POSITIVE OPERATORS AND THEIR DYNAMICS (S17)

JOCHEN GLUECK (WUPPERTAL), ANKE KALAUCH (DRESDEN)

Monday 14:00-16:00 (CHLT) chair: Jochen Glück

14:00-14:25 Florian Boisen
A generalization of Riesz homomorphisms on order unit spaces*

14:30-14:55 Janko Stennder
Markov operators on order unit spaces

15:00-15:25 Onno van Gaans
The order center and the algebraic center of a JB-algebra

15:30-15:55 Melchior Wirth
Symmetric quantum Markov semigroups and their generators

Monday 16:30-19:00 (CHLT) chair: Anke Kalauch

16:30-16:55 Christian Budde
Positive Desch-Schappacher perturbations of bi-continuous semigroups on AM-*spaces*

17:00-17:25 Marianna Porfido
Kernel estimates for parabolic systems of PDEs with unbounded coefficients

17:30-17:55 Sahiba Arora
Asymptotics of eventually positive semigroups

18:00-18:25 Jonathan Mui
Positivity as a stability condition

Tuesday 14:00-16:00 (SIBSR2) chair: Anke Kalauch

14:00-14:25 Alexander Dobrick
On buffered flows in infinite networks

14:30-14:55 Marianne Akian
Escape rate games and competitive spectral radii

15:00-15:25 Painos Chitanga
Hausdorff dimension of continued fractions and Perron-Frobenius operators

15:30-15:55 Julian Hölz
Uniform ergodicity of Banach lattice homomorphisms

Abstracts.

Marianne Akian, Inria and CMAP, École polytechnique, CNRS, IP Paris

Escape Rate Games and competitive spectral radii

Abstract. We consider a new class of repeated zero-sum games in which the payoff of one player is the escape rate of a dynamical system which evolves according to a nonexpansive nonlinear operator depending on the actions of both players. Considering order preserving finite dimensional linear operators over the positive cone endowed with Hilbert's projective (semi-)metric, we recover the matrix multiplication games introduced by Asarin et al. [1], which generalize the joint spectral radius of sets of nonnegative matrices. We establish a two-player version of Mañe's Lemma characterizing the value of the game in terms of a nonlinear eigenproblem. We deduce the existence of optimal strategies of both players. This is motivated by applications to population dynamics (growth maximization and minimization). This also provides a vector valued generalization of mean-payoff games.

This is a joint work with Stéphane Gaubert (Inria and CMAP, École polytechnique, CNRS, IP Paris), and Loic Marchesini (CMAP, École polytechnique, CNRS, IP Paris and Inria).

References

[1] E. Asarin, J. Cervelle, A. Degorre, C. Dima, F. Horn, and V. Kozyakin. Entropy games and matrix multiplication games. In 33rd Symposium on Theoretical Aspects of Computer Science, STACS 2016, February 17-20, 2016, Orléans, France, pages 11:1–11:14, 2016.
[2] M. Akian, S. Gaubert, J. Grand-Clément, and J. Guillaud. The operator approach to entropy games. Theory Comput. Syst., 63(5):1089–1130, 2019.
[3] R. Mañe. Generic properties and problems of minimizing measures of Lagrangian systems, Nonlinearity, 9(2):273–310, 1996.

Sahiba Arora, University of Twente

Asymptotics of eventually positive semigroups

Abstract. In many concrete applications, operator semigroups exhibit positivity, ensuring that a positive initial datum leads to a positive solution for all time $t \geq 0$. Consequently, positive semigroups have been extensively studied. However, recent years have witnessed a surge in the exploration of *eventually positive* semigroups, where a positive initial datum results in a solution that becomes and remains positive (only) for sufficiently large times.

This talk delves into the question of whether the favourable asymptotic properties observed in positive semigroups extend to eventual positivity. We explore the challenges encountered in adapting proofs and discuss strategies to overcome these obstacles.

Florian Boisen, Dresden University of Technology

A generalization of Riesz homomorphisms on order unit spaces*

Abstract. Riesz homomorphisms between vector lattices are generalized by van Haandel to Riesz* homomorphisms between pre-Riesz spaces. Riesz* homomorphisms are characterized, intrinsically, via a condition on finite sets. Originally, van Haandel claimed that sets with at most two elements are sufficient. In this talk, I illustrate that, even in the setting of finite-dimensional order unit spaces, this is not true, in general.

References

[1] F. Boisen, V.G. Hölker, A. Kalauch, J. Stennder, O. van Gaans, A generalization of Riesz* homomorphisms on order unit spaces, *Quaestiones Mathematicae*, (2024).

