RANDOM MATRICES AND FREE PROBABILITY (S10)

IAN CHARLESWORTH (CARDIFF), HÅKAN HEDENMALM (KTH STOCKHOLM), TORBEN KRÜGER (FAU ERLANGEN-NÜRNBERG), JANI VIRTANEN (READING)

Tuesday 14:00-16:00 (SIBSR6) chair: TBC

14:00-14:25 Daniel Perales
Finite free probability and the S-transform
14:30-14:55 Jacob Campbell
Even hypergeometric polynomials and finite free probability
15:00-15:25 Charles-Philippe Diez
TBA
15:30-15:55 Daniel Munoz
Asymptotic limit of cumulants of complex Wigner matrices

Wednesday 11:40-12:40 (PSR4) chair: Jani Virtanen

11:40-12:05 Tom Claeys
Large deviations for the log-Gamma polymer
12:10-12:35 Arno Kuijlaars
Orthogonal polynomials in a normal matrix model with two insertions

Wednesday 14:00–15:00 (PSR4) chair: Jani Virtanen

14:00-14:25 Santeri Miihkinen
Double-scaling limits of Toeplitz determinants with emerging Fisher-Hartwig singularities
14:30-14:55 Håkan Hedenmalm
Soft Riemann-Hilbert problems and random matrix theory

Thursday 14:00-16:00 (PSR4) chair: Ian Charlesworth

14:00-14:25 Paul Skoufranis
Bi-Free Independence and the Asymptotics of Tensor Random Matrices
14:30-14:55 Patrycja Hęćka
The Boolean quadratic forms and tangent law
15:00-15:25 Claus Köstler
Jones-Temperley-Lieb algebras from the viewpoint of distributional invariance principles
15:30-15:55 Natasha Blitvic
TBA

Thursday 16:30–18:30 (PSR4) chair: Håkan Hedenmalm

16:30-16:55 Yacin Ameur
The Coulomb gas near a spectral outpost
17:00-17:25 Oleksii Kolupaiev
Loschmidt echo for deformed Wigner matrices
17:30-17:55 Volodymyr Riabov
Eigenstate Thermalization for Wigner-type Matrices
18:00-18:25 Sang-Jun Park
Tensor-freeness and central limit theorem

Friday 14:00-16:00 (SIBLT3) chair: Torben Krüger

14:00-14:25 Johannes Alt
TBA

14:30-14:55 Félix Parraud
Interpolation between random matrices and free operators

15:00-15:25 Hong Chang Ji
Spectral edge of non-Hermitian random matrices

15:30-15:55 David Renfrew
Eigenvalues of minors of random matrices and roots of derivatives of random polynomials

Abstracts.

Yacin Ameur, University of Lund

The Coulomb gas near a spectral outpost

Abstract. We consider two-dimensional Coulomb systems in the regime when the droplet is connected, while the coincidence set for the obstacle problem contains an analytic Jordan curve outside of the droplet. A nontrivial (Heine-distributed) number of particles will tend to fall in the vicinity of this curve, which we denote "spectral outpost". Under the process of Laplacian growth, the outpost grows into a new ring-shaped component of the droplet, and the study of outposts is therefore closely related to the regime of disconnected droplets. We study among other things fluctuations of linear statistics. The talk is based on joint works with Joakim Cronvall and Christophe Charlier.

Jacob Campbell, University of Virginia

Even hypergeometric polynomials and finite free probability

Abstract. In 2015, Marcus, Spielman, and Srivastava realized that expected characteristic polynomials of sums and products of randomly rotated matrices behave like finite versions of Voiculescu's free convolution operations. In 2022, I obtained a similar result for commutators of such random matrices; one feature of this result is the special role of even polynomials, in parallel with the situation in free probability.

It turns out that a certain family of special polynomials, called hypergeometric polynomials, arises naturally in relation to convolution of even polynomials and finite free commutators. I will explain how these polynomials can be used to approach questions of real-rootedness and asymptotics for finite free commutators, and how they provide a systematic framework for the main examples. Based on arXiv:2209.00523 and ongoing joint work with Rafael Morales and Daniel Perales.

Tom Claeys, UCLouvain

Large deviations for the log-Gamma polymer

Abstract. I will present a conjecture about large deviations of the partition function of the log-Gamma polymer. I will show that the conjectured large deviation rate function matches with that of last passage percolation with exponential weights in the zero-temperature limit, and with the lower tail of the Tracy-Widom distribution for moderate deviations. We can nearly prove our result but there is one step in the proof for which we only have heuristic evidence. The talk will be based on joint work with Julian Mauersberger.

