SPECIAL MATRICES (S22)

NATALIA BEBIANO (UNIVERSITY OF COIMBRA), MIKHAIL TYAGLOV (SHANGHAI JIAO TONG UNIVERSITY)

Tuesday 16:30-19:00 (SIBSR3) chair: Sergei Grudsky

16:30-16:55 Jyoti Rani
Spectral properties of the Rhaly operator on weighted null sequence spaces and associated operator ideals

17:00-17:25 Sergei Grudsky
Asymptotics of eigenvalues and eigenvectors of Toeplitz matrices

17:30-17:55 Bibekananda Sitha
Generalized core-EP Inverse for Square Matrices

18:00-18:25 Natalia Bebiano
Real skew-symmetric and periodic skew-symmetric tridiagonal matrices with prescribed spectral data

Abstracts.

Natalia Bebiano, CMUC, Department of Mathematics, University of Coimbra

Real skew-symmetric and periodic skew-symmetric tridiagonal matrices with prescribed spectral data

Abstract. Skew-symmetric matrices have various applications in Physics, one of which is in the gyroscopic system. In this paper, we retrieve a unique real skew-symmetric tridiagonal matrix from the maximal and minimal imaginary parts of the eigenvalues of all its leading principal submatrices, and then we reconstruct a real periodic skew-symmetric tridiagonal matrix from these prescribed spectral data. The necessary and sufficient conditions for the existence of such matrices are given, and we show that the total number of possible n-by-n periodic skew-symmetric tridiagonal matrices is at most $2^{\lfloor \frac{n-s}{2} \rfloor}$, where s is the number of common elements in the two prescribed spectra. The proofs of the obtained results provide algorithmic procedures, which are supported by some illustrative numerical experiments.

Based on a joint work with Wei-Ru Xu and Yu Zeng, P.R. China

I would like to thank CMUC for the generous support.

Sergei Grudsky, CINVESTAV, Mexico

Asymptotics of eigenvalues and eigenvectors of Toeplitz matrices

Abstract. Analysis of the asymptotic behaviour of the spectral characteristics of Toeplitz matrices as the dimension of the matrix tends to infinity has a history of over 100 years. For instance, quite a number of versions of Szegö ‘s theorem on the asymptotic behaviour of eigenvalues and of the so-called strong Szegö theorem on the asymptotic behaviour of the determinants of Toeplitz matrices are known. Starting in the 1950s, the asymptotics of the maximum and minimum eigenvalues were actively investigated. However, investigation of the individual asymptotics of all the eigenvalues and eigenvectors of Toeplitz matrices started only quite recently: the first papers on this subject were published in 2009-2010. A survey of this new field is presented here.

Jyoti Rani, Indian Institute of Technology Bhilai

Spectral properties of the Rhaly operator on weighted null sequence spaces and associated operator ideals

Abstract. The research conducted in this work provides a thorough examination of the lower triangular terraced matrix, initially introduced by H. C. Rhaly, commonly referred to as the Rhaly matrix. In this work, we focused on continuity, compactness, and spectral properties of Rhaly matrix. For a sequence $a = \{a_n\}$ of real or complex numbers Rhaly [**?**] introduced the terraced matrix R_a also known as Rhaly matrix where

$$R_a = \begin{pmatrix} a_1 & 0 & 0 & 0 & \cdots \\ a_2 & a_2 & 0 & 0 & \cdots \\ a_3 & a_3 & a_3 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

