OPERATORS ON BANACH SPACES AND LATTICES (S20)

NIELS LAUSTSEN (LANCASTER), KEVIN BEANLAND (WASHINGTON & LEE UNIVERSITY)

Monday 14:00-16:00 (SIBSR2) chair: Niels Laustsen

14:00-14:25 Mitchell Taylor
Free Banach lattices
14:30-14:55 Krystian Kazaniecki
Martingale Type, the Gamlen–Gaudet Construction and a Greedy Algorithm

Tuesday 16:30-19:00 (SIBSR2) chair: Niels Laustsen

16:30-16:55 Jacek Chmieliński
Approximate smoothness of operators
17:00-17:25 Syamantak Das
Various generalizations of generalized centers and related geometric properties in Banach spaces
17:30-17:55 Tanmoy Paul
U-embedding of Banach spaces
18:00-18:25 Elroy Zeekoei
Some Dunford–Pettis-like properties in Banach lattices

Thursday 14:00-16:00 (SIBSR2) chair: Niels Laustsen

14:00-14:25 Richard Lechner
Haar multipliers on $L^p(L^1)$
14:30-14:55 Thomas Speckhofer
Factorization in Haar system Hardy spaces
15:00-15:25 James Smith
Closed ideals of operators on Baernstein spaces
15:30-15:55 Sukumar Daniel
Exponential spectrum commutes in $B(\ell_p \oplus \ell_q)$

Friday 14:00-16:00 (SIBSR2) chair: Niels Laustsen

14:00-14:25 Richard Smith
L-embeddability in Lipschitz-free spaces
14:30-14:55 Tomasz Kania
Polish spaces of Banach lattices
15:00-15:25 Jan Bíma
Nagata Dimension and Lipschitz Extensions Into Quasi-Banach Spaces
15:30-15:55 Natalia Maślany
On isometries and Tingley's problem for combinatorial Tsirelson spaces

Abstracts.

Jan Bíma, Charles University

Nagata Dimension and Lipschitz Extensions Into Quasi-Banach Spaces

Abstract. Given two metric spaces $\mathcal{N} \subseteq \mathcal{M}$ in inclusion and $0 < p \leq 1$, we wish to determine the smallest constant $\mathfrak{t}_p(\mathcal{N}, \mathcal{M})$ such that any Lipschitz map $f : \mathcal{N} \to Z$ into any p-Banach space Z can be extended to a Lipschitz map $f' : \mathcal{M} \to Z$ satisfying $\operatorname{Lip} f' \leq \mathfrak{t}_p(\mathcal{N}, \mathcal{M}) \cdot \operatorname{Lip} f$. In my talk, I will present a recent result showing that if $\mathcal{N}$ has finite Nagata dimension at most d with constant γ, then $\mathfrak{t}_p(\mathcal{N}, \mathcal{M}) \lesssim_p \gamma \cdot (d+1)^{1/p-1} \cdot \log(d+2)$ for all $0 < p \leq 1$. I will show that examples of spaces with finite Nagata dimension include doubling spaces, as well as minor-excluded metric graphs. Interestingly, I will also establish that the constant $\mathfrak{t}_p(\mathcal{N}, \mathcal{M})$ generally increases as p approaches zero.

References

[1] J. Bíma, "Nagata Dimension and Lipschitz Extensions Into Quasi-Banach Spaces," *arXiv preprint arXiv:2402.03189*, 2024. URL: https://arxiv.org/abs/2402.03189.

Jacek Chmieliński, UKEN Krakow

Approximate smoothness of operators

Abstract. We develop the notion of approximate smoothness introduced in [1] and apply it for spaces of operators acting on Banach spaces. Some results from [1] will be presented as well as new ones.

References

[1] J. Chmieliński, D. Khurana and D. Sain, *Approximate smoothness in normed linear spaces*, Banach J. Math. Anal. (2023), 17:41.

Syamantak Das, Indian Institute of Technology Hyderabad

Various generalizations of Generalized centers and related geometric properties in Banach spaces

Abstract. Veselý (1997) developed the idea of generalized centers for finite sets in Banach spaces. In this talk, we explore the concept of *restricted* $\mathcal{F}$*-center property* for a triplet $(X, Y, \mathcal{F}(X))$, where Y is a subspace of a Banach space X and $\mathcal{F}(X)$ is the family of finite subsets of X. In addition, we generalize the analysis to include all closed, bounded subsets of X. We show that Y has $n.X.I.P.$ in X for all natural numbers n if and only if $\operatorname{rad}_Y(F) = \operatorname{rad}_X(F)$ for all finite subsets F of Y. It then turns out that, for all continuous, monotone functions f, the f-radii viz. $\operatorname{rad}_Y^f(F), \operatorname{rad}_X^f(F)$ are the same whenever the generalized radii viz. $\operatorname{rad}_Y(F), \operatorname{rad}_X(F)$ are also the same, for all finite subsets F of Y. We establish a variety of characterizations of central subspaces of Banach spaces. With reference to an appropriate subfamily of closed and bounded subsets, we will see that a number of function spaces and subspaces exhibit the restricted weighted Chebyshev center property.

