FINITE AND INFINITE DIMENSIONAL MOMENT PROBLEMS (S11)

SALMA KUHLMANN (KONSTANZ), MARIA INFUSINO (CAGLIARI)

Monday 14:00-16:00 (SIBSR5) chair: Maria Infusino

14:00-14:25 Lawrence Fialkow
A core variety approach to the truncated moment problem on $y = x^d$

14:30-14:55 Dragu Atanasiu
Moment problems related to the intrinsic characterization of moment functionals in the compact case

15:00-15:25 Moritz Schick
Separation of the sums of squares cone and the sums of nonnegative circuit polynomials cone

15:30-15:55 Philipp di Dio
K-Positivity Preserver and their Generators

Tuesday 16:30-19:00 (SIBSR5) chair: Maria Infusino

16:30-16:55 Raul Curto
Mellin Transform and Exponential Polynomial Methods in the Study of the Square Root Problem for Positive Measures

17:00-17:25 Lorenzo Baldi
Trucated moment problems for genus one curves

17:30-17:55 Aljaž Zalar
Positive polynomials and the truncated moment problem on plane cubics

18:00-18:25 Pawel Pietrzycki
Hyperrigidity and the property of rigidity at zero

Wednesday 11:40-12:40 (W1-SR5, Woolf) chair: Maria Infusino

11:40-12:05 Andreas Debrouwere
The Stieltjes moment problem in Gelfand-Shilov spaces

12:10-12:35 Pier Luigi Novi Inverardi
Method of moments: a maxentropic re-examination of the Fréchet-Shohat theorem

Abstracts.

Dragu Atanasiu, University of Borås

Moment problems related to the intrinsic characterization of moment functionals

Abstract. In [2] is established an intrinsic characterization of moment functionals on compacts of characters of an unital commutative real algebra.
In this talk some applications of this characterization to the moment problem will be presented. Moreover, using results from [1] and [3], other intrinsic characterizations of moment functionals in the compact case will be obtained.
For example, with the definitions from [2], a short proof of the following theorem is given using the main result from [1].

Theorem. Let A be an unital commutative $\mathbb{R}$-algebra and $L : A \to \mathbb{R}$ be linear with $L(A^2) \subseteq [0, \infty)$ and $L(1) = 1$.

Then there exists a representing Radon measure for L with compact support if and only if the quadratic module Q_L is *Archimedean*.

References

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Lorenzo Baldi, Universität Leipzig

Trucated moment problems for genus one curves

Abstract. The study of Carathéodory numbers of the truncated moment cone is a classical topic, but exact results are rare in the literature. This is the number of Dirac measures necessary to represent all positive Borel measures supported on a fixed domain when restricted to polynomials of bounded degree.

In this talk, we study the Carathéodory number for projective genus one curves, and show how it is exactly determined by the topology of the curve. As an application, we study the truncated moment problem for affine plane cubics, and solve it proving sharp degree bounds on the flat extension degree.

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Raúl E. Curto, The University of Iowa

Mellin Transform and Exponential Polynomial Methods in the Study of the Square Root Problem for Positive Measures

Abstract. For recursively generated shifts, we provide definitive answers to two outstanding problems in the theory of unilateral weighted shifts: the Subnormality Problem (**SP**) (related to the Aluthge transform) and the Square Root Problem (**SRP**) (which deals with Berger measures of subnormal shifts). We use the Mellin Transform and the theory of exponential polynomials to establish that (**SP**) and (**SRP**) are equivalent if and only if a natural functional equation

holds for the canonically associated Mellin transform. For p-–atomic measures with $p \leq 6$, our main result provides a new and simple proof of the above-mentioned equivalence. Subsequently, we obtain an example of a 7-–atomic measure for which the equivalence fails. This provides a negative answer to a problem posed by G.R. Exner in 2009, and to a recent conjecture formulated by R.E. Curto et al in 2019.

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Andreas Debrouwere, Vrije Universiteit Brussel (VUB)

The Stieltjes moment problem in Gelfand-Shilov spaces

Abstract. In this talk, I will discuss the unrestricted Stieltjes moment problem in the setting of weighted spaces of smooth functions defined via weight sequences (= Gelfand-Shilov spaces). I will present a characterization of the surjectivity and the existence of a continuous linear right inverse of the Stieltjes moment mapping

$$f \mapsto \left(\int_0^\infty x^n f(x) dx \right)_{n \in \mathbb{N}}$$

on Gelfand-Shilov spaces in terms of their defining weight sequence. Furthermore, I will explain the connection between this problem and Borel type problems in ultradifferentiable classes.

