HARMONIC ANALYSIS AND RELATED AREAS (S1)

ALAN CHANG (WUSTL) AND JENNIFER DUNCAN (ICMAT)

Monday 16:30-19:00 (SIBSR4) chair: Jennifer Duncan

16:30-16:55 K. N. Sridharan
Orlicz space on Groupoids
17:00-17:25 Deborpita Biswas
Near-Riesz bases in Hilbert Spaces
17:30-17:55 Jane McDougall
The Rosette minimal surface, its deformation to the classical Enneper surface, and a non-Jenkins Serrin surface
18:00-18:25 Ahmad Al-Salman
Singular Integral Operators Along twisted Surfaces
18:30-19:00 Lyazzat Sarybekova
Some results on Fourier multipliers

Tuesday 16:30-19:00 (SIBSR4) chair: Alan Chang

16:30-16:55 Ritika Singhal
Paley inequality for the Weyl transform and its applications
17:00-17:25 Ferenc Weisz
Hardy-Littlewood-type theorems for multi-dimensional Fourier transforms
17:30-17:55 Himani Sharma
Vertical maximal functions on manifolds with ends
18:00-18:25 Ole Christensen
Frames and Redundancy

Wednesday 11:40-12:40 (SIBSR4) chair: Jennifer Duncan

11:40-12:05 Itamar Oliviera
A phase-space approach to weighted Fourier extension inequalities
12:10-12:35 Hrit Roy
Estimates for cone multipliers in $\mathbb{R}^3$ *associated with rough convex domains*

Wednesday 14:00–15:00 (SIBSR4) chair: Alan Chang

14:00-14:25 Lars Becker
Carleson Operators on Doubling Metric Measure Spaces
14:30-14:55 Carmelo Puliatti
On Fourier transforms of fractal measures on the parabola

Abstracts.

Ahmad Al-Salman, Sultan Qaboos University

Singular Integral Operators Along twisted Surfaces

Abstract. Singular integral operators on product domains have been introduced by R. Fefferman and E. M. Stein as a natural generalization of the double Hilbert transform on $\mathbb{R}^2$. Here, $\mathbb{R}^d$ $(d = n, m \geq 2)$ is the d-dimensional Euclidean space. Also, we let $\mathbb{S}^{d-1}$ be the unit sphere in $\mathbb{R}^d$ equipped with the normalized Lebesgue measure $d\sigma_d$. In this talk, we are interested in studying the L^p boundedness of a class of singular integrals on product domains along special surfaces. For an $\Omega \in L^1(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ satisfying

$$\Omega(tx, sy) = \Omega(x, y) \text{ for any } t, s > 0; \tag{1}$$

$$\int_{\mathbb{S}^{n-1}} \Omega(u, \cdot)\, d\sigma_n(u) = \int_{\mathbb{S}^{m-1}} \Omega(\cdot, v)\, d\sigma_m(v) = 0, \tag{2}$$

we let

$$K_\Omega(x, y) = \frac{\Omega(x', y')}{|x|^n\, |y|^m}$$

where $x' = x/|x|$ for $x \neq 0$ and $y' = y/|y|$ for $y \neq 0$. For mappings $\phi_1, \phi_2 : (0, \infty) \to \mathbb{R}$, we introduce the class of operators

$$T_\Omega^{(\phi_1 \hookleftarrow \phi_2)}(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - \phi_1(|v|)u, y - \phi_2(|u|)v) \frac{\Omega(u, v)}{|u|^n\, |v|^m} dudv. \tag{3}$$

When ϕ_1 and ϕ_2 are non zero constants, then the operator $T_\Omega^{(\phi_1 \hookleftarrow \phi_2)}$ reduces to the classical operator T_Ω given by

$$T_\Omega(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - u, y - v) \frac{\Omega(u, v)}{|u|^n\, |v|^m} dudv. \tag{4}$$

If Ω satisfies some regularity conditions, R. Fefferman and E. M. Stein showed that the special operator T_Ω is bounded on $L^p(\mathbb{R}^n \times \mathbb{R}^m)$ for all $1 < p < \infty$. Subsequently, J. Duoandikoetxea proved that T_Ω is bounded on L^p $(1 < p < \infty)$ provided that Ω satisfies the weaker condition $\Omega \in L^q(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$, $q > 1$. In 2006, A. Al-Salman, H. Al-Qassem, and Y. Pan showed that T_Ω is bounded on $L^p(\mathbb{R}^n \times \mathbb{R}^m)$ for all $1 < p < \infty$ provided that $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$, i.e.,

$$\int_{\mathbb{S}^{n-1} \times \mathbb{S}^{m-1}} |\Omega(u, v)| \log^2 (2 + |\Omega(u, v)|)\, d\sigma_n(u)\, d\sigma_m(v) < \infty. \tag{5}$$

