# LINEAR ALGEBRA AND CONTROL THEORY (S13)

ANDRE RAN (AMSTERDAM), VOLKER MEHRMANN (TU BERLIN)

**Tuesday 14:00-16:00** (CHLT) chair: André Ran

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| 14:00-14:25 | Sanne ter Horst<br>*Some classes of positive linear matrix maps that are also completely positive* |
| 14:30-14:55 | Froilan Dopico<br>*Strongly minimal linear polynomial system matrices of structured rational matrices* |
| 15:00-15:25 | Alicia Roca<br>*Row completion of polynomial and rational matrices* |
| 15:30-15:55 | Anton Arnold<br>*Short- and long-time behavior in evolution equations: the role of the hypocoercivity index* |

**Thursday 14:00-16:00** (CHLT) chair: Alicia Roca

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| 14:00-14:25 | André Ran<br>*Unbounded Toeplitz operators: invertibility and Riccati equations* |
| 14:30-14:55 | Mikael Kurula<br>*Canonical Wiener-Hopf factorization of dichotomous transfer functions on the unit circle* |
| 15:00-15:25 | Christian Mehl<br>*Computing the eigenvalues of singular Hermitian pencils* |
| 15:30-15:55 | Patryk Pagacz<br>*On four (quasi-)singular classes of linear pencils* |

**Thursday 16:30–18:30** (CHLT) chair: Christian Mehl

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| 16:30-16:55 | Dorothea Hinsen<br>*Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems - Part 1: An Overview* |
| 17:00-17:25 | Riccardo Morandin<br>*Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems - Part 2: On Time-Varying Storage Functions* |
| 17:30-17:55 | Hannes Gernandt<br>*Linear port-Hamiltonian DAE systems revisited once more* |
| 18:00-18:25 | Alexander Wierzba<br>*Towards BIBO stability of port-Hamiltonian systems* |

*Abstracts.*

**Anton Arnold, Vienna University of Technology**

*Short- and long-time behavior in evolution equations: the role of the hypocoercivity index*

**Abstract.** The *index of hypocoercivity* is defined via a coercivity-type estimate for the self-adjoint/skew-adjoint parts of the generator, and it quantifies 'how degenerate' a hypocoercive evolution equation is, both for ODEs and for evolutions equations in a Hilbert space. We show that this index characterizes the polynomial decay of the propagator norm for short time and illustrate these concepts for the Lorentz kinetic equation on a torus.

This talk is based on joint work with F. Achleitner, E. Carlen, E. Nigsch, and V. Mehrmann.

## References

[1] F. Achleitner, A. Arnold, E. Carlen, The Hypocoercivity Index for the short time behavior of linear time-invariant ODE systems, *J. of Differential Equations* **371**, (2023) 83–115.
[2] A. Arnold, B. Signorello, Optimal non-symmetric Fokker-Planck equation for the convergence to a given equilibrium, *Kinetic and Related Models* **15(5)**, (2022) 753–773.
[3] F. Achleitner, A. Arnold, V. Mehrmann, E. Nigsch, Hypocoercivity in Hilbert spaces, submitted, 2024.

**Froilán Dopico, Universidad Carlos III de Madrid, Spain**

*Strongly minimal linear polynomial system matrices of structured rational matrices*

**Abstract.** We consider three classes of rational matrices $R(z)$ that are Hermitian upon evaluation on (a) the real axis, (b) the imaginary axis, or (c) the unit circle. Our goal is to show how to construct linear polynomial system matrices for those $R(z)$ that preserve the corresponding structures and are strongly minimal, a property that guarantee that such polynomial system matrices allow for a complete recovery of the poles, zeros, and minimal indices of $R(z)$. This goal is fully achieved for the Hermitian structures on the real and on the imaginary axes, but for the Hermitian structure on the unit circle some obstacles arise, which require to modify the original problem at some extent and to construct a structured linear polynomial system matrix for the rational function $(1 + z)R(z)$ instead of for $R(z)$. The results presented in this talk are based on the references [1] and [2].