Christian Budde, University of the Free State

Positive Desch-Schappacher perturbations of bi-continuous semigroups on AM*-spaces*

Abstract. In this talk, we consider positive Desch-Schappacher perturbations of bi-continuous semigroups on AM-spaces with an additional property concerning the additional locally convex topology [2]. As an example, we discuss perturbations of the left-translation semigroup on the space of bounded continuous functions on the real line and on the space of bounded linear operators. As main reference serves the work of A. Batkái, B. Jacob, J. Wintermayr and J. Voigt [1] on positive Desch-Schappacher perturbations of strongly continuous operator semigroups.

References

[1] Bátkai, A., Jacob, B., Voigt, J., Wintermayr, J., Perturbations of positive semigroups on AM-spaces. *Semigroup Forum*, **96**:2, 333–347 (2018).
[2] Budde, C. Positive Desch-Schappacher perturbations of bi-continuous semigroups on AM-spaces. *Acta Sci. Math.*, **87**, 571–594 (2021).

Painos Chitanga, University of Kent

Hausdorff Dimension of continued fractions and Perron-Frobenius operators

Abstract. Given a subset E of $\mathbb{N}$ the set of continued fraction expansions is given by

$$J_E = \{x \in (0,1) \colon x = [a_1, a_2, a_3 \cdots] \text{ with } a_i \in E \text{ for all } i\},$$

where

$$[a_1, a_2, a_3, \cdots] = \cfrac{1}{a_1 + \cfrac{1}{a_2 + \frac{1}{a_3 + \cdots}}}.$$

These sets typically have a fractal nature and their Hausdorff dimension, denoted $\dim_{\mathcal{H}}(J_E)$, has been studied extensively. In this talk we will discuss the *dimension spectrum of* E,

$$\mathrm{DS}(E) = \{\dim_{\mathcal{H}}(J_F) \colon F \subseteq E\},$$

for different sets E. The structure of the dimension spectrum can be analysed using Perron-Frobenius operators, which are positive operators. Among other results we will see how the spectral theory of these operators can be used to answer a question raised by Chousionis, Leykekhman and Urbanski [1] concerning the dimension spectrum of sets of powers $E_k = \{n^k \colon k = 1, 2, 3, \ldots\}$.

Based on joint work with Bas Lemmens and Roger Nussbaum.

References

[1] V. Chousionis, D. Leykekhman, M. Urbanski, On the dimension spectrum of infinite subsystems of continued fractions, *Trans. Amer. Math. Soc.* **373**, (2019), 1009–1042.

Alexander Dobrick, CAU Kiel

On buffered flows in infinite networks

Abstract. We consider a transport problem on an infinite metric graph, focusing on its well-posedness and long-term behaviour, given that the mass flow is buffered in at least one vertex. In this context, we address the well-posedness of the flow by leveraging a recent boundary perturbation result on AL-spaces. Furthermore, we discuss the long-term behaviour of the flow using recent results about the convergence of stochastic semigroups that dominate a kernel operator. This is joint work with Florian G. Martin.

References

[1] Alexander Dobrick and Florian G. Martin. Well-posedness and long-term behaviour of buffered flows in infinite networks. *ArXiv:2404.14090*, 2024.
[2] Moritz Gerlach and Jochen Glück. Convergence of positive operator semigroups. *Trans. Amer. Math. Soc.*, 372(9):6603–6627, 2019.
[3] Jochen Glück and Florian G. Martin. Uniform convergence of stochastic semigroups. *Israel J. Math.*, 247(1):1–19, 2022.
[4] Marta Tyran-Kamińska. Transport equations and perturbations of boundary conditions. *Math. Methods Appl. Sci.*, 43(18):10511–10531, 2020.

Julian Hölz, University of Wuppertal

Uniform ergodicity of Banach lattice homomorphisms

Abstract. Measure preserving and topological dynamical systems can be associated with a homomorphism of a Banach lattice of functions by means of so-called composition operators. By studying properties of lattice homomorphisms, we thus show that the uniform ergodicity for topological dynamical systems implies eventual periodicity and for measure preserving system it implies periodicity.

Jonathan Mui, University of Wuppertal

Positivity as a stability condition

Abstract. This talk concerns C_0-semigroups and resolvent operators on Banach lattices which are not only positive, but even 'positivity improving' in a precise sense. This stronger property is often encountered in PDE problems, especially in the context of diffusion equations (e.g. [1]). Loosely speaking, the main goal is to demonstrate how positivity improvement is stable under bounded perturbations in an abstract setting which is motivated by the study of Schrödinger operators and applications to semilinear evolution equations [2].

This is joint work in progress with Daniel Daners and Jochen Glück.

References

[1] W. Arendt, A. F. M. ter Elst, J. Glück, *Strict positivity for the principal eigenfunction of elliptic operators with various boundary conditions*, Adv. Nonlinear Stud., **20** (2020), 633–650.
[2] W. Arendt, D. Daners, *Semilinear elliptic equations on rough domains*, J. Diff. Eq. **346** (2023), 376–415.