Here, we remark that $\Omega \in L^q(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1}) \subset L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ for all $q > 1$. Furthermore, the same authors proved that the condition $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ is nearly optimal in the sense that the exponent 2 in $L(\log L)^2$ can not be replaced by any smaller numbers.

We remark here that the general operator $T_\Omega^{(\phi_1 \hookleftarrow \phi_2)}$ arises naturally when considering singular integrals in the form

$$S_{\Omega,b}(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - u, y - v) \frac{b(|u|\,|v|)\Omega(u, v)}{|u|^n\, |v|^m} dudv \tag{6}$$

where b is bounded function. In particular, if $b(r) = r^{-m}\chi_{(1,\infty)}$, then by change of variables, it is radially seen that the operator $S_{\Omega,b}$ reduces to the operator

$$S_{\Omega,,b}(f)(x, y) = p.v. \int_{\mathbb{R}^n \times \mathbb{R}^m} f(x - |v|^{-1} u, y - v) \frac{b(|u|)\Omega(u, v)}{|u|^n\, |v|^m} dudv. \tag{7}$$

with $b(t) = \chi_{(1,\infty)}$. It is shown by A. Al-Salman that if $\varphi(t) = t$ or $\phi(t) = t$, then the corresponding operator may fail to be bounded on L^p for any $1 < p < \infty$. Furthermore,

it is shown that the operator $T_\Omega^{(\phi_1 \hookleftarrow \phi_2)}$ is bounded on L^p for $1 < p < \infty$ provided that $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{m-1})$ and that the functions φ and ϕ belong to the class $\mathcal{F}$ of smooth functions $\Phi : (0, \infty) \to \mathbb{R}$ which satisfy $\Phi(0) = 0$ and the following growth conditions:

$$\sup_{0<t<\infty} t^{-d_\Phi} |\Phi(t)| \le C_1 \text{ and } \inf_{0<t<\infty} t^{2-d_\Phi} \left|\Phi''(t)\right| \ge C_2 \tag{8}$$

for some $d_\Phi \neq 0$ where C_1 and C_2 are positive constants.

In this talk, we are interested in investigating the boundedness of the operator $T_\Omega^{(\phi_1 \hookleftarrow \phi_2)}$ under weak conditions on the kernel Ω. In fact, we shall consider the following question:

Question. *Suppose that $\phi_1, \phi_2 \in \mathcal{F}$. Under what conditions on the kernel function Ω, the corresponding operator $T_\Omega^{(\phi_1 \hookleftarrow \phi_2)}$ is bounded on $L^p(\mathbb{R}^n \times \mathbb{R}^m)$ for some $1 < p < \infty$?*

Lars Becker, University of Bonn

Carleson Operators on Doubling Metric Measure Spaces

Abstract. A classical theorem of Carleson states that the maximally modulated Hilbert transform is bounded on L^2. More famous might be its corollary, the pointwise convergence of Fourier series of functions in L^2. We give a new generalization of Carleson's theorem, in which the Hilbert transform is replaced by a singular integral operator on a doubling metric measure space. Our theorem unifies various generalizations of Carleson's theorem that are already in the literature. One application is pointwise convergence of expansions in certain orthogonal polynomials.

This is joint work with Floris van Doorn, Asgar Jamneshan, Rajula Srivastava and Christoph Thiele.

Deborpita Biswas, Clemson University

Near-Riesz bases in Hilbert Spaces

Abstract. James R. Holub, in one of his papers in 1994, introduced the influential concept of near-Riesz bases as frames which become Riesz bases after removal of finitely many terms. We recently extended his definition of near- Riesz basis to sequences which are not frames. In this talk I will present a characterization of our extended near-Riesz bases in terms of the Fredholmness of their associated synthesis operator. I will also present some perturbation results for our near-Riesz bases.

