OPERATOR THEORY ON ANALYTIC FUNCTION SPACES 2 (S24)

BAS LEMMENS, (KENT), ANA LOUREIRO (KENT), MARCO MARLETTA (CARDIFF), IAN WOOD (KENT)

Wednesday 11:40-12:40 (SIBSR3) chair: TBD

11:40-12:05 Mohana Rahul Nandan
A study of multiplicative maps on Reproducing Kernel Hilbert spaces

12:10-12:35 Jaikishan
Multiplicativity of linear functionals on function spaces on an open disc

Wednesday 14:00-15:00 (SIBSR3) chair: TBD

14:00-14:25 Bharti Garg
J-contractive operator valued functions and vector valued de Branges spaces

14:30-14:55 Christian Emmel
A Generalization of Krein's extension formalism for simple symmetric operators with deficiency index (1, 1)

Thursday 16:30-18:30 (SIBSR2) chair: TBD

16:30-16:55 Chong Zhao
Essential normality of quotient modules vs. Hilbert-Schmidtness of submodules in $H^2(\mathbb{D}^2)$

17:00-17:25 Golla Ramesh
Denseness of a subclass of norm attaining operators

Friday 14:00-16:00 (SIBLT1) chair: TBD

14:00-14:25 Reid Johnson
Pullback Operators on Bargmann Spaces

14:30-14:55 Andrew Graven
On the Uniqueness of Generalized Quadrature Domains via the Faber Transform

15:00-15:25 Hui Dan
Gaussian Dirichlet series with periodic coefficients

15:30-15:55 Axel Renard
Criteria of contractivity for small size matrices and characterization of model operators

Abstracts.

Hui Dan, Sichuan University

Gaussian Dirichlet series with periodic coefficients

Abstract. In this talk, we consider Gaussian Dirichlet series $\sum_{n=1}^{\infty} X_n n^{-s}$ with periodic coefficients ($X_{n+q} = X_n$). These random series are Gaussian analytic functions on the half plane $\operatorname{Re} s > 1$. Some situations are discussed concering the probability of this Dirichlet series having zeros in $\operatorname{Re} s > 1$.

Christian Emmel, Stockholm University

A Generalization of Krein's extension formalism for simple symmetric operators with deficiency index $(1,1)$

Abstract. Simple symmetric operators with deficiency index $(1,1)$ can be realized as multiplication operators by the independent variable on suitable reproducing kernel Hilbert spaces. Their self-adjoint extensions are then characterized by Krein's extension formalism. In this talk, we extend this formalism to characterize all, not necessarily self-adjoint, extensions using comparably simple function-theoretic arguments. This is a special case of a result obtained for arbitrary symmetric operators with deficiency index $(1,1)$.

Bharti Garg, Indian Institute of Technology Ropar

J-contractive operator valued functions and vector valued de Branges spaces

Abstract. This talk aims to give an overview of the reproducing kernel Hilbert spaces (RKHS) constructed from J-contractive operator valued analytic functions on the upper half-plane. First, we discuss the Potapov-Ginzburg transform of a certain class of bounded linear operators, investigate some of its properties, and outline sufficient conditions for a J-contractive operator to be J-bicontractive. Finally, a construction of the de Branges space of vector valued analytic functions denoted by $\mathcal{H}_{\mathfrak{S}_\infty}(U)$ is proposed with the help of J-contractive operator valued analytic functions.
This is part of an ongoing work with my Ph.D. supervisor Dr. Santanu Sarkar.

Andrew Graven, Caltech

On the Uniqueness of Generalized Quadrature Domains via the Faber Transform

Abstract. We study the theory of generalized quadrature domains (GQDs) and describe an approach to questions of uniqueness, involving the Faber transform and ideas from logarithmic potential theory. By GQDs, we mean quadrature domains with respect to weighted area measures and those with Abelian quadrature functions (i.e. arc integral terms in the quadrature identity). Connections to the integrability of the Hele-Shaw flow are also discussed. This is joint work with Nikolai Makarov.

I would like to thank the DOD National Defense Science and Engineering Graduate Fellowship Program (NDSEG) for their generous support.

Jaikishan, Shiv Nadar Institution of Eminence, Delhi NCR, India

Multiplicativity of linear functionals on function spaces on an open disc

Abstract. Gleason-Kahane-Żelazko (GKZ) theorem characterizes all the multiplicative linear functionals on complex unital Banach algebras. Recently, the GKZ theorem has been extended by Javad Mashreghi and Thomas Ransford to function spaces that are not algebras.

In this talk, we present a general version of the GKZ theorem. First, we characterize a class of linear functionals as point evaluations on the vector space of all complex polynomials $\mathcal{P}$. We then apply this characterization to present a version of the GKZ theorem for a vast class of topological spaces of complex-valued functions, including the Hardy, Bergman, Dirichlet, and many other well-known function spaces. Also, we use the GKZ theorem for polynomials to obtain a version of the GKZ theorem for strictly cyclic weighted Hardy spaces.

References

[1] J. Mashreghi and T. Ransford. *A Gleason-Kahane-Żelazko theorem for modules and applications to holomorphic function spaces.* Bull. Lond. Math. Soc., 47(6):1014-1020, 2015.
[2] J.-P. Kahane and W. Żelazko. *A characterization of maximal ideals in commutative Banach algebras.*, Studia Math., 29:339-343, 1968.
[3] A. M. Gleason. *A characterization of maximal ideals.* J. Analyse Math., 19:171-172, 1967.

I would like to thank the Shiv Nadar Institution of Eminence for providing me travel support to attend this conference.