Marianna Porfido, TU Bergakademie Freiberg

Kernel estimates for parabolic systems of PDEs with unbounded coefficients

Abstract. In this talk we consider a class of systems of nondegenerate elliptic partial differential equations with unbounded coefficients with possibly unbounded diffusion coefficients which may vary equation by equation. In particular, we deal with a vector-valued elliptic operator $\boldsymbol{\mathcal{A}}$ in divergence form defined on smooth functions $\boldsymbol{f} : \mathbb{R}^d \to \mathbb{R}^m$ by

$$(\boldsymbol{\mathcal{A}}\boldsymbol{f})_h = \operatorname{div}(Q^h \nabla f_h) + \langle b^h, \nabla f_h \rangle - (V\boldsymbol{f})_h$$

for $h = 1, \dots, m$, where $Q^h : \mathbb{R}^d \to \mathbb{R}^{d\times d}$, $b^h : \mathbb{R}^d \to \mathbb{R}^d$ for every $h = 1, \dots, m$ and $V : \mathbb{R}^d \to \mathbb{R}^{m\times m}$. Under suitable assumptions, we prove pointwise upper bounds for the transition kernels of the semigroup associated in $C_b(\mathbb{R}^d; \mathbb{R}^m)$ with the operator $\boldsymbol{\mathcal{A}}$. The idea is to adapt and generalize to our setting the techniques exploited in the scalar case based on time-dependent

Lyapunov functions for the parabolic operator $D_t + \boldsymbol{\mathcal{A}}$. We finally illustrate our results in case of polynomially and exponentially growing coefficients.

References

[1] D. Addona, L. Lorenzi, M. Porfido, Kernel estimates for parabolic systems of partial differential equations with unbounded coefficients, preprint.

Janko Stennder, TU Dresden

Markov operators on order unit spaces

Abstract. Let Ω be a topological space. An operator T on $\mathrm{C}(\Omega)$ is called a Markov operator if T is positive and maps the constant 1 function to itself. It is well-known that T is an extreme point in the set of Markov operators if and only if T is a Riesz homomorphism. This was also shown in the more general context of unital f-algebras. We investigate Markov operators in the setting of order unit spaces. By means of the functional representation order unit spaces can be represented as order dense subspaces of $\mathrm{C}(\Omega)$ for some compact Hausdorff space Ω. We present a similar result as above in the case that the order unit space is a subalgebra of its functional representation.

Onno van Gaans, Leiden University

The order center and the algebraic center of a JB-algebra

Abstract. The vector space of all self-adjoint operators on a Hilbert space with the Jordan product $A \circ B = (AB + BA)/2$ is a typical example of a JB-algebra. A JB-algebra is both a Jordan algebra and a Banach space with with a suitably compatible norm. A JB-algebra is commutative but typically not associative. The set of all squares in a JB-algebra is a closed cone and, with the induced order, a JB-algebra is an Archimedean directed partially ordered vector space.

There are two natural notions of a center in a JB-algebra. The algebraic center of a JB-algebra is defined to be the set of those elements whose corresponding left multiplication operator commutes with all other left multiplication operators. The order center is the subspace of all linear operators on the JB-algebra consisting of the operators that are below and above a multiple of the identity operator, where the space of operators is ordered by the cone of positive operators. We will show that the order center and the algebraic center of a unital JB-algebra are isomorphic.

References

[1] A. Kalauch, M. Roelands, and O. van Gaans, Order theoretical structures in atomic JBW-algebras: disjointness, bands, and centres, *Positivity* **28** (2024).

Melchior Wirth, Institute of Science and Technology

Symmetric Quantum Markov Semigroups and Their Generators

Abstract. A quantum Markov semigroup is a point-weak* continuous semigroup of unit-preserving, completely positive maps on a von Neumann algebra. For such semigroups there are several non-equivalent notions of symmetry, including GNS and KMS symmetry. In this talk I will report on recent progress in the project to characterize the generators of GNS- and KMS-symmetric quantum Markov semigroups and discuss some open questions. (This is partly joint work with Matthijs Vernooij.)

References

[1] M. Wirth, The Differential Structure of Generators of GNS-symmetric Quantum Markov Semigroups, *arXiv:2207.09247* (2022).
[2] M. Vernooij and M. Wirth, Derivations and KMS-Symmetric Quantum Markov Semigroups, *Communications in Mathematical Physics* (2023).
[3] M. Wirth, Christensen–Evans theorem and extensions of GNS-symmetric quantum Markov semigroups, *Journal of Functional Analysis* (2024).
[4] M. Wirth, Modular Completely Dirichlet forms as Squares of Derivation *International Mathematics Research Notices* (2024).

The author was funded by the Austrian Science Fund (FWF) under the Esprit Programme [ESP 156].