References

[1] Syamantak Das, Tanmoy Paul, *A study on various generalizations of Generalized centers* (**GC**) *in Banach spaces*, arXiv:2311.15818.

[2] Libor Veselý, *Generalized centers of finite sets in Banach spaces*, Acta Math. Univ. Comenian. (N.S.) **66**(1) (1997), 83–115.

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Jorge González Camus, Universidad Tecnológica Metropolitana

Representation of Solution for Fractional Damped Heat and Wave Equation on an Infinite Lattice Via Subordination Techniques and Banach Algebras.

Abstract. In this talk, we present a study of the non-local in-time damped wave and heat equations on an infinite lattice in the linear case. Under suitable assumptions, we establish a representation of the solutions in terms of subordinators, as well as an explicit formula via discrete Fourier transform, on the Banach Algebras framework. Moreover, we establish sufficient conditions to guarantee the solution as a probability distribution, and we point out the differences to its continuous counterpart. The discrete maximal regularity of the non-homogeneous cases on $\ell^p(\mathbb{Z})$ also are presented.

References

[1] L. Abadias, M. de León-Contreras, J.L. Torrea. *Non-local fractional derivatives. Discrete and continuous.* J. Math. Anal. Appl., **449**(1) (2017) 734–755.

[2] F. Alegría, V. Poblete and J. C. Pozo. *Nonlocal in-time telegraph equation and telegraph processes with random time.* J. Differential Equations, Vol 347,(2023),310–347.

[3] H. Bateman. *Some simple differential difference equations and the related functions.* Bull. Amer. Math. Soc. 49 (1943), 494–512.

[4] O. Ciaurri, T.A. Gillespie, L. Roncal, J.L. Torrea and J.L. Varona. Harmonic analysis associated with a discrete Laplacian, J. d'Analyse Mathématique, 132 (2017), 109–131.

[5] J. González-Camus, C. Lizama and P.J. Miana. Fundamental solutions for semi discrete evolution equations via Banach algebras. Adv Differ Equ, 35 (2021).

[6] E. Orsingher and L. Beghin. *Time-fractional telegraph equations and telegraph processes with Brownian time* Probab. Theory Relat. Fields 128(1) (2004) 141–160.

[7] J.C. Pozo, V. Vergara, A non-local in time telegraph equation, Nonlinear Anal. 193 (2020) 111411.

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Tomasz Kania, Jagiellonian University

Polish spaces of Banach lattices

Abstract. Using admissible topologies on spaces of closed subspaces of the universal Banach space C[0,1], introduced by Godefroy and Saint-Raymond in 2018, we develop the descriptive set theory of Polish spaces whose points are separable Banach spaces with extra structure such as Banach lattices. To achieve this, we exploit the recent construction of a free Banach lattice. Within this framework we will show how to bound the Borel complexity of various Banach lattice properties.

Krystian Kazaniecki, Johannes Kepler University Linz

Martingale Type, the Gamlen-Gaudet Construction and a Greedy Algorithm

Abstract. A Banach space X satisfies Martingale Type p if there exists $C = C_p$ such that any filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$ gives rise to the upper ℓ^p estimates,

$$(*) \qquad \|f\|^p_{L^p(\Omega,X)} \leq C^p \left(\|\mathbb{E}(f|\mathcal{F}_0)\|^p_{L^p(\Omega,X)} + \sum_{n=1}^{\infty} \|\boldsymbol{\Delta}_n f\|^p_{L^p(\Omega,X)} \right)$$

where $\boldsymbol{\Delta}_n f = \mathbb{E}(f\big|\mathcal{F}_n) - \mathbb{E}(f\big|\mathcal{F}_{n-1})$.

Assuming that $\mathcal{F}_n$ is a sequence of increasing purely atomic sub-σ-algebras of $\mathcal{F}$ and $\mathcal{F} = \sigma(\bigcup \mathcal{F}_n)$ we will identify precisely all filtered probability spaces $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$, that are able to determine the martingale type of Banach space X. We associate explicit intrinsic conditions on the filtration $(\mathcal{F}_n)$ which determine that the upper ℓ^p estimates $(*)$ imply the martingale type p.
Theorem 1. For each fixed $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$ the following dichotomy holds true: Either, there exists $C > 0$ such that for any Banach space X and any $f \in L^p(\Omega, \mathcal{F}, \mathbb{P}, X)$

$$(**) \qquad \|f\|^p_{L^p(\Omega,X)} \leq C \left(\|\mathbb{E}(f|\mathcal{F}_0)\|^p_{L^p(\Omega,X)} + \sum_{n=1}^{\infty} \left\|\mathbb{E}(f\big|\mathcal{F}_n) - (f\big|\mathcal{F}_{n-1})\right\|^p_{L^p(\Omega,X)} \right)$$

or the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_n), \mathbb{P})$ and upper ℓ^p estimates $(**)$ already determine that the Banach space X is of martingale type p.