Ole Christensen, Technical University of Denmark

Frames and Redundancy

Abstract. A frame in a Hilbert space can be considered as an "redundant basis:" every element in the Hilbert space has an expansion in terms of the frame element, but the corresponding coefficients might not be unique. We first give a short introduction to general frame theory, and then discuss redundancy properties for the so-called Carleson frames constructed by Aldroubi et al. in 2016; they have the form $\{T^k\varphi\}_{k=0}^\infty$ for a bounded linear operator on the underlying Hilbert space. We show that such frames have a number of remarkable features that have not been identified for any other frames in the literature. Most importantly, the subfamily obtained by selecting each Nth element from the frame is itself a frame, regardless of the choice of $N \in \mathbf{N}$. Furthermore, the frame property is kept upon removal of an arbitrarily finite number of elements. The new results are joint work with M. Hasannasab, F. M. Philipp, and D. Stoeva.

Jane McDougall, Colorado College

The Rosette Minimal Surface, its deformation to the classical Enneper surface, and a non-Jenkins Serrin Surface

Abstract. A harmonic mapping f is a complex valued univalent harmonic function defined on a region in the complex plane. Rosette harmonic mappings are generalizations of the polynomial harmonic mappings $z + \overline{z}^{n-1}/(n-1)$ through modifying the canonical decomposition with hypergeometric ${}_2F_1$ factors. These mappings were discovered through a fortuitous application of Clunie and Sheil-Small's famous shear construction. For appropriate parameters, the harmonic mapping 'lifts', via the Weierstrass Enneper equations, to a Triply Periodic Minimal Surface (TPMS) known as the Rosette Minimal Surface. We describe the continuous deformation of this TPMS into the classical Enneper surface, using a further generalization of the rosette harmonic mappings. In contrast, the Poisson integral formula, applied to piecewise constant functions on the boundary of the unit disk, frequently lifts to a Jenkins Serrin minimal surface (for which the classical doubly-periodic Schwarz surface is a prototypical example). Poisson extensions of piecewise constant functions have in common with the rosette harmonic mappings that there are arcs of constancy. However we explore examples where the dilatation is not a power function, and the corresponding minimal surface is not a Jenkins Serrin surface.

I would like to thank the Science Division Research and Development fund at Colorado College for their generous support

Itamar Oliveira, University of Birmingham

A phase-space approach to weighted Fourier extension inequalities

Abstract. The goal of the talk is to present a certain *ray bundle representation* of the Fourier extension operator in terms of the Wigner transform to investigate two longstanding conjectures in the restriction theory of the Fourier transform, namely *Stein's* and the *Mizohata-Takeuchi conjecture*. In joint work with Bennett, Gutierrez and Nakamura, we show how Sobolev estimates for the Wigner transform can be converted into certain tomographic bounds for the Fourier extension operator to the paraboloid, which imply weaker variants of these conjectures. We are also able to extend this analysis to a wide class of hypersurfaces, a step that requires finding and understanding a good "geometric" replacement for the classical Wigner transform. Our results do not depend on lower bounds for the Gaussian curvature of these manifolds, which contrasts the intuition behind the classical Fourier restriction conjecture. If time allows, we will make a connection between our results and Flandrin's conjecture in the plane.

References

[1] J. Bennett, S. Gutierrez, S. Nakamura, I. Oliveira, *A phase-space approach to weighted Fourier extension inequalities*, preprint 2024.
[2] J. Bennett, S. Nakamura, I. Oliveira, *Weighted Strichartz estimates for the Schrödinger equation: euclidean versus periodic*, in preparation.

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Carmelo Puliatti, Universitat Autònoma de Barcelona

On Fourier transforms of fractal measures on the parabola

Abstract. I will discuss L^p bounds for the Fourier transform of Borel measures supported on the parabola $\mathbb{P} = \{(x, x^2) : x \in [-1, 1]\}$ satisfying an s-dimensional Frostman condition, $s \in [0, 1]$. These estimates are closely connected to incidence problems of points and lines and, in particular, rely on K. Ren and H. Wang's recent solutions to the Furstenberg set conjecture

in the plane. Additionally, I will talk about related lower bounds for the Hausdorff dimension of the n-fold sum-set $nK = K + \cdots + K$ of a Borel subset K of the parabola.

This is a joint work with T. Orponen and A. Pyörälä.

