HERGLOTZ FUNCTIONS AND COMPOSITE MATERIALS (S12)

VICTOR VINNIKOV (BEN GURION), JOE BALL (VIRGINIA TECH), AARON WELTERS (FLORIDA TECH)

Monday 14:00-16:00 (SIBSR3) chair: TBD

14:00-14:25 Aaron Welters
Perspectives on perspective functions, part 1: Effective operators in the theory of composites

14:30-14:55 Aaron Welters
Perspectives on perspective functions, part 2: Kubo-Ando means

15:00-15:25 James Pascoe
The inverse problem for kernel means

15:30-15:55 Ryan Tully-Doyle
Graphs and representations of analytic functions

Monday 16:30-19:00 (SIBSR3) chair: TBD

16:30-16:55 Annemarie Luger
About the integral representation for Herglotz-Nevanlinna functions

17:00-17:25 Joseph Ball
Representations for Herglotz-Agler functions

17:30-17:55 Alexander Kiselev
Negative group velocity in doubly-porous media via sharp norm-resolvent estimates

Tuesday 14:00-16:00 (SIBSR3) chair: TBD

14:00-14:25 Elena Cherkaev
Herglotz functions in forward and inverse homogenization

14:30-14:55 Yury Grabovsky
Optimal recovery of Stieltjes functions from their noisy measurements

15:00-15:25 Oranelle Mattei
On bounding the response in time of viscoelastic composites with a void phase

15:30-15:55 Daniel Alpay
Hyperpositive functions

Abstracts.

Daniel Alpay, Chapman University

Hyperpositive functions

Abstract. Given a strictly positive matrix $T \in \mathbb{C}^{n\times n}$, we study matrix-valued functions for which the kernel

$$\frac{F(z) + F(w)^* - T - F(z)TF(w)^*}{z + \overline{w}}$$

is positive definite on the open right half-plane. These functions are associated to absolute stability (the Lurie problem). We will in particular present state-space characterization of these functions through a corresponding Kalman-Yakubovich-Popov Lemma. Links with de Branges $H(S)$ spaces will be discussed.

This is joint work with Izchak Lewkowicz (Ben-Gurion University of the Negev, Beer-Sheva, Israel).

References

[1] D. Alpay and I. Lewkowicz. Quantitatively Hyper-Positive Real Functions. Linear Algebra and Its Applications, vol. 623 (2021) 316-334.

[2] D. Alpay and I. Lewkowicz. D. Alpay and I. Lewkowicz. Quantitatively Hyper-Positive Real Rational Functions II. Linear Algebra and its Applications. Accepted. To appear.

[3] D. Alpay and I. Lewkowicz. Quantitatively Hyper-Positive Real Functions III. Preprint.

Joseph Ball, Virginia Tech

Representations for Herglotz-Agler-class functions

Abstract. By the d-variable Herglotz-Agler class we mean functions F mapping the polydisk $\mathbb{D}^d$ holomorphically into the right half plane having the additional property that, for any commuting d-tuple $(T_1, \ldots, T_d)$ of Hilbert space contraction operators, the operator $F(T_1, \ldots, T_d)$ (defined e.g. via the Riesz-Dunford functional calculus) has positive real part (in the standard Loewner partial order for Hilbert-space operators). We discuss kernel decompositions and representations associated with such functions and indicate how, for the case $d = 1$, one can recover the standard Herglotz integral representation for a single-variable Herglotz function.

References

[1] J. A. Ball and D. S. Kaliuzhnyi-Verbovetskyi, Schur-Agler and Herglotz-Agler classes of functions: Positive-kernel decompositions and transfer-function realizations, *Adv. Math.* **280**, (2015), 121-167.

