SCHEDULE

TITLES AND ABSTRACTS FOR PLENARY AND SEMI-PLENARY SPEAKERS

	Monday	Tuesday	Wednesday	Thursday	Friday	
08:30	Registration (Woolf)					
09:00		**Erdős** (Woolf LT)	**Kurasov** (Woolf LT)	**Freitag** (Woolf LT)	**Kaltenbacher** (Woolf LT)	9:00
09:15	Opening (Woolf LT)					
10:00	**Kutyniok** (Woolf LT)	**Pramanik** (Woolf LT)	**Embree** (Woolf LT)	**Haragus** (Woolf LT)	**Gaubert** (Woolf LT)	10:00
11:00	Coffee	Coffee	Coffee	Coffee	Coffee	11:00
11:40	**Claeys** (SIBLT3) **Gross** (SIBLT1) **Lee** (CHLT)	**Bhat** (SIBLT3) **Laurent** (SIBLT1) **Martin** (CHLT)	S1, S2, S3, S5 S6, S8, S10, S11 S14, S15, S16, S21 S24	**Chandler-Wilde** (SIBLT3) **ter Horst** (SIBLT1) **Tylli** (CHLT)	**Boegli** (SIBLT3) **Dopico** (SIBLT1) **Mashreghi** (CHLT)	11:40
12:40	Lunch	Lunch	Lunch	Lunch	Lunch	12:40
14:00	S2, S4, S6 S8, S9, S11 S12, S17, S18 S20, S21, S23	S3, S4, S6 S7, S8, S9 S10, S12, S13 S17, S19, S21	S1, S2, S3, S5, S8, S10 S14, S15 S16, S24	S2, S3, S4 S5, S10, S13 S15, S16, S19 S20, S21	S10, S15 S20, S24	14:00
15:00						15:00
16:00	Coffee	Coffee	Canterbury Walk	Coffee	Closing	16:00
16:30	S1, S2, S4 S6, S7, S8 S9, S12, S17 S21, S23	S1, S3, S5, S6 S8, S9, S11 S14, S15, S18 S20, S21, S22		S2, S3, S4 S5, S10, S13 S15, S16, S19 S23, S24	Coffee	16:30
						17:30
18:30						18:30
19:00				Drinks Reception		19:00
19:30				Conference Dinner		19:30

Monday

9:15-9:55	Opening: Ian Wood (Main Organizer) J. William Helton (Chair of the IWOTA Steering Committee) Shane Weller (Deputy-Vice Chancellor for Research and Innovation)		
10:00-11:00	Gitta Kutyniok *An Operator Theoretical Perspective on Reliable AI*	(Woolf LT)	chair: Felix Schwenninger
11:40-12:40	Tom Claeys *Fredholm determinants and determinantal point processes*	(SIBLT3)	chair: Ana Loureiro
	David Gross *Quantum network correlations and polynomial optimization over states*	(SIBLT1)	chair: Igor Klep
	Woo Young Lee *Backward-shift invariant subspaces*	(CHLT)	chair: Raul Curto

0.0.1. *Abstracts.*

Gitta Kutyniok, LMU Munich

An Operator Theoretical Perspective on Reliable AI

Abstract. The new wave of artificial intelligence is impacting industry, public life, and the sciences in an unprecedented manner. In industrial and applied mathematics, it has by now already led to paradigm changes in several areas. A particular emphasis is on graph data due to the importance of application settings such as recommender systems, social media, or molecular dynamics. However, one current major drawback is the lack of reliability as well as the enormous energy problem.

The goal of this lecture is to first provide an introduction into this new vibrant research area. We will then survey recent advances from an operator theoretical perspective, in particular, concerning performance guarantees and explainability methods, which are key to ensure reliability. Finally, we will discuss fundamental limitations in terms of computability, which seriously affect diverse aspects of reliability, and reveal a surprising connection to novel computing approaches such as neuromorphic computing, which also bear the potential to ensure sustainability of future AI systems.

