical University in Prague

Maps preserving Jordan decompositions of functionals in preduals of JBW algebras

Abstract. Let M be a JBW algebra with predual M_*. Any element $\varrho \in M_*$ has a canonical Jordan decomposition, $\varrho = \varrho_+ - \varrho_-$, where ϱ_+ and ϱ_- are mutually orthogonal positive functionals. Linear maps between preduals preserving these decompositions, called orthogonally decomposable homomorphisms, have been studied in the framework of von Neumann algebras by Araki, Bunce and Wright, and Lau and Wong [1,3]. We continue this line of the research in the more general framework of JBW algebras, which requires some new approaches [2]. We also sharpen hitherto known results for von Neumann algebras and C*-algebras. Among others we characterize continuous orthogonally decomposable homomorphism $\Phi : M_* \to N_*$ in term of its dual map, Φ^*, by showing that Φ^* is a Jordan isomorphism from the support $N_\Phi \subset N$ of the range of Φ onto certain direct summand of M followed by multiplication by a central positive element.

All descriptions of orthogonally decomposable maps [1,3], known so far, assume their continuity. In contrast to this, our new results on order topology on preduals of Jordan algebras (which are of independent interest) imply that in case of order decomposable bijections continuity obtains automatically. As a consequence bijective order decomposable maps arise as Jordan isomorphisms multiplied by central invertible elements. This means that the structure of Jordan decompositions in preduals is a complete invariant for JBW algebras. Various consequences (notably for duals of JB algebras) will be discussed.

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Curt Healey, University of Malta

Every Symmetric Kubo-Ando connection has the order determining property

Abstract. Recently, in [1], [3] and [4] considerable attention was given to the study of extending a surjective map ϕ which preserves the norm of a mean σ to a Jordan $*$-isomorphism. Namely, $\phi : \mathcal{A}^{++} \to \mathcal{B}^{++}$ acting on the positive definite cones of the unital C^*-algebras $\mathcal{A}, \mathcal{B}$ such that $||A\sigma B|| = ||\phi(A)\sigma\phi(B)||$ where $A, B \in \mathcal{A}^{++}$. The means in consideration fall under the general category of *Kubo-Ando means*. A binary operation σ on the positive definite cone $\mathcal{B}(H)^+$ of $\mathcal{B}(H)$ is called a *Kubo-Ando connection* if it satisfies the following properties:

(i) If $A \leq C$ and $B \leq D$, then $A\sigma B \leq C\sigma D$.
(ii) $C(A\sigma B)C \leq (CAC)\sigma(CBC)$.
(iii) If $A_n \downarrow A$ and $B_n \downarrow B$ strongly, then $A_n\sigma B_n \downarrow A\sigma B$ strongly.

A Kubo-Ando mean is a Kubo-Ando connection with the normalization condition $I\sigma I = I$.

Before proving that ϕ can be extended, the weaker condition that ϕ is an *order isomorphism* is first proven. The most direct way in proving the above is showing that σ is Order Determining (**OD**). Let $A, B \in \mathcal{A}^{++}$, then σ is (**OD**) if

$$A \leq B \iff ||A\sigma X|| \leq ||B\sigma X||, \quad \forall\, X \in \mathcal{A}^{++}.$$

It is easy to show that if σ is (**OD**), then a bijective map ϕ preserving the norm of σ is an order isomorphism. In our presentation, we shall show that every symmetric Kubo-Ando connection is (**OD**) [2, Theorem 6].

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Bas Lemmens, University of Kent

Horofunction compactifications of Hermitian symmetric spaces and JB*-triples

Abstract. From seminal works by Kaup and Loos we know that the open unit balls of finite dimensional JB*-triples are precisely the noncompact type Hermitian symmetric spaces. In this talk I will discuss the horofunction compactification of the open unit ball D in a finite dimensional JB*-triple V equipped with Carathéodory distance. The Carathéodory distance is a length metric with a Finsler structure, where the norm in the tangent space corresponds to the JB*-triple norm. Among other things we shall see that the geometry and global topology of the horofunction compactification coincides with the closed dual unit ball of the JB*-triple V. Moreover, we shall give explicit descriptions of the horofunction compactifications of D and of the JB*-triple V as a normed space, and see that the exponential map $\exp_0 \colon V \to D$ extends as a homeomorphism to the horofunction compactification of these spaces. The talk is based on joint work with Cho-Ho Chu and Maria Cueto Avellaneda

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Michael Mackey, University College Dublin

Fixed points of holomorphic automorphisms in spin factors

Abstract. A result of Hayden and Suffridge provides that biholomorphic automorphisms of the open unit ball of a Hilbert space have a fixed point when extended to the closed ball. Spin factors form a key test space in the generalisation of this result. The approach used for Hilbert space fails because non-linear automorphisms of a spin factor are never weakly continuous and so different techniques are required. We will present some conditions which guarantee that an automorphism of the unit ball of a spin factor has a fixed point.

This is joint work with Pauline Mellon.