Elena Cherkaev, University of Utah

Herglotz functions in forward and inverse homogenization

Abstract. Herglotz and Stieltjes functions play a central role in the homogenization of microstructured media, linking composite microgeometry to the spectral properties of a related

self-adjoint operator. The talk will discuss the Stieltjes/ Herglotz function integral representations for the homogenized transport coefficients of composites and matrix Pade approximations of the resolvents of operators arising in homogenization problems. I will show that the matrix spectral measure in the integral representation of the effective properties of the composite can be uniquely reconstructed; this uniqueness provides a basis for the inverse homogenization problem of recovering information about the microgeometry of the medium. Pade approximations of the spectral measure lead to efficient computational methods for wave propagation in composite materials, allow constructing bounds on the effective properties of composites in forward homogenization, and result in spectrally matched geometries in inverse homogenization, linking it to the inverse spectral problem.

Yury Grabovsky, Temple University

Optimal recovery of Stieltjes functions from their noisy measurements

Abstract. In many applications, such as electrochemical impedance spectroscopy, remote sensing, analysis of composite materials, etc., the response of materials can be described in terms of a Stieltjes function. This function can be measured experimentally at a number of points in the complex plane. The problem of its identification is then of central importance. In this talk I will address the question of quantitative measure of feasibility of such an identification. I will also examine this problem by placing it in a general Hilbert space framework, deriving optimality conditions, error estimates, and proposing an algorithm for its solution that comes with a certificate of optimality. This is a joint work with Dr. Narek Hovsepyan (Rutgers University) and Henry J. Brown (Temple University).

References

[1] Y. Grabovsky, N. Hovsepyan, On the feasibility of extrapolation of the complex electromagnetic permittivity function using Kramers-Kronig relations. *SIAM J. Math. Anal.*, **53**, No. 6, (2021) 6993—7023.
[2] Y. Grabovsky, Reconstructing Stieltjes functions from their approximate values: a search for a needle in a haystack. *SIAM J. Appl. Math.*, **82**, No. 4, (2022) 1135–1166.
[3] H. J. Brown, Y. Grabovsky, On feasibility of extrapolation of completely monotone functions. *arXiv:2401.15178* (2024).

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Alexander V. Kiselev, University of Bath

Negative group velocity in doubly-porous media via sharp norm-resolvent estimates

Abstract. I will discuss the possibility for doubly-porous media to exhibit negative group velocities, or, in other words, to support metamaterial regimes of wave propagation.

On the one hand, I will argue that the norm-resolvent limits of such media cannot exhibit negative behaviour (see, e.g., [1] for the context of linear elasticity). On the other hand, following the rationale introduced in [2], I will show that the next-order term of the resolvent asymptotics, if taken into account, leads to negative properties of the effective material.

The model I will consider in detail will be based on a periodic graph, which is itself seen as a norm-resolvent limit of a PDE on a thin network (see [3]). Unlike [4], where a metamaterial-type behaviour is demonstrated in a periodic medium under a strong enough magnetic field, or a number of other works, where the effect is essentially due to some special assumptions

on the components constituting the composite, I will consider the simplest possible, although high-contrast, scalar setup.

References

[1] Kirill Cherednichenko, Alexander Kiselev, Igor Velčić, Josip Žubrinić, Effective behaviour of critical-contrast PDEs: micro-resonances, frequency conversion, and time dispersive properties. II. *arXiv:* 2307.01125

[2] Cherednichenko, K.D., Ershova, Y.Y., Kiselev, A.V., Ryzhov, V.A., Silva, L.O. (2023). Asymptotic Analysis of Operator Families and Applications to Resonant Media. In: Brown, M., et al. *From Complex Analysis to Operator Theory: A Panorama. Operator Theory: Advances and Applications*, vol 291. Birkhäuser, Cham.

[3] Cherednichenko, Kirill D., Yulia Yu. Ershova, and Alexander V. Kiselev (2024). Norm-Resolvent Convergence for Neumann Laplacians on Manifold Thinning to Graphs. *Mathematics* 12, no. 8: 1161.

[4] Alexander V. Kiselev and Kirill Ryadovkin (2024). Phase Transition in a Periodic Tubular Structure. *SIAM Journal on Applied Mathematics* 84:3, 890-914.

Annemarie Luger, Stockholm University

About the integral representation for Herglotz-Nevanlinna functions

Abstract. In this talk we will give an overview on what is known about the integral representation for Herglotz-Nevanlinna functions in several variables.