Patrycja He cka, Wrocław University of Science and Technology

The Boolean quadratic forms and tangent law

Abstract. Ejsmont and Lehner [3] study the limit sums of free commutators and anticommutators and show that the generalized tangent function

$$\frac{\tan z}{1 - x \tan z}$$

describes the limit distribution. This is the generating function of the higher order tangent numbers of Carlitz and Scoville [1] which arose in connection with the enumeration of certain permutations. I will talk about the limit of weighted sums of Boolean commutators and anticommutators and I will show that the shifted generalized tangent function appears in a limit theorem. In order to do this, I shall provide an arbitrary cumulants formula of the quadratic

form. I will also apply this result to show several theorems in a Boolean probability theory. The talk is based on joint work [2] with Wiktor Ejsmont.

References

[1] L. Carlitz, R. Scoville, Tangent numbers and operators, *Duke Math. J.* **39** (1972), 413–429.
[2] W. Ejsmont, P. Hećka, The Boolean quadratic forms and tangent law, *Random Matrices: Theory and Applications* **13** (2024), no. 1, Paper No. 2450004.
[3] W. Ejsmont, F. Lehner, The free tangent law, *Adv. Appl. Math.* **121** (2020), 102093.

Håkan Hedenmalm, KTH and Reading

Soft Riemann-Hilbert problems and random matrix theory

Abstract. It has been known since the 2007 paper by Its and Takhtajan that the orthogonal polynomials with respect to an exponentially varying weight in the plane are characterized in terms of a 2x2 matrix dbar-problem. In the recent breakthrough by Hedenmalm and Wennman the asymptotics of these planar orthogonal polynomials was found (Acta Math, 2021). However, the connection with the 2x2 dbar-problem was left open for further investigation. It turns out that there is a nice algorithm to find the asymptotics of the planar orthogonal polynomials in terms of the dbar-problem, which also has the benefit of supplying better error terms. This algorithm will be presented here. The work was published in CPAM 2024.

References

[1] H. Hedenmalm, Soft Riemann-Hilbert problems and planar orthogonal polynomials, *Comm. Pure Appl. Math.* **77**, (2024), 2413–2451.

Hong Chang Ji, Institute of Science and Technology Austria

Spectral edge of non-Hermitian random matrices

Abstract. We report recent progresses on spectra of so-called deformed i.i.d. matrices. They are square non-Hermitian random matrices of the form $A+X$ where X has centered i.i.d. entries and A is a deterministic bias, and A and X are on the same scale so that their contributions to the spectrum of $A+X$ are comparable. Under this setting, we present two recent results concerning universal (over A) patterns arising in eigenvalue statistics of $A+X$ around its boundary, on macroscopic and microscopic scales. The first result shows that the macroscopic eigenvalue density of $A+X$ typically has a jump discontinuity around the boundary, which is a distinctive feature of the eigenvalue density of X by *circular law*. The second result shows universality for the local eigenvalue statistics of $A+X$ around a typical (jump) boundary point, i.e. that the local statistics match with that of a Ginibre matrix X with i.i.d. standard Gaussian entries. Based on joint works with A. Campbell, G. Cipolloni, and L. Erdős.

References

[1] L. Erdős and H.C. Ji, Density of Brown measure of free circular Brownian motion, *arXiv:2307.08626.*
[2] A. Campbell, G. Cipolloni, A. Campbell, and H.C. Ji, On the spectral edge of non-Hermitian random matrices, *arXiv:2404.17512* .

Oleksii Kolupaiev, Institute of Science and Technology Austria

Loschmidt echo for deformed Wigner matrices

Abstract. We will discuss recent results on sensitivity of a quantum evolution to perturbations. Consider two self-adjoint Hamiltonians $H_1 \approx H_2$ and an initial quantum state ψ_0. First, evolve ψ_0 under the Hamiltonian H_1 from time zero to t, and then consider the backward evolution from t to zero under H_2, resulting in ψ_0'. The question is how precisely the initial state is recovered. One possible way to measure this revival is the Loschmidt echo $|\langle \psi_0, \psi_0' \rangle|^2$. We model H_1, H_2 by deformed Wigner matrices and show that the Loschmidt echo follows a universal behavior as a function of time. Our proof relies on a two-resolvent global law, i.e. a concentration estimate for products of resolvents of H_1 and H_2. The talk is based on a joint work with Laszlo Erdős and Joscha Henheik.