Tom Claeys, UCLouvain

Fredholm determinants and determinantal point processes

Abstract. Determinantal point processes are point processes with a specific determinantal structure arising for instance in random matrix theory, tiling models, and random growth models. The Laplace functionals of such point processes can be written as Fredholm series. I will explain how this can be used to study asymptotic behavior in determinantal point processes and in particular to obtain large deviation type estimates. I will illustrate these general considerations with concrete examples related to random matrices and polymer models.

David Gross, University of Cologne

Quantum network correlations and polynomial optimization over states

Abstract. The problem of deciding whether observed correlations are compatible with a quantum model has been extensively studied and is now well-understood. In particular, in the "commuting operator model" of locality, there is a family of semi-definite programs (the *NPA hierarchy*), whose feasible region converges to the set of compatible correlations from the outside. We are interested in analogous results for scenarios that have a non-trivial "network" or "causal" structure. The most basic example is the *bilocal scenario*, in which two independent bipartite quantum systems are distributed among three observers, one pair between Alice and Bob, and the other pair between Bob and Charlie. The independence condition between the two pairs gives rise to polynomial constraints in the global quantum state, which previous techniques could not handle. Addressing this issue, we have developed a convergent hierarchy of SDPs for the problem of optimizing over the state space of a universal C^*-algebra given by generators and relations, subject to polynomial constraints in the state. (The construction is based on the *quantum inflation technique* by Wolfe et al. and the convergence proof makes use of the *quantum de Finetti theorem* as stated by Raggio and Werner. Simultaneously, Klep, Magron, Volčič, and Wang laid out a different approach under the name of *state polynomial optimization*). The method achieves our goal for the quantum bilocal scenario in the sense that it gives rise to a family of SDPs whose feasible region converges to the set of compatible correlations from the outside. The situation is more complex for general causal scenarios, for which I will state partial results and open questions.

References

[1] L.T. Ligthart, M. Gachechiladze, D. Gross, A convergent inflation hierarchy for quantum causal structures, *Comm. Math. Phys.* **401**, (2003), 2673-2714.
[2] L.T. Ligthart, D. Gross, The inflation hierarchy and the polarization hierarchy are complete for the quantum bilocal scenario, *J. Math. Phys.* **64**, (2023), 072201.

Woo Young Lee, Korea Institute for Advanced Study

Backward-shift invariant subspaces

Abstract. Let $S_{\mathbb{C}^n}$ be the shift operator on $\mathbb{C}^n$-valued Hardy space $H^2_{\mathbb{C}^n}$, i.e., $(S_{\mathbb{C}^n} f)(z) = zf(z)$ for each $f \in H^2_{\mathbb{C}^n}$. The Beurling-Lax Theorem states that every invariant subspace of $S_{\mathbb{C}^n}$ is of the form $\Delta H^2_{\mathbb{C}^r}$ for some $n \times r$ inner matrix function Δ. Equivalently, every invariant subspace of the backward shift operator $S^*_{\mathbb{C}^n}$ is of the form $\mathcal{H}(\Delta) := H^2_{\mathbb{C}^n} \ominus \Delta H^2_{\mathbb{C}^r}$, which is often called a model space. Thus, for a subset F of $H^2_{\mathbb{C}^n}$, if E^*_F denotes the smallest $S^*_{\mathbb{C}^n}$-invariant subspace containing F, i.e., $E^*_F := \bigvee\{(S^*_{\mathbb{C}^n})^i F : i \geq 0\}$, then $E^*_F = \mathcal{H}(\Delta)$ for some inner matrix function Δ. In this context, for a given inner matrix function Δ, we may ask: What is the smallest number of vectors in F satisfying $E^*_F = \mathcal{H}(\Delta)$? This question is closely related to a decomposition problem of matrix-valued L^2-functions. In this talk, we give a canonical decomposition of matrix-valued L^2-functions, which reduces to the Douglas-Shapiro-Shields factorization for a special case of bounded type functions. This idea invites a new notion of the "Beurling degree" of inner matrix functions. Eventually, we answer the above question in terms of the Beurling degree. The talk is based on the recent research with Raúl E. Curto and In Sung Hwang. Korea