Hrit Roy, University of Edinburgh

Cone multipliers associated to general convex domains

Abstract. In this talk, we shall discuss variants of Mockenhaupt's cone multiplier in $\mathbb{R}^3$ associated to cones generated by arbitrary planar convex domains Ω. We obtain estimates for general cones described in terms of a fractal dimension κ_Ω introduced by Seeger–Ziesler, which measures the flatness of the boundary $\partial\Omega$. Our argument relies on an L^4 square function estimate for general cones, which is a generalization of the Guth–Wang–Zhang square function estimate for the light cone.

Himani Sharma

Vertical maximal functions on manifolds with ends

Abstract. We consider a class of Riemannian manifolds with ends, denoted by $\mathcal{M}$, which is obtained by taking the connected sum of a finite number of N-dimensional Riemannian manifolds of the form $\mathbb{R}^{n_i} \times \mathcal{M}_i$, where $\mathcal{M}_i$ is a compact manifold, with product metric. An interesting case of these manifolds occurs when the Euclidean dimensions n_i are not equal, making $\mathcal{M}$ a non-doubling space. This talk is based on the $L^p(\mathcal{M})$ boundedness of the vertical maximal function operator $M^{res,\nabla} f(x) := \sup_{t>0} |\sqrt{t}\nabla(1 + t\Delta)^{-m} f(x)|$, where $m \geq 1$ and Δ is the Laplace-Beltrami operator. We show that $M^{res,\nabla}$ is weak type (1,1) and bounded on $L^p(\mathcal{M})$ for $1 < p \leq n^*$ where $n^* = \min_i\{n_i\}$. The techniques we use here come from the papers of Hassell-Sikora and Bailey-Sikora on non-doubling manifolds with ends. We also show Fefferman-Stein inequality for the vector-valued version of the vertical maximal function on $L^p(\mathcal{M})$ for $1 < p < n^*$.

This talk is based on my joint work with Adam Sikora.

References

[1] J. Bailey, A. Sikora, Vertical and horizontal square functions on a class of non-doubling manifolds, *J. Differ. Equations* **358**, (2023), 41–102.
[2] A. Hassell, A. Sikora, Riesz transforms on a class of non-doubling manifolds, *Commun. Partial Differ. Equ.* **44(11)**, (2019), 1072–1099.
[3] H. Sharma, A. Sikora, Vertical maximal functions on manifolds with ends, *J. Evol. Equ.* **24**, 49, 31p, (2024).

Ritika Singhal, Indian Institute of Technology Delhi, India

Paley inequality for the Weyl transform and its applications

Abstract. Our aim is to prove the classical Paley inequality in the context of the Weyl transform. As the Weyl transform maps function spaces to bounded operator, we could prove several versions of this inequality. As for some applications, we prove a version of the Hörmander's multiplier theorem to discuss L^p-L^q boundedness of the Weyl multipliers. We also proved the Pitt's inequality for the Weyl transform. In this talk, the results from the following article will be discussed:

References

[1] Ritika Singhal, and N Shravan Kumar, *Paley inequality for the Weyl transform and its applications*, Forum Mathematicum, 2024, https://doi.org/10.1515/forum-2023-0302

K. N. Sridharan, Indian Institute of Technology Delhi

Orlicz space on Groupoids

Abstract. Let G be a locally compact second countable groupoid with a fixed haar system $\lambda = \{\lambda^u\}_{u \in G^0}$ and (Φ, Ψ) be a complementary pair of N-functions satisfying the Δ_2-condition. We introduce the continuous field of Orlicz space (L_0^Φ, Δ_1), and provide a sufficient condition for the space of continuous sections vanishing at infinity E_0^Φ, to be an algebra under a suitable convolution. The condition for a closed $C_b(G^0)$-submodule I of E_0^Φ to be a left ideal is established. We provide a groupoid version of the result that characterizes the space of convolutors of Morse-Transue space for locally compact groups.

This is a joint work with Dr. N. Shravan Kumar.

Ferenc Weisz, Eötvös Loránd University

Hardy-Littlewood-type theorems for higher dimensional Fourier transforms

Abstract. We obtain Fourier inequalities in the weighted L_p spaces for any $1 < p < \infty$ involving the Hardy-Cesàro and Hardy-Bellman operators. We extend these results to product Hardy spaces for $p \leq 1$. Moreover, boundedness of the Hardy-Cesàro and Hardy-Bellman operators in various spaces (Lebesgue, Hardy, BMO) is discussed.