OPERATOR SEMIGROUPS AND EVOLUTION EQUATIONS (S18)

LYONELL BOULTON (HERIOT-WATT), CHRISTIAN BUDDE (FREE STATE)

Monday 14:00-16:00 (SIBLT3) chair: Lyonell Boulton

14:00-14:25	Alexander Dobrick *Towards a general framework for queueing and reliability theory*
14:30-14:55	David Seifert *A Katznelson-Tzafriri theorem for analytic Besov functions*
15:00-15:25	Felix Schwenninger *On the solvability of the radiative transfer equations with polarization*
15:30-15:55	Christian Budde *Well-posedness of non-autonomous transport equation on metric graphs*

Tuesday 16:30-19:00 (SIBLT3) chair: Christian Budde

16:30-16:55	Hannes Gernandt *On a class of dissipative boundary control systems and networks*
17:00-17:25	François Genoud *Finite time blow-up for the nonlinear Schrödinger equation on a star graph*
17:30-17:55	Lyonell Boulton *Weak revivals in time-evolution models*

Abstracts.
Lyonell Boulton *Weak revivals in time-evolution models*

Abstract. The mysterious phenomenon of *revivals* in linear dispersive periodic equations, was discovered first experimentally in optics in around 1834, then rediscovered several times by theoretical investigations. While the term has been used systematically and consistently by many authors, there is no consensus on a rigorous definition. Several have described it by stating that a given periodic time-dependent boundary value problem exhibits *revivals at rational times*, if the solution evaluated at a certain dense subset of times is given by finite superposition of translated copies of the initial conditions. When this initial condition has jump discontinuities at time zero, these discontinuities propagate and remain present in the solution at each rational time but disappear completely at irrational times.

In this talk, I will report on the presence of revivals in three distinct models of parabolic differential equations: (1) non-local equations that arise in water wave theory and are defined by convolution kernels [1]; (2) Schrödinger equations with different types of boundary conditions [2] or with complex potentials [3]; (3) dislocated Laplacian time-evolution equations [4]. As we shall see, in all cases the solution is given explicitly by finite combination of translations, dilations and scaling of the initial datum, plus additional regular terms. When present, these extra terms can be interpreted as a weak manifestation of the classical revivals phenomenon.

The research has been conducted jointly with George Farmakis (London South Bank University), Peter Olver (University of Minnesota), Beatrice Pelloni (Heriot-Watt University) and David Smith (Yale NUS).

References

[1] *Stud. Appl. Math.* **147** (2021) p.1209.

[2] *Proc. Royal Soc. A.* **477** (2021) p.2251.

[3] Preprint ArXiV:2308.09961. To appear in *ZAA* (2024).

[4] Preprint ArXiV:2403.01117.

Christian Budde

Well-posedness of non-autonomous transport equation on metric graphs

Abstract. We consider transport processes on metric graphs with time-dependent velocities and show that, under continuity assumption of the velocity coefficients, the corresponding non-autonomous abstract Cauchy problem is well-posed by means of evolution families and evolution semigroups.

Consider a finite network (i.e., of pipelines) where some material is transported along its branches (i.e., pipes). The velocity of the transport depends on a given branch but may also change in time. We would like to know under which condition such a system can be modelled in a way that for any given initial distribution we are able to predict the state of the system in any time. We would also like to obtain stable solutions that continuously depend on the initial state. In this case we will call our problem well-posed.

Such transport problems on networks have already been studied by several authors. The operator theoretical approach by means of abstract Cauchy problems on Banach spaces was initiated by the second author and Sikolya [3]. However, the majority of the publications concentrates on time-independent transport and hence autonomous abstract Cauchy problems. A first attempt to non-autonomous problems of this kind was performed by Bayazit et al. [1]. They considered transport on networks with boundary conditions changing in time. The advantage of such an approach is that the corresponding operator does not change its action on the Banach space, only its domain changes in time. Our aim is to consider also the non-autonomous operator, that is, we study transport problems on finite metric graphs with time-dependent velocities along the edges. We use evolution families and evolution semigroups as studied by Nickel [4] and show that the abstract Cauchy problem, which can be associated to the transport equation on these graphs, is well-posed.

This talk is based on joint work with M. Kramar Fijavž [2].

References

[1] Bayazit, F., Dorn, B, Kramar Fijavž, M:. Asymptotic periodicity of flows in time-depending networks. *Netw. Heterog. Media*, **8**(4), 843–855 (2013).
[2] Budde, C., Kramar Fijavž, M. Well-posedness of non-autonomous transport equation on metric graphs. *Semigroup Forum*, **108**, 319–334 (2024).
[3] Kramar, M., Sikolya, E.: Spectral properties and asymptotic periodicity of flows in networks. *Math. Z.*. **249**(1), 139–162 (2005).
[4] Nickel, G.: Evolution semigroups for nonautonomous Cauchy problems. *Abstr. Appl. Anal.*, **2**(1–2), 73–95 (1997).