Institute for Advanced Study, Seoul, 02455, Korea, E-mail address: wylee@snu.ac.kr

Tuesday

9:00-10:00	Laszlo Erdős *Universality phenomena for random matrices*	(Woolf LT) chair: Bill Helton
10:00-11:00	Malabika Pramanik *Circles, projections and operators*	(Woolf LT) chair: Alan Chang
11:40-12:40	V Rajarama Bhat *Dilation theory of quantum dynamical maps*	(SIBLT3) chair: Baruch Solel
	Monique Laurent *Performance Analysis of Sums of Squares Approximations in Polynomial Optimization*	(SIBLT1) chair: Hugo Woerdeman
	Rob Martin *Operator realizations of non-commutative functions*	(CHLT) chair: Nicholas Young

0.0.2. *Abstracts.*

Laszlo Erdős, IST Austria

Universality phenomena for random matrices

Abstract. Large random matrices tend to exhibit universal fluctuations. Beyond the well-known Wigner-Dyson and Tracy-Widom eigenvalue distributions, we overview other universality results for Hermitian and non-Hermitian matrices. We discuss the emergence of normal distribution involving eigenvectors, especially the random matrix version of quantum unique ergodicity. We also explain why results on non-Hermitian random matrices are much harder than their Hermitian counterparts and highlight our new methods to tackle them.

Supported by ERC Advanced Grants *RANMAT* and *RMTBeyond*.

Malabika Pramanik, University of British Columbia

Circles, projections and operators

Abstract. How large can a planar set be if it contains a circle of every radius? This is the quintessential example of a curvilinear Kakeya problem, central to many areas of harmonic analysis and incidence geometry.

Large sets in Euclidean space should have large projections in most directions. Projection theorems in geometric measure theory make this intuition precise, by quantifying the words "large" and "most". What do circles and projections have in common?

The talk will survey a few landmark results in these areas and point to a newly discovered connection between the two, in the form of a geometric maximal operator.

B V Rajarama Bhat, Indian Statistical Institute, Bangalore

Dilation theory of quantum dynamical maps

Abstract. It is a recurring theme in mathematics to view complicated structures as parts or components of larger but simpler looking objects. Sz. Nagy dilation of contractions on Hilbert spaces to unitaries is a great example of the same. Here we dilate unital completely positive maps on C^*-algebras to unital endomorphisms of larger algebras. Not only such dilations exist, but also there exist a *minimal dilation* which is unique up to unitary isomorphism ([2-3]).

The theory can also be developed in a von Neumann algebra setting and it can be extended to one parameter semigroups of unital completely positive maps. Then the dilations are one parameter semigroups of endomorphisms or E-semigroups in the sense of Arveson [1] and Powers. It is a useful tool for constructing E_0-semigroups of type I factors [8]. It has applications in the classification theory of type II factors [7].

In the context of quantum theory of open systems the dilation presented above amounts to a construction of a quantum Markov process of a given quantum dynamical semigroup [4]. Recently it has been demonstrated that non-commutative peripheral Poisson boundaries admit very simple descriptions through dilation theory [6].

References

[1] W. Arveson, *Noncommutative dynamics and E-semigroups.* Springer, (2003). [2] B. V. Rajarama Bhat, An index theory for quantum dynamical semigroups, *Trans. Amer. Math. Soc.* **3**48(2), (1996), 561–583.
[3] –, Minimal dilations of quantum dynamical semigroups to semigroups of endomorphisms of C^*-algebras, *J. Ramanujan Math. Soc.* **1**4(2), (1999), 109-124.
[4] B. V. Rajarama Bhat and K. R. Parthasarathy, Markov dilations of nonconservative dynamical semigroups and a quantum boundary theory, *Ann. Inst. H. Poincaré Probab. Statist.* , **3**1(4), (1995) 601–651.
[5] B. V. Rajarama Bhat and M. Skeide, Tensor product systems of Hilbert modules and dilations of completely positive semigroups, *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* **3**(4), (2000), 519–575.
[6] B. V. Rajarama Bhat, S. Kar and B. Talwar, Peripheral Poisson boundary, https://arxiv.org/abs/2209.07731 (To appear in the Israel J. of Math.)
[7] S. Das and J. Peterson, Poisson boundaries of II_1 factors, Compositio Mathematica, 158(8), (2022),1746–1776.
[8] R.T. Powers, Construction of E_0-semigroups of $\mathscr{B}(\mathcal{H})$ from CP-flows, in: Advances in Quantum Dynamics, in: Contemp. Math., vol. 335, AMS, Providence, RI, (2003), 57–97.