If we consider the sequence $a_n = \frac{1}{n}$ for $n \in \mathbb{N}$, the operator R_a simplifies to the well-known Cesàro operator. In the specific case where $a_n = \frac{1}{n^p}$ with $p \in \mathbb{R}$, it reduces to the p-Cesàro

operator. While numerous researchers have investigated the spectrum of Rhaly operators in various classical sequence spaces like c_0, ℓ_p, bv_0, etc. and over Hardy spaces. To the best of our knowledge, no investigation is carried out so far on the spectral properties of Rhaly operators over the weighted sequence spaces. This study aims to fill that gap. Within this article, we delve into the topics of boundedness, compactness, and various spectral properties of the Rhaly operator when it operates on the weighted sequence space $c_0(s)$, where $s = \{s_n\}$ represents the weight vector. We obtained that, for a bounded decreasing sequence $s = s_n$ of positive real numbers such that $R_a \in \mathcal{B}(c_0(s))$,(set of all bounded linear operators on $c_0(s)$), the spectrum $\sigma(R_a, c_0(s))$ satisfies the following relation

$$\sigma(R_a, c_0(s)) \subseteq \left\{\lambda \in \mathbb{C} \setminus \bar{S} : \left|\lambda - \frac{\mu}{2}\right| \leq \frac{\mu}{2}\right\} \cup \bar{S},$$

where $\mu = \lim_{n\to\infty} na_n$ with $0 \leq \mu < \infty$ and $S = \{a_n : n \in \mathbb{N}\}$. Under similar assumptions, we have also obtained, the point spectrum $\sigma_p(R_a, c_0(s))$, continuous spectrum $\sigma_c(R_a, c_0(s))$ and residual spectrum $\sigma_r(R_a, c_0(s))$ of R_a as follows,

(i) $\sigma_p(R_a, c_0(s)) = A_1$,
(ii) $\sigma_r(R_a, c_0(s)) = (A_2 \cup S) \setminus A_1 = A_2 \cup (S \setminus A_1)$.

In addition, if the sequence $\{s_n\}$ is a decreasing sequence then

(iii) $\sigma(R_a, c_0(s)) \subseteq \left\{\lambda \in \mathbb{C} \setminus \bar{S} : \left|\lambda - \frac{\mu}{2}\right| \leq \frac{\mu}{2}\right\} \cup \bar{S}$,
(iv) $\{0\} \subseteq \sigma_c(R_a, c_0(s)) \subseteq (\left\{\lambda \in \mathbb{C} \setminus \bar{S} : \left|\lambda - \frac{\mu}{2}\right| \leq \frac{\mu}{2}\right\} \cup \bar{S}) \setminus (A_2 \cup S)$,

where

$$\begin{aligned} A_1 &= \left\{\lambda \in S : \lim_{n\to\infty} a_n s_n n^{\alpha\chi} = 0\right\}, \\ A_2 &= \left\{\lambda \in \mathbb{C} \setminus (S \cup \{0\}) : \sum_{n=1}^{\infty} \frac{1}{s_n n^{\alpha\chi}} < \infty\right\}, \end{aligned}$$

where $\alpha = \Re(\frac{1}{\lambda})$.

As as application, A novel class of operator ideals, denoted as $\chi^{(s)}_{c_0(r)}$, associated with the Rhaly operator acting on weighted c_0 spaces has been introduced. $\chi^{(s)}_{c_0(r)}$ is defined as follows:

$$\chi^{(s)}_{c_0(r)} = \left\{\phi \in \mathcal{B} : \lim_{i\to\infty} \left(a_i \sum_{j=1}^{i} s_j(\phi)\right) r_i = 0\right\},$$

where $\mathcal{B}$ denotes the class of all bounded linear operators between any pair of Banach spaces and $s_j(\phi)$ denotes the sequence of s-numbers. It has been demonstrated that, under certain assumptions on the sequence $\{a_n\}$, this class forms a closed quasi-Banach operator ideal.

Bibekananda Sitha, BITS Pilani Goa Campus

Generalized core-EP Inverse for Square Matrices

Abstract. In this paper, we introduce two new types of inverses for complex square matrices by using an inner inverse and core-EP inverse, called ICEP, and its dual called CEPI inverse. Further, we extend the notion of P-core inverse for square matrices with arbitrary index. A few equivalent characterizations of these inverses have been derived. In addition, the representations of these inverses are established via core-EP and HS decomposition. Moreover, we introduce a binary relation for these inverses and a few derived properties. An application of these inverses in solving linear systems also discussed.