Talk is based on a joint work with Paul F.X. Müller [1].

References

[1] K. Kazaniecki, P. F. X. Müller, *Martingale type, the Gamlen-Gaudet construction and a greedy algorithm.* J. Funct. Anal., 286(9):Paper No. 110369, 43, 2024.

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Richard Lechner, Johannes Kepler University Linz

Haar multipliers on $L^p(L^1)$

Abstract. Let (h_I) denote the standard Haar system on $[0,1]$ and let $h_I \otimes h_J$ denote the tensor product $(s,t) \mapsto h_I(s)h_J(t)$. Given $1 \leq p, q < \infty$, we define the *biparameter Lebesgue space* $L^p(L^q)$ as the completion of $\operatorname{span}\{h_I \otimes h_J\}$ under the norm

$$\Big\|\sum_{I,J} a_{I,J} h_I(s) h_J(t)\Big\|_{L^p(L^q)} = \Big\| s \mapsto \Big\| t \mapsto \sum_{I,J} a_{I,J} h_I(s) h_J(t)\Big\|_{L^q}\Big\|_{L^p}.$$

We say that $D\colon L^p(L^q) \to L^p(L^q)$ is a Haar multiplier if $D h_I \otimes h_J = d_{I,J} h_I \otimes h_J$, where $d_{I,J} \in \mathbb{R}$. The decisive representative of Haar multipliers is the Capon projection $\mathcal{C}\colon L^p(L^q) \to L^p(L^q)$ given by $\mathcal{C} h_I \otimes h_J = h_I \otimes h_J$ if $|I| \leq |J|$, and $\mathcal{C} h_I \otimes h_J = 0$ if $|I| > |J|$, as our main result highlights:

For any bounded Haar multiplier $D\colon L^p(L^q) \to L^p(L^q)$, there exist $\lambda, \mu \in \mathbb{R}$ such that

$$\lambda\mathcal{C} + \mu(\mathrm{Id} - \mathcal{C}) \text{ factors through } D,$$

i.e., there exist bounded operators A, B so that $\lambda\mathcal{C} + \mu(\mathrm{Id} - \mathcal{C}) = ADB$. Additionally, if $\mathcal{C}$ is unbounded on $L^p(L^q)$ (which is true if $p = 1$ or $q = 1$), then $\lambda = \mu$ and the identity operator Id either factors through D or $\mathrm{Id} - D$.

This constitutes an important step towards establishing the primarity of $L^p(L^1)$—which is among the most prominent examples of classical Banach spaces for which primarity is still open.

This talk is based on joint work Motakis, Müller and Schlumprecht.

References

[1] R. Lechner, P. Motakis, P. F. X. Müller, Th. Schlumprecht, Multipliers on bi-parameter Haar system Hardy spaces, *Mathematische Annalen*, https://doi.org/10.1007/s00208-024-02887-9.

Tanmoy Paul, IIT Hyderabad

U-embedding of Banach spaces

Abstract. Given two Banach spaces X, Y, I will discuss some instances when there exists an into isometry $\Phi : Y \to X$ such that $\Phi(Y)$ is a U-subspace of X. We study the cases when X is of the form $C(K)$, the continuous function spaces on a compact Hausdorff space. I will discuss some necessary and sufficient conditions Y must satisfy for this case. I will discuss that even if we consider a particular subclass of functionals on Y have unique norm preserving extensions over $C(K)$ then Y must satisfy some geometric properties similar to $C(K)$.

References

[1] T. S. S. R. K. Rao, *A geometric Jordan decomposition theorem*, Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser.,(2024), 118(68) 1–8.
[2] Francis Sullivan, *Geometric properties determined by the higher duals of Banach spaces*, Illinois J. Math.,(1977) 21(2), 315–331.

James Smith, Lancaster University

Closed ideals of operators on Baernstein spaces

Abstract. In the presence of a basis for a Banach space X, $\mathcal{L}(X)$ contains at most $2^{|\mathbb{R}|}$ closed ideals. In this talk, we discuss how the Baernstein spaces B_p (where $1 < p < \infty$) can be added to the growing list of spaces in which this upper bound is attained. Along the way, we explain how subsequences of the unit vectors in B_p are equivalent precisely when they are equivalent in the Schreier space, a somewhat surprising outcome. Finally, we deduce that $\mathcal{L}(B_p)$ contains at least $|\mathbb{R}|$ maximal closed ideals, and whether $2^{|\mathbb{R}|}$ many could exist. This is joint work with Niels Laustsen.