I would like to thank SERB(India) for their generous support through J C Bose Fellowship.

Monique Laurent, Centrum Wiskunde & Informatica (CWI) Amsterdam / Tilburg University, The Netherlands

Performance Analysis of Sums of Squares Approximations in Polynomial Optimization

Abstract. We discuss hierarchies of approximations for polynomial optimization, based on using sums of squares of polynomials as a tractable proxy to certify polynomial positivity. Our focus will be on analyzing the quality of these approximations as a function of the degrees of the involved sums of squares. We will present recent state-of-the-art results for polynomial optimization over various classes of semi-algebraic sets and some of the main techniques used to obtain these results, which include Fourier analysis, reproducing kernels, and extremal roots of orthogonal polynomials.

Rob Martin, University of Manitoba

Operator realizations of non-commutative functions

Abstract. Realization theory is powerful tool in non-commutative (NC) function theory. Here, a *realization* is a triple, (A, b, c), consisting of a $d-$tuple, $A = (A_1, \cdots, A_d)$, of bounded linear operators on a separable, complex Hilbert space, $\mathcal{H}$, and vectors $b, c \in \mathcal{H}$. Any such realization defines a (uniformly analytic) NC function in an open neighbourhood of the origin, $0 = (0, \cdots, 0)$, of the *NC universe* of $d-$tuples of square matrices of any fixed size, via the formula $h(X) = I_n \otimes b^*(I_n \otimes I_{\mathcal{H}} - \sum X_j \otimes A_j)^{-1} I_n \otimes c$.

It is well-known that an NC function has a finite–dimensional realization if and only if it is a non-commutative rational function that is defined at 0. Such finite realizations contain valuable information about the NC rational functions they generate. By considering more general, infinite–dimensional realizations we study, construct and characterize more general classes of NC functions. In particular, we show that an NC function is (uniformly) entire, if and only if it has a jointly compact and quasinilpotent realization. Restricting our results to one-variable shows that an analytic Taylor series extends to an entire or meromorphic function on the complex plane if and only if it has a realization whose component operator is compact and quasinilpotent, or compact, respectively. This then motivates our definition of the set of global uniformly meromorphic NC functions as the (universal) skew field (of fractions) generated by NC rational expressions in the (semi-free ideal) ring of NC functions with jointly compact realizations.

This is joint work with Méric L. Augat (James Madison) and Eli Shamovich (Ben-Gurion).

Wednesday

9:00-10:00	Pavel Kurasov *From metric graphs to Fourier quasicrystals*	(Woolf LT) chair: Ian Wood
10:00-11:00	Mark Embree *Contour Integral Eigensolvers through the Lens of System Identification*	(Woolf LT) chair: Sanne Ter Horst

0.0.3. *Abstracts.*

Pavel Kurasov, Stockholm University

From metric graphs to Fourier quasicrystals

Abstract. Spectra of Laplacians on metric graphs have been intensively studies in recent years due to possible applications in nano-physics. Consider finite metric graphs formed from compact intervals e_n, $n = 1, 2, \ldots, N$, of lengths ℓ_n and Laplace operators $-\frac{d^2}{dx^2}$ with standard vertex conditions:

- the function is continuous at the vertex (continuity condition);
- the sum of outgoing first derivates at the vertex is equal to zero (Kirchhoff condition).