## References

[1] F. Dopico, M.C. Quintana, P. Van Dooren, Strongly minimal self-conjugate linearizations for polynomial and rational matrices, *SIAM J. Matrix Anal. Appl.* **43**, (2022), 1354–1381.
[2] F. Dopico, V. Noferini, M.C. Quintana, P. Van Dooren, Para-Hermitian rational matrices, submitted (arXiv:2407.13563).

This work has been partially supported by the Agencia Estatal de Investigación of Spain MCIN/AEI/10.13039/501100011033/ through grants PID2019-106362GB-I00 and RED2022-134176-T.

This is joint work with V. Noferini, M.C. Quintana, and P. Van Dooren.

**Hannes Gernandt, University of Wuppertal**

*Linear port-Hamiltonian DAE systems revisited once more*

**Abstract.** In this talk, we prove a one-to-one correspondence between the geometric formulation of port-Hamiltonian (pH) systems defined by Dirac structures, Lagrange structures, maximal resistive structures, and external ports and a state-space formulation by means of port-Hamiltonian descriptor systems, i.e., differential algebraic equations (DAE) with inputs and outputs. Furthermore, assuming that the Lagrange structure is nonnegative, we are able to show that the resulting pH descriptor system that results from the geometric formulation is positive real. Furthermore, we compare our findings with recent results from [1], [3], and [4].

This talk is based on the preprint [2] with Friedrich Philipp, Till Preuster and Manuel Schaller (TU Ilmenau).

**References**

[1] H. Gernandt, F. E. Haller, and T. Reis, A linear relation approach to port-Hamiltonian differential-algebraic equations, SIAM J. Matrix Anal. Appl. 42, pp. 1011-1044, 2021.
[2] H. Gernandt, F. Philipp, T. Preuster and M. Schaller, On the equivalence of geometric and descriptor representations of linear port-Hamiltonian systems, arXiv:2305.08270.
[3] A. van der Schaft and B. Maschke, Generalized port-Hamiltonian DAE systems Systems Control Lett. 121, pp. 31-37, 2018.
[4] A. van der Schaft and V. Mehrmann, Linear port-Hamiltonian DAE systems revisited, Systems Control Lett. 177, 105564, 2023.

**Dorothea Hinsen, Technische Universität Berlin**

*Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems - Part 1: An Overview*

**Abstract.** In this talk, we study the relationship between passivity, having a nonnegative supply, and port-Hamiltonian representations for continuous-time linear time-varying systems. The previous results are surveyed and the subtle differences between the concepts are analyzed in detail. Furthermore, the connection to positive semidefinite solutions of the Kalman-Yakubovich-Popov inequality is investigated.

**Mikael Kurula, Åbo Akademi University, Finland**

*Canonical Wiener-Hopf factorization of dichotomous transfer functions on the unit circle*

**Abstract.** The talk is on joint work with Sanne ter Horst and André Ran. We wish to obtain the following result:

**Theorem** Let $F$ be a Hilbert-space operator-valued function of the form

$$W(z) = I + F(z), \qquad F(z) = D + zC(I - zA)^{-1}B, \qquad \sup_{z\in\mathbb{T}} \|F(z)\| < 1,$$

where the operators $A, B, C, D$ are bounded and $A$ is *dichotomous*, i.e., the resolvent set $\mathrm{res}\,(A)$ of $A$ contains the complex unit circle $\mathbb{T}$.

Then $W$ has a *left canonical Wiener-Hopf factorization*, i.e., $W(z) = V_+(z)\,V_-(z)$, where neither $V_+$ nor $V_+^{-1}$ has any poles in the closed unit disk $\overline{\mathbb{D}}$, while neither $V_-$ nor $V_-^{-1}$ has any poles in the closed complement $\overline{\mathbb{E}}$ of the unit disk.

Via symmetry, one also gets a *right canonical Wiener-Hopf factorization*, i.e., $W(z) = W_-(z)\,W_+(z)$, where $W_+$ and $W_-$ have the same properties as $V_+$ and $V_-$ above, respectively.