Reid Johnson, UCLA

Pullback Operators on Bargmann Spaces

Abstract. We characterize boundedness and compactness of pullback operators under holomorphic maps between Bargmann spaces of entire holomorphic functions with quadratic strictly plurisubharmonic exponential weights, extending a result of Carswell-MacCluer-Schuster obtained in the case of the radial quadratic weight. We also show that the pullback operator between Bargmann spaces is compact precisely when it is of trace class, with sub-exponentially decaying singular values.

References

[1] B. J. Carswell, B. D. MacCluer, and A. Schuster, *Composition operators on the Fock Space*, Acta Sci. Math., **69**, (2003), 871–887.
[2] R Johnson, Pullback operators on Bargmann spaces, *Proc. Am. Math. Soc.*, to appear, arXiv:2403.13227.

Mohana Rahul Nandan, Indian Institute of Technology Hyderabad

A study of multiplicative maps on Reproducing Kernel Hilbert spaces

Abstract. The study of multiplicative linear maps in Banach algebras is a well-explored area in the mathematical literature. A key theorem in this context is the Gleason-Kahane-Żelazko (GKZ) theorem, which provides insights into the multiplicative properties of linear functionals. Similarly, the Kowalski-Słodkowski theorem addresses conditions under which a functional is both linear and multiplicative.

Recently, Cheng Chu, Michael Hartz, Javad Mashreghi, and Thomas Ransford have extended the GKZ theorem to reproducing kernel Hilbert spaces (RKHS) with normalized complete Pick kernels. In this talk, we will discuss about generalizing linearity in the hypothesis of the above theorem. Also, we will discuss about generalizing the Kowalski-Słodkowski theorem for RKHS with normalized complete Pick kernel.

References

[1] Chu, C., Hartz, M., Mashreghi, J., Ransford, T.: Gleason-Kahane-Żelazko theorem for reproducing kernel Hilbert space, *Bull. Lond. Math. Soc.* **54(3)**, (2022), 1120–1130.
[2] Sebastian, G., Daniel, S.: A weaker Gleason-Kahane-Żelazko theorem for modules and applications to Hardy spaces, Colloq. Math. **164(2)**, (2021), 273–282.
[3] Mashreghi, J., Ransford, T.: A Gleason-Kahane-Żelazko theorem for modules and applications to holomorphic function spaces, *Bull. Lond. Math. Soc.* **47(6)**, (2015), 1014–1020.

I would like to thank the Indian Institute of Technology Hyderabda for their generous support.

Golla Ramesh, Indian Institute of Technology

Denseness of a subclass of norm attaining operators

Abstract. Let H be a complex Hilbert space and $\mathcal{B}(H)$ denote the space of all bounded linear operators on H. We say $T \in \mathcal{B}(H)$ is norm attaining if there exists $x \in H$ with $\|x\| = 1$ such that $\|Tx\| = \|T\|$. We define a new class

$$\beta(H) := \{T \in \mathcal{B}(H) : \, T \text{ attains norm on every reducing subspace of } T\}.$$

In this talk we discuss the invariant subspace of operators in $\beta(H)$ and denseness of $\beta(H)$ in $\mathcal{B}(H)$ with respect to the operator norm.

This is a joint work with Hiroyuki Osaka and Shanola S. Sequeira.

References

[1] Ramesh, Golla; Osaka, Hiroyuki; On a subclass of norm attaining operators; Acta Sci. Math. (Szeged) 87(2021), no.1-2, 247–263.
[2] Ramesh, G.; Osaka, H.; On operators which attain their norm on every reducing subspace; Ann. Funct. Anal. 13(2022), no.2, Paper No. 19, 13 pp.
[3] Ramesh, G.; Osaka, H and S.; Shanola; Denseness of a subclass of norm attaining operators; Preprint, 2024.

Axel Renard, University of Lille

Criteria of contractivity for small size matrices and characterization of model operators

Abstract. It is quite known in the literature that the Schwarz-Pick inequality can be obtained as a particular case of the von Neumann inequality, applied to a well chosen 2×2 matrix. Trying to generalize this observation to matrices of higher sizes leads us to the issue of estimating the (Euclidean) norm of a $n \times n$ upper-triangular matrix T_n, while the computations using the spectral radius of $T_n^* T_n$ become too intricate to give a useful criterion in practice. I will give an answer to this issue for 3×3 and 4×4 matrices. Then, this leads to a nice characterization of the matrix representation of the compressed shift acting on the model space $H^2(\mathbb{D}) \ominus uH^2(\mathbb{D})$, where u is a finite Blaschke product of degree $n \geq 2$.

References

[1] C. Badea, A. Renard, "Schwarz-Pick type inequalities from an operator theoretical point of view", *Journal of Mathematical Analysis and Applications* **540** (2024).
[2] A. Renard, "A criterion of contractivity for four by four matrices and applications to model

operators", hal-04565458f (2024), Preprint.

This talk is based on a joint work with Catalin Badea (University of Lille).

Chong Zhao, Shandong University

Essential normality of quotient modules vs. Hilbert-Schmidtness of submodules in $H^2(\mathbb{D}^2)$

Abstract. I would like to talk our latest work on the essential normality of quotient modules over the polydiscs, and the Hilbert-Schmidtness of submodules. We prove that all the quotient modules in $H^2(\mathbb{D}^2)$, associated to the finitely generated submodules containing a distinguished homogenous polynomial, are essentially normal, which is the first result on the essential normality of non-algebraic quotient modules in $H^2(\mathbb{D}^2)$. Moreover, we obtain the equivalence of the essential normality of a quotient module and the Hilbert-Schmidtness of its associated submodule in $H^2(\mathbb{D}^2)$, in the case that the submodule contains a distinguished homogenous polynomial. As an application, we prove that each finitely generated submodule containing a polynomial is Hilbert-Schmidt, which partially gives an affirmative answer to a conjecture of R. Yang.