One of the most interesting results in the area is the trace formula connecting the spectrum $\lambda_j = k_j^2$ to geometric and topologic properties of the metric graph [10,3,5,4]:

$$\underbrace{2\delta(k) + \sum_{k_n \neq 0} (\delta(k - k_n) + \delta(k + k_n))}_{\text{spectral information}} = \underbrace{\underbrace{(1 - \beta_1)}_{= \chi} \delta(k) + \frac{\mathcal{L}}{\pi} + \frac{1}{\pi} \sum_{p \in \mathcal{P}} l(\text{prim}\,(p)) S_{\text{v}}(p) \cos k\ell(p)}_{\text{geometric/topologic information}}$$

where

- $\mathcal{L} = \sum_{n=1}^{N} \ell_n$ - the total length of the graph;
- χ – Euler characteristic of Γ;
- β_1 – number of independent cycles in Γ;
- $\mathcal{P}$ – the set of closed oriented paths p on Γ;
- $\ell(p)$ – length of the closed path p;
- $S_{\text{v}}(p)$ – product of all vertex scattering coefficients along the path p.

This formula can be seen as a direct generalisation of the classical Poisson summation formula and coincides with it if the graph is just one interval with Neumann conditions at the end points. In contrast to Chazarian-Duistermaat-Guilemin-Melrose trace formulas the obtained formula is exact.

It appears that this formula is extremely interesting for Fourier analysis since it provides explicit examples of crystalline measure, which can be defined following Y. Meyer as [8]:

A discrete measure μ is **crystalline** *if it is a tempered distribution and if the measure itself and its Fourier transform $\hat{\mu}$ are sums of delta functions with discrete supports:*

$$\mu = \sum_{\lambda \in \Lambda} a_\lambda \delta_\lambda \qquad \hat{\mu} = \sum_{s \in S} b_s \delta_s.$$

Collecting all delta functions to the left hand side the trace formula can be written as:

$$(2 - \chi)\delta(k) + \sum_{k_n \neq 0} (\delta(k - k_n) + \delta(k + k_n)) = \frac{\mathcal{L}}{\pi} + \frac{1}{\pi} \sum_{p \in \mathcal{P}} l(\text{prim}\,(p)) S_{\text{v}}(p) \cos k\ell(p).$$

One may obtain similar summation formulas starting from multivariate stable polynomials [6]. The supports of the corresponding measures are described as zeroes of trigonometric polynomials. If the multivariate polynomials in addition are symmetric, *i.e.* invariant under involution $z_j \mapsto 1/z_j$, then the trigonometric polynomials have only real zeroes and the corresponding measures are crystalline measures. It appears that all one-dimensional crystalline measures are given by real rooted trigonometric polynomials [9]. It was proven recently that all such polynomials can be obtained using our construction via multivariate stable polynomials [1]. Spectral properties of Laplacians on metric graphs are further described in [7,4].

This is a joint work with Peter Sarnak.

Described approach is generalised in collaboration with L. Alon, M. Kummer, and C. Vinzant to obtaine crystalline measures in several dimensions.

References

[1] L. Alon, A. Cohen, C. Vinzant, Every real-rooted exponential polynomial is the restriction of a Lee-Yang polynomial, preprint arXiv:2303.03201.
[2] L. Alon, M. Kummer, P. Kurasov, C. Vinzant, Higher Dimensional Fourier Quasicrystals from Lee-Yang Varieties, preprint arXiv:2407.11184.
[3] B. Gutkin, U. Smilansky, Can one hear the shape of a graph?, J. Phys. A **34** (2001), no. 31, 6061Ð6068.
[4] P. Kurasov, *Spectral geometry of graphs*, Birkhäuser (2024) (Operator Theory Advances and Applications, **293**).
[5] P. Kurasov, M. Nowaczyk, Inverse spectral problem for quantum graphs, J. Phys. A **38** (2005), no. 22, 4901Ð4915.
[6] P. Kurasov, P. Sarnak, Stable polynomials and crystalline measures, J. Math. Phys. *61* (2020), no. 8, 083501.
[7] P. Kurasov, P. Sarnak, The additive structure of the spectrum of a Laplacian on a metric graph, preprint (2024).
[8] Y. Meyer, Measures with locally finite support and spectrum, Proc. Natl. Acad. Sci. USA **113** (2016), no. 12, 3152Ð3158.
[9] A. Olevskii, A. Ulanovskii, Fourier quasicrystals with unit masses, C. R. Math. Acad. Sci. Paris **358** (2020), no. 11-12, 1207Ð1211.
[10] J.-P. Roth, Le spectre du laplacien sur un graphe (French), Théorie du potentiel (Orsay, 1983), 521Ð539, Lecture Notes in Math., **1096**, Springer, Berlin, 1984.