Pauline Mellon, University College Dublin

The Wolff hull of a compact holomorphic map

Abstract. Let Z be a complex Banach space with open unit ball B and $f : B \to B$ be a compact holomorphic fixed-point free map. For example, for Z the complex plane and B the unit disc Δ, the classic Wolff Theorem gives that $(f^n)_n$ converges to (a Wolff point) ξ on the

boundary of the disc. However, $(f^n)_n$ does not generally converge even in finite dimensions (and for the infinite dimensional Hilbert ball compactness of f is necessary for convergence).

The more general aim therefore must be to locate the accumulation points $\Gamma(f)$ of $(f^n)_n$ for a suitable topology, or indeed to ask for $g \in \Gamma(f)$, where does $g(B)$ lie? This information is captured if we could locate the target set of f, namely,

$$T(f) = \bigcup_{g \in \Gamma(f)} g(B).$$

To this end, we introduce the concept of the *Wolff hull*, $W(f)$, of f and prove that this hull $W(f)$ is proximal in a precise way to $T(f)$. In particular, the Wolff hull generalises the concept of a Wolff point, where such a point can no longer be uniquely determined, and it coincides with the Wolff point if Z is a Hilbert space. We do this for the large class of Banach spaces known as the JB^*-triples.

This is joint work with Michael Mackey.

Lajos Molnár, University of Szeged

*Characterizations of Jordan *-isomorphisms between C^*-algebras by relative entropy preserving maps*

Abstract. We consider several concepts of (not only numerical valued) relative entropies on positive cones in C^*-algebras. We present recent results showing that bijective transformations between such cones that respect any of those quantities necessarily originate from Jordan *-isomorphisms between the underlying full algebras. Therefore, each of the considered relative entropies to some extent determines the full Jordan structure of C^*-algebras.

Michiya Mori, University of Tokyo

Nonexpansive and noncontractive mappings on the set of quantum pure states

Abstract. Wigner's theorem characterizes isometries of the set of all rank one projections on a Hilbert space. In metric geometry, nonexpansive maps and noncontractive maps are well studied generalizations of isometries. I will explain that under certain conditions Wigner symmetries can be characterized as nonexpansive or noncontractive maps on the set of all projections of rank one. The assumptions required for such characterizations are injectivity or surjectivity and they differ in the finite and the infinite-dimensional case. Motivated by a recently obtained optimal version of Uhlhorn's generalization of Wigner's theorem, I also give a description of nonexpansive maps which satisfy a condition that is much weaker than surjectivity. Such maps do not need to be Wigner symmetries. The optimality of the results is shown by (counter)examples.

This talk is based on a joint work [1] with Peter Šemrl (University of Ljubljana).

References

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Matthew Neal, Denison University

Metric linear characterizations of algebraic properties in operator spaces

Abstract. In the last several decades, many algebraic structures on operator spaces have been characterized in terms of the matrix norm and linear structure of the operator space, which we call the "metric-linear" or "geometric" structure. One famous early result is the characterization of unital operator algebras up to complete isometry among operator spaces that are Banach

algebras. Recently, David Blecher and I have given many other such characterizations, including the first metric-linear characterizations of C*-algebras up to complete isometry. All of these metric-linear characterizations will be explained in the talk. Our ultimate goal is to give a metric-linear characterization of JC*-triples among operator spaces. In addition to this, we will give some bonus results on holomorphic characterizations of algebraic structures on operator spaces, inspired of course by the holomorphic characterization of JB*-triples. We will also throw in one affine geometric characterization of JB*-triples among Banach spaces that references the dual space (inspired by the results of Alfsen and Shultz).

Shiho Oi, Niigata University, Japan

Isometries between groups of invertible elements in Fourier-Stieltjes algebras

Abstract. In this talk, we describe the structure of isometric real algebra isomorphisms between Fourier-Stieltjes algebras. By applying it, we see that if open subgroups of the groups of invertible elements in two Fourier-Stieltjes algebras are isometric as metric spaces, the underlying locally compact groups are topologically isomorphic.

This talk is supported by JSPS KAKENHI Grant Numbers JP24K06754.

Antonio M. Peralta, Universidad de Granada

Cartan factors as an ideal mathematical model for Wigner's theorem

Abstract. In 2002, L. Molnár established an analogue to the celebrated Piron-Wigner theorem by showing that, for each complex Hilbert space H with $\dim(H) \geq 3$, every bijective transformation on the lattice of partial isometries on H which preserves the partial ordering and orthogonality between partial isometries in both directions, and is norm continuous at a single non-zero partial isometry, extends to a real-linear triple isomorphism on $B(H)$ [2]. The novelty consisted in replacing the lattice of projections in $B(H)$ by the lattice of partial isometries.

The Banach space $B(H)$ is a particular case of type 1 Cartan factors. Cartan factors are useful to represent, via a Gelfand-Naimark type theorem, each abstract JB*-triple as a JB*-subtriple of an ℓ_∞-sum of Cartan factors (the latter objects are called atomic JBW*-triples). This talk will be devoted to presenting some recent extensions to the case of bijective transformations between lattices of atomic JBW*-triples preserving the natural partial ordering in both directions and orthogonality (in one direction) and enjoying a mild continuity property [1].