Arno Kuijlaars, KU Leuven, Belgium

Orthogonal polynomials in a normal matrix model with two insertions

Abstract. The talk is about the asymptotic behavior of polynomials $P_{n,N}$ with orthogonality in the complex plane

$$\int_{\mathbb{C}} P_{n,N}(z)\overline{z}^k |z^2+a^2|^{2cN} e^{-N|z|^2} dA(z) = 0, \quad k = 0, \dots, n-1,$$

with $c, a > 0$ and $dA(z)$ denotes planar Lebesgue measure. These polynomials are connected with a normal matrix model with external potential $N|z|^2 - 2cN \log|z^2+a^2|$ which is a modification of the Ginibre ensemble with two logarithmic singularities. The eigenvalues of the random matrices fill out a bounded region in the complex plane as $n, N \to \infty$ with $n/N \to t > 0$. We prove, that for a certain regime of parameters a, c, t, the zeros of the orthogonal polynomials tend to an interval on the real line, with an asymptotic density that is characterized by a vector equilibrium problem.

Our analysis essentially relies on the reformulation of the planar orthogonality as non-Hermitian multiple orthogonality and on a steepest descent analysis of the associated Riemann-Hilbert problem of size 3×3.

This is based on joint work with Mario Kieburg and Sampad Lahiry.

References

[1] M. Kieburg, A.B.J. Kujlaars, and S. Lahiri, Orthogonal polynomials in the normal matrix model with two insertions, preprint.

Santeri Miihkinen, University of Reading

Double-scaling limits of Toeplitz determinants with emerging Fisher-Hartwig singularities

Abstract. Asymptotic behaviour of determinants $D_n(f)$ of Toeplitz matrices generated by Fisher-Hartwig (F-H) symbols f has been under active investigation since the late 1970s by many authors. An interesting trend is to consider so-called double-scaling limits of Toeplitz determinants $D_n(f_t)$ with the F-H symbol f_t depending on some external parameter t (corresponding to e.g. temperature in applications.) A goal is to find an asymptotic formula for $D_n(f_t)$ when n is large and t is close to some critical value. In this talk, I review some background on this topic and present our recent results on computing the double-scaling limits of Toeplitz determinants with $m \geq 1$ fixed Fisher-Hartwig singularities located on the unit circle away from $z = 1$. They describe the transition between the asymptotic regimes of m singularities and $m+1$ singularities, where one singularity emerges at $z = 1$ as $t \to 0$. We consider both cases for the seminorm associated to the singularities of f_t being less than one or equal to one. This talk is based on joint work with Reham Alahmadi and Jani Virtanen (University of Reading).

Daniel Munoz, City University of Hong Kong

Asymptotic limit of cumulants of complex Wigner matrices

Abstract. During this talk we will look at the asymptotic limit of normalized cumulants of Traces of powers of complex Wigner matrices. We will prove that as the dimension of the matrices goes to infinity the large limit converges and its limit can be characterized by some graphs that result to be trees. If we regard this limit as moment sequences indexed by the powers of the Wigner matrices then another natural question is to find their corresponding higher order free cumulants. This question has already been answered in the first order case by the well known Wigner semicircle law and for second and third order in [1] and [2] respectively. We will see that the trees that show up in the limit can be counted using the set of non-crossing partitioned permutations which permit us to find the higher order free cumulants up to order 4. If time permits we will see that our methods can be used to find a convergence rate for the trace of a polynomial on GUE variables to the normal distribution.

References

[1] M. Camille, J.A. Mingo, S. Péché, R. Speicher, Joint Global Fluctuations of complex Wigner and deterministic Matrices, *Random Matrices: Theory Appl.* Vol. 11, No. 02, 2250015 (2022).
[2] J.A. Mingo, D. Munoz, Third order cumulants of Complex Wigner matrices, submitted. Arxiv preprint. 2205.13081
[3] J.A. Mingo, D. Munoz, Asymptotic limit of cumulants and higher order free cumulants of Complex Wigner matrices. Arxiv preprint 2407.17608

Sang-Jun Park, CNRS, Laboratoire de Physique Théorique

Tensor-freeness and central limit theorem

Abstract. Voiculescu's notion of asymptotic free independence applies to a wide range of random matrices, including those that are independent and unitarily invariant. In this talk, we generalize this notion by considering random matrices with a tensor product structure that are invariant under the action of local unitaries. We show that, given the existence of 'local moments' described by tuples of permutations, an independent family of locally unitarily invariant random matrices satisfies a new kind of freeness in the limit, which we will call 'local freeness'. This can be defined via vanishing mixed 'local-free cumulants', allowing the joint local moments of local-free elements to be described in terms of that of individual elements. Furthermore, we propose a local-free version of the central limit theorem, which extends and recovers several previous results. This is a joint work with Ion Nechita.