Ornella Mattei, San Francisco State University

On bounding the response in time of viscoelastic composites with a void phase

Abstract. We are interested in the quasi-static shear antiplane response of those composite materials composed by three phases: a void phase, an elastic phase and a viscoelastic phase. As an example, consider the case of a 3d-printed polymer-based composite reinforced by carbon fibers, where the presence of voids in the microstructure of the composite is either by design or a result of the printing process. By exploiting the analytic properties of the effective tensor as a function of the shear moduli of the phases, we propose bounds on the antiplane shear effective tensor of the composite in the complex plane, in the following two scenarios: (i) no information is provided regarding the microstructure of the composite; (ii) the volume fraction of the three phases is known. The bounds consist in curves in the complex plane that denote the lens-shaped region where the effective tensor takes value: as expected the region corresponding to the second scenario (the volume fractions are known) lies within the region corresponding to the first one (no information is available). Furthermore, we provide bounds in the time domain, when the material is subject to a relaxation experiment. Specifically, we determine the minimum and maximum values of the antiplane shear stress in time in the two aforementioned scenarios. Very interestingly, the bounds incorporating the volume fractions are incredibly tight at certain moments of time, allowing us to use them in an inverse fashion to determine the volume fraction of voids. Furthermore, by comparing such bounds to the ones obtained for the case when there are no voids, one can quantify the effects of the presence of voids in the shear response of the composite.

James Eldred Pascoe, Drexel University

The inverse problem for kernel means

Abstract. We give an overview of the kernel embedding of measures and discuss the inverse problem for kernel means. That is, when is an element of a reproducing kernel Hilbert space an embedded positive measure? We will give insights into the answer over various spaces with varying levels of systematization. Finally, we will discuss obstructions to an integral Herglotz-type representation theory in several variables

Ryan Tully-Doyle, Cal Poly San Luis Obispo

Graphs and representations of analytic functions

Abstract. Given a self-adjoint matrix A and a vector α, the map

$$f(z) = \langle (A - zI)^{-1}\alpha, \alpha \rangle,$$

produces a rational self-map of the complex upper half plane. One interesting class of self-adjoint matrices that can be studied via this function representation is adjacency matrices of simple undirected graphs. Given a simple undirected graph G with adjacency matrix A, one can view the matrix $A - zI$ as the adjacency matrix of a graph that puts self-loops of weight z onto the existing structure of the graph G. In this talk, we'll look extending examples studied by Pascoe, by Bickel, Pascoe, and Sola, and in an honors thesis of Hong, connecting graph structures to the behavior of a two variable analogue of the classical representation, which allows for more complicated boundary behavior to occur. The central matrix becomes $(A - zY - w(I - Y))$, where Y is, for example, a diagonal matrix with diagonal entries 0 and 1. Reformulating these function theoretic ideas in linear algebraic terms turns out to provide straightforward arguments that, for example, extend known one-variable ideas about representation of certain graph products into two-variables. One interesting consequence of this perspective is a method for creating compositions of representations.

I would like to thank the NSF for their generous support.

Aaron Welters, Florida Institute of Technology

Perspectives on perspective functions, part 1: Effective operators in the theory of composites

Abstract. In the first part of my talk, I will give a brief overview of the definition of effective operator for an n-phase isotropic composite in the theory of composites. Then I will highlight their analytic properties as a function of the phases and, in particular, show that they belong to a class of maps known as perspective functions. As a quintessential concrete example, we will consider the problem of periodic conductivity in d-dimensions ($d = 2, 3$) and effective conductivity in that setting.

Aaron Welters, Florida Institute of Technology

Perspectives on perspective functions, part 2: Kubo-Ando means

Abstract. In the second part of my talk, I will use the perspective of perspective functions to bridge a connection between effective operators of 2-phase composites and Kubo-Ando means of pairs of positive operators. Special attention will be given to the arithmetic, harmonic, and geometric means as examples within this context. I will conclude with some recent results I have obtained with my collaborator, Graeme Milton (Univ. of Utah), to illustrate the effectiveness of such perspectives.