Supported by the Swedish Research Council.

Mark Embree, Virginia Tech

Contour Integral Eigensolvers through the Lens of System Identification

Abstract. In the past two decades contour integral methods have emerged as an intriguing technology for numerically computing eigenvalues within some target region of the complex plane. These algorithms are remarkably flexible, applying to self-adjoint and non-self-adjoint operators, pencils, and (thanks to a classical theorem of Keldysh) even to nonlinear eigenvalue problems. These methods, such as the Sakurai–Sugiura algorithm, typically reduce the problem to an equivalent low-dimensional Hankel matrix pencil, whose elements are matrix moments that are computed approximately with quadrature. We will show how such methods can be seen as Ho-Kalman system identification problems with noisy data. This connection to systems theory suggests new algorithms that leverage modern system identification tools based on the Loewner framework for rational interpolation [1]. In theory, these approaches yield equivalent matrix pencils with the exact desired eigenvalues – *provided the data is exact*. Yet these

spectrally equivalent pencils can have starkly different perturbation properties, giving different degrees of sensitivity when operating on the inexact data that is inevitable in practical problems. We will frame this discussion in terms of pseudospectra of matrix pencils [2] and the location of Loewner interpolation points [3], and demonstrate use of these contour methods to solve nonlinear eigenvalue problems arising from networks of vibrating strings [4].

References

[1] M.C. Brennan, M. Embree, S. Gugercin, Contour integral methods for nonlinear eigenvalue problems: a systems theoretic approach, *SIAM Review* **65** (2023) 439–470.

[2] M. Embree, A.C. Ionita, Pseudospectra of Loewner matrix pencils, in *Realization and Model Reduction of Dynamical Systems: A Festschrift in Honor of the 70th Birthday of Thanos Antoulas*, Springer, Cham, 2022, pp. 59–78.

[3] N.A. Garcia Hilares, *Mathematical modeling and dynamic recovery of power systems*, PhD dissertation, Virginia Tech, 2023.

[4] J.P. Baker, *Vibrations of mechanical structures: source localization and nonlinear eigenvalue problems for mode calculation*, PhD dissertation, Virginia Tech, 2023.

I would like to thank the US National Science Foundation for their support (DMS-1720257).

Thursday

9:00-10:00	Melina Freitag *Solving parameter dependent eigenvalue problems using Taylor series and Chebyshev expansion*	(Woolf LT) chair: Marco Marletta
10:00-11:00	Mariana Haragus *Spectral and semigroup methods for hydrodynamic and optical waves*	(Woolf LT) chair: Marco Marletta
11:40-12:40	Simon Chandler-Wilde *On Spectral Inclusion Sets and Computing the Spectra and Pseudospectra of Bounded Linear Operators*	(SIBLT3) chair: Lyonell Boulton
	Sanne ter Horst *Unbounded Toeplitz operators with rational symbols*	(SIBLT1) chair: Andre Ran
	Hans-Olav Tylli *Closed ideals and subideals associated to the algebra of compact-by-approximable operators*	(CHLT) chair: Niels Laustsen

0.0.4. *Abstracts.*

Melina Freitag, University of Potsdam

Solving parameter dependent eigenvalue problems using Taylor series and Chebyshev expansions

Abstract. We discuss two approaches to solving the parametric (or stochastic) eigenvalue problem. One of them uses a Taylor expansion and the other a Chebyshev expansion. The parametric eigenvalue problem assumes that the matrix A depends on a parameter μ, where μ might be a random variable. Consequently, the eigenvalues and eigenvectors are also functions of μ. We compute a Taylor approximation of these functions about μ_0 by iteratively computing the Taylor coefficients. The complexity of this approach is $\mathcal{O}(n^3)$ for all eigenpairs, if the derivatives of $A(\mu)$ at μ_0 are given. The Chebyshev expansion works similarly. We first find an initial approximation iteratively which we then refine with Newton's method. This second method is more expensive but provides a good approximation over the whole interval of the expansion instead around a single point. We present numerical experiments confirming the complexity and demonstrating that the approaches are capable of tracking eigenvalues at intersection points. Further experiments shed light on the limitations of the Taylor expansion approach with respect to the distance from the expansion point μ_0.