Daniel Perales, Texas A& M University

Finite free probability and the S-transform

Abstract. The finite free additive and multiplicative convolutions are binary operations of polynomials that behave well with respect to the roots. On one hand, they can be understood as a finite analogue of free probability that involves only discrete measures. On the other hand, these operations can be realized as expected characteristic polynomials of adding (or multiplying) two randomly rotated matrices.

In the first half of this talk we will introduce these polynomial convolutions and mention their basic properties.

For the second part of the talk we will focus on an ongoing project where we define a new finite S-transform. Among other things, this transform provides a simple way to obtain the limiting spectral distribution of a sequence of polynomials (with increasing degree) directly using their coefficients. As one of many applications, we can easily compute the limiting S-transform of hypergeometric polynomials. These large class of polynomial contains many of the important families of polynomials that naturally appear in finite free probability, such as Laguerre, Hermite and Jacobi.

References

[1] Octavio Arizmendi, Katsunori Fujie, Daniel Perales, Yuki Ueda, S-transform in finite free probability (work in progress).

David Renfrew, University of Binghamton

Eigenvalues of minors of random matrices and roots of derivatives of random polynomials

Abstract. I will describe the Brown measure of the free compression of R-diagonal operators, which also gives limiting behavior of the eigenvalues of minors of large bi-unitarily random matrices, and the roots of derivatives of polynomials with independent, random coefficients. These processes are given by fractional convolutions powers of the additive convolution of R-diagonal operators introduced by Kösters and Tikhomirov. This is joint work with Andrew Campbell and Sean O'Rourke.

References

[1] A. Campbell, S. O'Rourke, D. Renfrew, The fractional free convolution of R-diagonal elements and random polynomials under repeated differentiation, *IMRN* , (2024).

Volodymyr Riabov, ISTA

Eigenstate Thermalization for Wigner-type Matrices

Abstract. Following the typicality paradigm, random matrix models are ubiquitously used in studying properties of chaotic complex quantum systems. One such property is the Eigenstate Thermalization Hypothesis (ETH) positing that the eigenfunctions of a sufficiently chaotic quantum system become uniformly distributed in the phase space. Following the recent surge in understanding of the multi-resolvent local laws, the validity of ETH was rigorously established for the simplest model of a chaotic quantum system - the Wigner random matrix [1,2]. However, standard Wigner matrices (with essentially i.i.d. entries) only model quantum systems with no underlying spatial structure.

To mimic physically more realistic systems, we consider a much more general class of random matrices - the Wigner-type ensembles. The entries of a Wigner-type matrix follow a non-trivial variance profile, with potentially large vanishing blocks, which allows to encode a non-trivial spatial structure.

In this talk, we present our result on the Eigenstate Thermalization Hypothesis in the bulk spectrum of general Wigner-type matrices. Based on [3], joint work with László Erdős.

References

[1] G. Cipolloni, L. Erdős, and D. Schröder. Eigenstate Thermalization Hypothesis for Wigner Matrices. *Commun. Math. Phys.* **388**, (2021), 1005–1048.

[2] G. Cipolloni, L. Erdős, and J. Henheik, Eigenstate Thermalisation at the Edge for Wigner Matrices, arXiv:2309.05488, (2023).

[3] L. Erdős, R., Eigenstate Thermalization Hypothesis for Wigner-type Matrices, arXiv:2403.10359, (2024).

Supported by the ERC Advanced Grant “RMTBeyond” No. 101020331.

Paul Skoufranis, York University

Bi-Free Independence and the Asymptotics of Tensor Random Matrices

Abstract. It has been 10 years since the first paper on bi-free independence was published by Voiculescu. Since then, the theory of bi-free probability has been well developed. In this talk, a basic introduction to the main concepts of bi-free probability will be provided and work connecting bi-free probability to the asymptotics of tensor products of random matrices will be discussed.

References

[1] C. Lacien, P. O. Santos, P. Youssef, *Central Limit Theorem for tensor products of free variables*, preprint at arXiv:2404.19662, (2024), 25 pages.
[2] P. Skoufranis, Central Limit Theorem for Tensor Products of Free Variables via Bi-Free Independence, preprint at arXiv:2405.19216, (2024), 20 pages.

I would like to thank the Natural Sciences and Engineering Research Council of Canada for their generous support.