We shall also present some recent advances on preservers of triple transition pseudo-probabilities between minimal tripotents of two atomic JBW*-triples [3,4].

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Kieran Power, University of Kent

The horofunction compactification of symmetric cones equipped with the Hilbert metric

Abstract. The horofunction compactifaction of a metric space is a well known concept that goes back to Gromov, with many applications in dynamics, geometry, and complex variables. In general the horofunction compactification is hard to compute, and its global geometry and topology is not well understood. In certain classes of finite dimensional normed spaces a duality phenomenon has been observed. In work with Bas Lemmens we have shown that this duality phenomenon occurs for a variety of classes of Finsler metric spaces. In this talk we will focus specifically on symmetric cones under the Hilbert metric, which originated as a way to generalise the metric for Klein's model of hyperbolic geometry. We exploit the interplay between the Hilbert metric and the JB-algebra structure of the cone in order to explicitly construct the horofunction compactification for these spaces. We begin the talk with a brief discussion of hyperbolic models of geometry and introduce the horofunction compactification. We proceed to introduce the machinery needed to show an explicit homeomorphism from the horoboundary onto the closed unit ball of the dual of the tangent space at the base point.

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Lina Oliveira, University of Lisbon

Matrix operator algebras of JB-triples*

Abstract. In this talk we describe the Tits–Kantor–Koecher Lie algebras of JB*-triples and characterise their representations as matrix operator algebras.

Tamás Titkos, Corvinus University and Rényi Institute

Lattice properties of strength functions

Abstract. We investigate an important functional representation of the cone of bounded positive semidefinite operators. It is known that the representation by strength functions turns the Löwner order into the pointwise order. However, very little is known about the structure of strength functions. We are going to show that the representation behaves naturally with the infimum and supremum operations. More precisely, we show that the pointwise minimum of two strength functions f_A and f_B is a strength function if and only if the infimum of A and B exists. The cornerstone of each argument in this talk is a recent discovery of Molnár and Ramanantoanina, namely that the strength function of the parallel sum $A : B$ (which is half of the harmonic mean) equals the parallel sum of the strength functions f_A and f_B. As a byproduct of this fact, in some special cases, we describe the strength function of the so-called (generalized) short.

This is a joint work with Andriamanankasina Ramanantoanina.

Harald Upmeier, University of Marburg

Hilbert Modules and Jordan Algebras

Abstract. Commuting tuples of non-selfadjoint operators can often be modelled by Hilbert modules of holomorphic functions on bounded domains D in $E = \mathbb{C}^n$. If D is a bounded symmetric domain, then E carries the structure of a Jordan algebra or, more generally, a hermitian

Jordan triple. Based on the Peter-Weyl decomposition of polynomials on E, under the Jordan triple automorphism group K, we construct a Hilbert module with reproducing kernel function, for each fixed integer partition of length r, the rank of E. As our main result, we determine the so-called eigenbundle of this Hilbert module, which is a singular vector bundle with respect to a natural stratification on the domain D, in terms of the Peirce decomposition under certain tripotents. This eigenbundle determines the Hilbert module up to unitary equivalence and corresponds to a coherent analytic sheaf, in contrast to the real-analysis methods in the self-adjoint case. Generalizations to the vector-valued case are also discussed shortly.

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Dániel Virosztek, Alfréd Rényi Institute of Mathematics

Quantum Wasserstein distances: metric properties and isometries

Abstract. Although the theory of classical optimal transport has been playing an important role in mathematical physics (especially in fluid dynamics) and probability since the late 80s, concepts of optimal transportation in quantum mechanics have emerged only very recently. First, we briefly review two such concepts: one relying on quantum channels (pioneered by De Palma and Trevisan) and one relying on quantum couplings (pioneered by Caglioti, Golse, Mouhot, and Paul). Then, we report on our progress in proving a conjecture of De Palma and Trevisan, saying that a smart modification of channel-based quantum Wasserstein distances gives rise to genuine metrics on quantum state spaces [1]. Finally, we describe the isometries of the qubit state space endowed with distinguished quantum Wasserstein distances [2].

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Cormac Walsh, INRIA Saclay and CMAP École Polytechnique Paris

Isometric embeddings of Hermitian symmetric spaces

Abstract. We study the rigidity of maps between Hermitian symmetric spaces that preserve the Carathéodory (or equivalently the Kobayashi) distance. We show that for such maps the rank of the codomain must be at least as great as the rank of the domain. When the two ranks are the same, and the domain is irreducible, we show that the map is either holomorphic or antiholomorphic. These results generalise those of Kim and Seo, who worked in the setting of C^1-smooth maps. Our metric geometry techniques allow us to dispense with any smoothness assumptions on the map. The main tool we use is the horofunction boundary, which for Hermitian symmetric spaces was described by Chu, Cueto-Avellaneda, and Lemmens.

This is joint work with Bas Lemmens.