We follow the so-called *matching subspaces* approach [1] to Wiener-Hopf factorization, using the Dichotomous Bounded Real Lemma [2] and Kreĭn-space theory [3].

In [1], canonical Wiener-Hopf factorizations are established for $F$ a strictly proper matrix valued function which is contractive on the real line. The theorem above is an extension to Hilbert space operator valued functions which are instead strictly contractive on the complex unit circle.

### References

[1] I. Gohberg and A.C.M. Ran, On pseudo-canonical factorization of rational matrix functions. *Indag. Math. (N.S.)* 4 (1993), 51-63.
[2] J.A. Ball, G.J. Groenewald and S. ter Horst, Standard versus strict bounded real lemma with infinite-dimensional state space III: The dichotomous and bicausal cases. In: *Operator theory, analysis and the state space approach*, Oper. Theory Adv. Appl. 271, Birkhäuser/Springer, Cham, 2018, 23-73.
[3] T. Ya. Azizov and I.S. Iokhvidov, *Linear operators in spaces with an indefinite metric*, John Wiley, New York, London, 1989.

**Christian Mehl, Technische Universität Berlin**

*Computing the eigenvalues of singular Hermitian pencils*

**Abstract.** The solution of singular eigenvalue problems has been a challenge for many years. In this talk, we show how theoretical results from perturbation theory can be exploited to compute the eigenvalues of singular pencils with symmetry structures. Special focus is laid on singular Hermitian pencils. The developed method for solving the corresponding eigenvalue problem has the advantage that it relies on structure-preserving transformations. As a consequence important structural invariants like the so-called sign characteristic are preserved.

This is joint work with Michiel Hochstenbach and Bor Plestenjak.

**Riccardo Morandin, Technische Universität Berlin**

*Dissipativity Concepts for Linear Time-Varying Port-Hamiltonian Systems – Part 2: On Time-Varying Storage Functions*

**Abstract.** In this talk we study more in detail the characterization of storage functions for linear time-varing systems, and how to use their properties to connect the different dissipativity concepts presented in Part 1.

**Patryk Pagacz, Jagiellonian University**

*On four (quasi-)singular classes of linear pencils.*

**Abstract.** During my talk I will discuss the relation between the following four kinds of linear pencils:

(1) linear pencils with the spectrum equal to the whole complex plane,
(2) linear pencils with the numerical range equal to the whole complex plane,
(3) linear pencils such that $(0, 0)$ belongs to the Taylor spectrum of their coefficients,
(4) linear pencils such that (0,0) belongs to the joint numerical range of their coefficients.

The direct motivation for our research was two questions: "Is it true that the class (1) is equal to (3)?" arise in [1] and "Is it true that the class (2) is equal to (4)?" arise in [2]. The answers of these questions differ depending on the specific assumptions. For example there are no equivalences relations between (1)-(4) for linear pencils of operators. We will also show that linear pencils from class (2) do not have to belong to (4), even if matrix coefficients commute. However, for matrix linear pencils with coefficients that have positive semidefinite hermitian

parts conditions (2) and (4) means the same. Moreover, we will show that the first question has an affirmative answer also for matrix polynomials. The talk is based on the joint works with Vadym Koval.

## References

[1] S. V. Djordjevic, J. Kim, J. Yoon, Spectra of the spherical Aluthge transform, the linear pencil, and a commuting pair of operators, *Linear and Multilinear Algebra*, **70**, (2022), 2533–2550.
[2] C. Mehl, V. Mehrmann, M. Wojtylak, Matrix pencils with coefficients that have positive semidefinite hermitian part, *SIAM J. Matrix Anal. Appl.*, **43**, (2022), 1186–1212.

**André Ran, Vrije Universiteit Amsterdam adn North West University, South Africa**

*Unbounded Toeplitz operators: invertibility and Riccati equations*

**Abstract.** For a class of unbounded block-Toeplitz operators it will be shown how invertibility is connected to factorization of the symbol, and to existence of a particular solution to an unsymmetric algebraic Riccati equation. This is motivated by a similar result for bounded Toeplitz operators due to Rien Kaashoek, Art Frazho and the speaker. In turn, that result was inspired by a paper by Peter Lancaster, Leiba Rodman and the speaker.