References

[1] M.A. Freitag, T. Mach, Solving Parameter Dependent Eigenvalue Problems Using Taylor Series and Chebyshev Expansions, `https://doi.org/10.48550/arXiv.2302.03661`

Mariana Haragus, University of Franche-Comté, France

Spectral and semigroup methods for hydrodynamic and optical waves

Abstract. Motivated by two concrete examples from optics and hydrodynamics we present some general results for linear differential operators with periodic coefficients. These examples share the property that the underlying mathematical models possess stationary periodic solutions. We are interested in the stability of these periodic solutions under spatially localized perturbations.

The first example is a nonlinear Schrödinger equation with damping and forcing that arises in nonlinear optics. The linear operator obtained by linearizing the equation at a stationary periodic solution is a matrix differential operator with periodic coefficients. We discuss its spectral

properties and analyze the decay of the evolution semigroup. The result for the semigroup holds for differential operators with periodic coefficients under some rather general hypotheses.

The second example is the classical water-wave problem in hydrodynamics. Linear operators arising in the study of water waves typically have a product structure JL in which J and L are respectively skew- and self-adjoint operators. Classical counting results show that, under suitable conditions, the number of unstable eigenvalues of the operator JL is bounded by the number of nonpositive eigenvalues of the self-adjoint operator L. We extend these results by showing that the operator L can be replaced by another self-adjoint operator K, provided the operators JL and JK commute.

This talk is based on joint works with Mat Johnson, Wesley Perkins, Björn de Rijk, Jin Li and Dmitry Pelinovsky.

Simon Chandler-Wilde, University of Reading

On Spectral Inclusion Sets and Computing the Spectra and Pseudospectra of Bounded Linear Operators

Abstract. In this talk, based substantially on our recent paper [2], we derive novel families of inclusion sets for the spectrum and pseudospectrum of large classes of bounded linear operators, and establish convergence of particular sequences of these inclusion sets to the spectrum or pseudospectrum, as appropriate. Our results apply, in particular, to bounded linear operators on a separable Hilbert space that, with respect to some orthonormal basis, have a representation as a bi-infinite matrix that is banded or band-dominated. More generally, our results apply in cases where the matrix entries themselves are bounded linear operators on some Banach space. In the scalar matrix entry case we show that our methods, given the input information we assume, lead to a sequence of approximations to the spectrum, each element of which can be computed in finitely many arithmetic operations, so that, with our assumed inputs, the problem of determining the spectrum of a band-dominated operator has solvability complexity index one, in the sense of Ben-Artzi et al. [1]. As a concrete application, we apply our methods to the determination of the spectra of non-self-adjoint bi-infinite tridiagonal matrices that are pseudoergodic in the sense of Davies [3].

References

[1] J. Ben-Artzi, M. J. Colbrook, A. C. Hansen, O. Nevanlinna, M. Seidel, Can everything be computed? - On the Solvability Complexity Index and Towers of Algorithms, arXiv:1508.03280, 2020.

[2] S. N. Chandler-Wilde, R. Chonchaiya, M. Lindner, On spectral inclusion sets and computing the spectra and pseudospectra of bounded linear operators, *J. Spectr. Theory* **14**, (2024), 719–804.

[3] E. B. Davies, Spectral theory of pseudo-ergodic operators, *Commun. Math. Phys.* **216**, (2001), 687–704.