Richard Smith, University College Dublin, Ireland

L-embeddability in Lipschitz-free spaces

Abstract. Let $\mathrm{Lip}_0(M)$ denote the Banach space of real-valued Lipschitz functions on a complete metric space (M, d) that vanish at a point $0 \in M$. It has a natural isometric predual which is sometimes called the Lipschitz-free space over M, and denoted $\mathcal{F}(M)$. We consider the question of when $\mathcal{F}(M)$ is L-embedded in its bidual $\mathrm{Lip}_0(M)^*$, that is, when there exists a projection $P : \mathrm{Lip}_0(M)^* \to \mathcal{F}(M)$ such that $\|\psi\| = \|P\psi\| + \|(I - P)\psi\|$, $\psi \in \mathrm{Lip}_0(M)^*$. The question of L-embeddability, or more generally complementability of $\mathcal{F}(M)$ in $\mathrm{Lip}_0(M)^*$, has implications for the non-linear geometry of Banach spaces, which we discuss. We give an explicit and natural description of a projection P as above when M is proper and purely 1-unrectifiable. This is done using the De Leeuw transform, which is an important tool for representing elements of $\mathrm{Lip}_0(M)^*$ by signed Radon measures on a certain compact space.

This talk is based on joint work with Ramón Aliaga (Universitat Politècnica de València) and Eva Pernecká (Czech Technical University, Prague).

Thomas Speckhofer, Johannes Kepler University Linz

Factorization in Haar system Hardy spaces

Abstract. A Haar system Hardy space is the completion of the linear span of the Haar system $(h_I)_I$, either under a rearrangement-invariant norm $\|\cdot\|$ or under the associated square function norm given by

$$\Big\|\sum_I a_I h_I\Big\|_* = \Big\|\Big(\sum_I a_I^2 h_I^2\Big)^{1/2}\Big\|.$$

Apart from L^p, $1 \le p < \infty$, the class of these spaces includes all separable rearrangement-invariant function spaces on $[0,1]$ and also the dyadic Hardy space H^1.

Using a unified and systematic approach, we prove that every Haar system Hardy space X with $X \neq C(\Delta)$ (where $C(\Delta)$ denotes the continuous functions on the Cantor set) has the following property: For every bounded linear operator T on X, the identity I_X factors either through T or through $I_X - T$. Moreover, if T has large diagonal with respect to the Haar system, then the identity factors through T. In particular, we obtain that

$$\mathcal{M}_X = \{T \in \mathcal{B}(X) : I_X \text{ does not factor through } T\}$$

is the unique maximal ideal of the algebra $\mathcal{B}(X)$ of bounded linear operators on X. Finally, we establish analogous factorization results for the spaces $\ell^p(X)$, $1 \le p < \infty$, and we use Pełczyński's decomposition method to show that these spaces are primary.

Based on joint work with Richard Lechner [1].

References

[1] R. Lechner, T. Speckhofer, Factorization in Haar system Hardy spaces. Preprint available on arXiv:2212.06723, October 2023.

Sukumar Daniel, Indian Institute of Technology Hyderabad

Exponential spectrum commutes in $B(\ell_p \oplus \ell_q)$

Abstract. The exponential spectrum does not commute, as shown by Klaja and Ransford. In literature, the operator algebra $\mathcal{B}(\ell^p \oplus \ell^q)$ was also anticipated to be a potential algebra of the same type. But, we show that the exponential spectrum commutes in this algebra.

References

[1] Hubert Klaja and Thomas Ransford, Non-commutativity of the exponential spectrum, *J. Funct. Anal.* **272**, (2017), no. 10, 4158–4164. MR3626037.

This is a joint work with Dr. Arindam Ghosh.

Mitchell Taylor, ETH Zurich

Free Banach lattices

Abstract. Given a Banach space E, one may construct a Banach lattice $\mathrm{FBL}[E]$ with the property that every bounded linear operator from E into a Banach lattice X extends uniquely to a lattice homomorphism from $\mathrm{FBL}[E]$ into X. We will discuss the structure of $\mathrm{FBL}[E]$ and its larger role in the theory of Banach spaces and lattices.

Elroy Zeekoei, North-West University

Some Dunford–Pettis-like operators on Banach lattices

Abstract. The notion of a p-convergent operator on a Banach space was originally introduced in 1993 by Castillo and Sánchez in the paper entitled “Dunford-Pettis-like properties of continuous vector function spaces”. In this talk we consider the p-convergent operators on Banach lattices as well as the notion of a weak p-convergent operator. We also discuss the notion of a disjoint p-convergent operator on Banach lattices and apply it to a study of the positive Schur property of order p are considered.