Philip J. di Dio, University of Konstanz

K-Positivity Preserver and their Generators

Abstract. *joint work with Konrad Schmüdgen*

We study K-positivity preservers for closed sets $K \subseteq \mathbb{R}$, i.e., linear maps $T : \mathbb{R}[x_1, \ldots, x_n] \to \mathbb{R}[x_1, \ldots, x_n]$ such that $T\mathrm{Pos}(K) \subseteq \mathrm{Pos}(K)$ holds where $\mathrm{Pos}(K)$ is the set of all polynomials non-negative on K [1-3]. We give a full description of K-positivity preservers for general closed K [2]. Borcea gave a complete classification for $K = \mathbb{R}^n$ [4] and Netzer only gave a partial description for arbitrary closed K [5].

We use the technique of regular Fréchet Lie groups and Lie algebras to define what e^{tA} means for a linear operator $A : \mathbb{R}[x_1, \ldots, x_n] \to \mathbb{R}[x_1, \ldots, x_n]$. We characterize all such maps A such that e^{tA} is a K-positivity preserver on $\mathbb{R}$ [1,2].

References

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Lawrence Fialkow, SUNY

A core variety approach to the truncated moment problem on $y = x^d$

Abstract. We discuss a framework for studying the truncated moment problem on the curve $y = x^d$ based on the core variety. This permits us to give a short new proof to the solution for $d = 3$ first proved in [TAMS 363(2011), 3133-3165].

Pier Luigi Novi Inverardi, University of Trento

Method of moments: a maxentropic re-examination of the Fréchet-Shohat theorem

Abstract. This talk reconsiders the Fréchet-Shohat theorem (1931) in the special case where the random variables involved are absolutely continuous with the entire real line as support (also known as the Hamburger moment problem). Using recent results from information theory, maximum entropy and convergence in the entropy of maximum entropy densities, we obtain, as expected, stronger modes of convergence than in distribution. As a first result, an alternative proof of such a theorem is given. Furthermore, due to the flexibility of the maximum entropy formalism, the new proof can be easily extended to the case of the Stieltjes moment problem.

References

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Paweł Pietrzycki, Jagiellonian University

Hyperrigidity and the property of rigidity at zero

Abstract. Motivated both by the fundamental role of the Choquet boundary in classical approximation theory, and by the importance of approximation in the contemporary theory of operator algebras, Arveson introduced hyperrigidity as a form of 'noncommutative' approximation that captures many important operator-algebraic phenomena.

The concept of rigidity at 0 was introduced by Salomon, who studied hyperrigid subsets of Cuntz-Krieger algebras.

In this talk, we will discuss new results on hyperrigid sets that have the property of rigidity at zero.

References

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Moritz Schick, University of Konstanz

Separation of the sums of squares cone and the sums of nonnegative circuit polynomials cone

Abstract. An n-variate homogeneous polynomial (form) is positive semidefinite (PSD), if it attains nonnegative values over the whole $\mathbb{R}^n$. The set of all such forms of fixed degree $2d$ is a convex cone, called the PSD cone. Studying subcones of the PSD cone is a rich topic in Real Algebraic Geometry with applications in polynomial optimization. Two prominent subcones of the PSD cone, are the sums of squares (SOS) cone and the sums of nonnegative circuit polynomials (SONC) cone. In this talk, we formally introduce the three cones, and study their set-theoretic relations. We then show how to find forms separating two of the cones, respectively. Therefore, we especially focus on methods studying the involved Newton polytopes. Moreover, it was recently shown by Reznick, that the odd powers of the Motzkin form M are not SOS. In this context, we show that M^k is SONC if and only if $k = 1$.

Aljaž Zalar, University of Ljubljana

Positive polynomials and the truncated moment problem on plane cubics

Abstract. Given a linear functional $L : \mathbb{R}[x, y]_{\leq 2k} \to \mathbb{R}$ on all bivariate polynomials of total degree at most $2k$ and a cubic curve C, the truncated moment problem (C–TMP) asks to characterize the existence of a positive Borel measure μ on $\mathbb{R}^2$ with support in C such that $L(p) = \int_{\mathbb{R}^2} p \, d\mu$. In the talk concrete solutions to the C–TMP will be presented for C being one of the following: (i) vanishing set of a polynomial of the form $g(x, y) = y^2 - r(x)$, $\deg r = 3$; (ii) rational curve; (iii) reducible curve of a special form. Certificates for positivity of polynomials with best possible degree bounds are crucial ingredients in the proofs of (i),(ii).

This is joint work with Abhishek Bhardwaj, Mario Kummer and Seonguk Yoo.

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