Alexander Dobrick, CAU Kiel

Towards a general framework for queueing and reliability theory

Abstract. Building on Greiner's result on boundary perturbations, we develop an abstract framework within the context of AL-spaces. This framework allows proving a generation result for C_0-semigroups arising from various examples from the queueing and reliability theory. Leveraging recent results of Glück, Gerlach and Martin, it further allows investigating the long-term behaviour of these semigroups without requiring an in-depth analysis of the spectrum of their generators. Finally, we sketch an extension of the framework that can be used to discuss buffered transport problems on infinite networks.

References

[1] Günther Greiner. Perturbing the boundary conditions of a generator. *Houston J. Math.*, 13(2):213–229, 1987.
[2] Piotr Gwiżdż and Marta Tyran-Kamińska. Positive semigroups and perturbations of boundary conditions. *Positivity*, 23(4):921–939, 2019.
[3] Abdukerim Haji and Geni Gupur. Asymptotic property of the solution of a reliability model. *Int. J. Math. Sci.*, 3(1):161–165, 2004.
[4] Marta Tyran-Kamińska. Transport equations and perturbations of boundary conditions. *Math. Methods Appl. Sci.*, 43(18):10511–10531, 2020.

François Genoud, EPFL

Finite time blow-up for the nonlinear Schrödinger equation on a star graph

Abstract. The construction of a finite time blow-up solution for a nonlinear Schrödinger equation (NLS) on a star graph will be presented. The simplest configuration of a graph with two branches corresponds to the NLS on the line with a delta potential at the origin. The general case involves a one-dimensional Laplace operator on the graph with Robin boundary conditions at the vertex. The blow-up analysis relies on the resolution of the nonlinear Cauchy problem within the domain of the corresponding linear operator. This is joint work with Stefan Le Coz and Julien Royer.

Hannes Gernandt, University of Wuppertal

On a class of dissipative boundary control systems and networks

Abstract. In this talk, we consider a class of dissipative boundary control systems whose dynamics is generated by a 2x2 block operator in a Hilbert space that has a bounded dissipative diagonal and a possibly unbounded skew-adjoint off-diagonal. Sufficient conditions for the strong and exponential stability of the underlying semigroup generators are provided along with the derivation of a power balance equation for classical solutions of the boundary control system. Furthermore, we consider interconnections of several such dissipative boundary control systems and show that Kirchhoff-type interconnections preserve the underlying structure of the considered block operators, and thus, of the aforementioned stability and passivity properties. The results are illustrated for a power network connecting several prosumers via distributed transmission lines that are modeled based on the telegraph equations.

This talk is based on the preprint [1] with Dorothea Hinsen (TU Berlin)

References

[1] H. Gernandt, D.Hinsen, Stability and passivity for a class of distributed port-Hamiltonian networks, arXiv:2212.02792.

Felix L. Schwenninger, University of Twente

On the solvability of the radiative transfer equations with polarization

Abstract. The well-posedness of the radiative transfer equation with polarization and varying refractive index is investigated using semigroup techniques. This includes non-homogeneous boundary value problems on bounded spatial domains, which requires the analysis of suitable trace spaces. Additionally, we discuss positivity, Hermiticity, and norm-preservation of the matrix-valued solution. This is joint work with M. Schlottbom and V. Bosboom (Twente).

References

[1] V. Bosboom, M. Schlottbom, F.L. Schwenninger, On the unique solvability of radiative transfer equations with polarization, *J. Diff. Eq.* **393**, (2024), 174–203.

David Seifert, Newcastle University

A Katznelson–Tzafriri theorem for analytic Besov functions

Abstract. Let $-A$ be the generator of a bounded C_0-semigroup $(T(t))_{t\geq 0}$, and suppose that A admits a bounded functional calculus with respect to an algebra $\mathcal{A}$ of holomorphic functions on the the open right half-plane $\mathbb{C}_+$. Theorems of Katznelson–Tzafriri type provide sufficient conditions under which

$$\lim_{t\to\infty} \|T(t)f(A)\| = 0 \qquad (*)$$

for suitable functions $f \in \mathcal{A}$. Such theorems play an important role in the asymptotic theory of C_0-semigroups, and in particular have been used to give an alternative proof of the famous countable spectrum theorem. The main result to be presented in this talk is a new Katznelson–Tzafriri theorem for operators A admitting a bounded functional calculus with respect to a certain algebra $\mathcal{B}$ of analytic Besov functions. The theorem states that $(*)$ holds for all $f \in \mathcal{B}$ such that f vanishes on the boundary spectrum $\sigma(A) \cap i\mathbb{R}$ of A and $|f(z)| \to 0$ as $|z| \to \infty$ with $z \in \mathbb{C}_+$. The talk is based on joint work with Charles Batty.