This is joint work with Jacob Jaftha (University of Cape Town), Gilbert Groenewald and Sanne ter Horst (both North West University).

## References

[1] G.J. Groenewald, S. ter Horst, J.J. Jaftha, and A.C.M. Ran. A Toeplitz-like operator with rational matrix symbol having poles on the unit circle: invertibility and Ricatti equations *Journal of Mathematical Analysis and Applications*, 532 (2024) Paper no. 127925, 15 pp.

The research was partially supported by the National Research Foundation of South Africa (NRF, Grant Numbers 118513, 127364 and 145688) and the DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS).

**Alicia Roca, Universitat Politécnica de Valéncia**

*Row completion of polynomial and rational matrices*

**Abstract.** An important problem in Matrix Theory is the *matrix completion problem*. It consists in characterizing the existence of a matrix with certain properties when a submatrix is prescribed. This work is devoted to the row completion problem for polynomial and rational matrices.

We characterize the existence of a polynomial matrix when its complete structural data (the invariant factors, the invariants orders at infinity, and the column and row minimal indices) and some of its rows are prescribed. This problem was solved in [1] when the polynomial matrix has the same degree as the prescribed submatrix. Here we remove this restriction.

The same problem is also solved for rational matrices.

Obviously, the results obtained hold for the corresponding column completion problems.

## References

[1]A. Amparan, I. Baragaña, S. Marcaida, A. Roca, Row or column completion of polynomial matrices of given degree, *SIAM J. Matrix Anal. Appl.* **45(1)**, (2024), 478–503.

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**Sanne ter Horst, North-West University**

*Some classes of positive linear matrix maps that are also completely positive*

**Abstract.** Motivated by a Nevanlinna-Pick type interpolation problem, the question arose whether linear matrix maps of the form $\mathcal{L}_B \circ \mathcal{L}_A^{-1} : \mathbb{C}^{n\times n} \to \mathbb{C}^{n\times n}$ are always completely positive whenever they are positive, for matrices $A, B \in \mathbb{C}^{n\times n}$ with $A$ Lyapunov regular (eigenvalues $\lambda_1, \ldots, \lambda_n$ satisfy $\lambda_i + \overline{\lambda}_j \neq 0$ for all $i, j$) and $B$ in the double commutant of $A$. Here for $E \in \mathbb{C}^{n\times n}$, $\mathcal{L}_E$ is the Lyapunov operator $\mathcal{L}_E(X) = XE + E^*X$, $X \in \mathbb{C}^{n\times n}$. While "there are many more positive maps than completely positive maps" [3], we identify certain classes of linear matrix maps for which positivity implies complete positivity, and in doing so provide a positive answer to the question. The talk is based on work with Alma van der Merwe in [1,2].

## References

[1] S. ter Horst and A. van der Merwe, Linear matrix maps for which positivity and complete positivity coincide, *Linear Algebra Appl.* 628 (2021) 140–181.
[2] S. ter Horst and A. van der Merwe, A Hill-Pick matrix criterion for the Lyapunov order, *J. Math. Anal. Appl.* 515 (2022), 126401.
[3] I. Klep, S. McCullough, K. Sivic, and A. Zalar, There are many more positive maps than completely positive maps, *Int. Math. Res. Not.* 11 (2019), 3313–3375.

**Alexander A. Wierzba, University of Twente**

*Towards BIBO stability of port-Hamiltonian systems*

**Abstract.** Port-Hamiltonian systems (pHS) provide a useful tool for modelling physical systems such as e.g. flexible beams within mechanical systems. In this contribution we consider the question of when a distributed port-Hamiltonian system is bounded-input bounded-output (BIBO) stable, continuing recent research on subtleties of this classical notion for infinite-dimensional systems. Analysing and utilizing the special structure of the transfer function of this system class, we provide sufficient conditions for BIBO stability for a sub-class of pHS.

This contribution is based upon joint work with Felix L. Schwenninger.