Sanne ter Horst, North-West University

Unbounded Toeplitz operators with rational symbols

Abstract. Although there are some earlier sources on Unbounded Toeplitz operators, starting in the 1950s and 1960s, it was not until the 2008 paper by Sarason [3], in which he studies a class of unbounded Toeplitz operators in the context of truncated Toeplitz operators, that the topics gained in popularity. We begin with a brief overview of some types of unbounded Toeplitz operators that have appeared in the recent literature, after which we shall focus on a class of unbounded Toeplitz operators with rational symbols that have poles on the unit circle [1]. In that case, more concrete results can be obtained regarding the spectrum and Fredholm theory of such operators. If time permits, the case of rational matrix symbols will be discussed, in which further challenges arise [2].

References

[1]G.J. Groenewald, S. ter Horst, J. Jaftha and A.C.M. Ran, A Toeplitz-like operator with rational symbol having poles on the unit circle I: Fredholm properties, *Oper. Theory Adv. Appl.* **271** (2018), 239–268.
[2] G.J. Groenewald, S. ter Horst, J. Jaftha and A.C.M. Ran, A Toeplitz-like operator with rational matrix symbol having poles on the unit circle: Fredholm properties, *Complex Anal. Oper. Theory* **15** (2021), article 1.
[3] D. Sarason, Unbounded Toeplitz operators, *Integr. Equ. Oper. Theory* **61** (2008), 281–298.

Hans-Olav Tylli, University of Helsinki

Closed ideals and subideals associated to the algebra of compact-by-approximable operators

Abstract. I will describe recent work [1] with Henrik Wirzenius (Tampere), where we construct explicit non-trivial closed ideals $\mathcal{A}(X) \subset \mathcal{J} \subset \mathcal{K}(X)$ for various Banach spaces X that fail to have the approximation property. Here $\mathcal{K}(X)$ is the Banach algebra of compact operators on X, and the ideal of the approximable operators $\mathcal{A}(X) := \overline{\mathcal{F}(X)}$ is the uniform closure of the bounded finite rank operators $\mathcal{F}(X)$. Our work was motivated by longstanding problems about the structure of the elusive compact-by-approximable algebra $\mathcal{K}(X)/\mathcal{A}(X)$, and it complements current work on closed ideals of the bounded operators $\mathcal{L}(Z)$ for various classical Banach spaces Z.

Results include a Banach space Z together with an uncountable lattice $\{\mathcal{J}_\alpha : \alpha \in \Lambda\}$ of closed ideals of $\mathcal{K}(Z)$, which are not ideals of the algebra $\mathcal{L}(Z)$ of bounded operators. This family has the unexpected property that $\mathcal{J}_\alpha$ and $\mathcal{J}_\beta$ are isomorphic as Banach algebras whenever $\alpha \neq \beta$, which is not possible for closed ideals of $\mathcal{L}(Z)$. Time permitting I will describe further examples from [2] of non-trivial closed subideals of $\mathcal{L}(X)$. (Here $\mathcal{J}$ is a non-trivial closed subideal of $\mathcal{L}(X)$ if $\mathcal{J} \subset \mathcal{I} \subset \mathcal{L}(X)$, where $\mathcal{J}$ is a closed ideal of $\mathcal{I}$ and $\mathcal{I}$ is a closed ideal of $\mathcal{L}(X)$, but J is not an ideal of $\mathcal{L}(X)$.)

References

[1] Hans-Olav Tylli, Henrik Wirzenius: Closed ideals in the algebra of compact-by-approximable operators, *J. Funct. Anal.* **282** (2022), Paper No. 109328
[2] Hans-Olav Tylli, Henrik Wirzenius: Exotic closed subideals of algebras of bounded operators, *Proc. Amer. Math. Soc.* **152** (2024), 987–1002.

Friday

9:00-10:00	Barbara Kaltenbacher *Inverse problems for fractional PDEs*	(Woolf LT) chair: Irene Sabadini
10:00-11:00	Stéphane Gaubert *Tropical geometry, operators and games*	(Woolf LT) chair: Bas Lemmens
11:40-12:40	Sabine Bögli *On the discrete eigenvalues of Schrödinger operators with complex potentials*	(SIBLT3) chair: Christiane Tretter
	Froilán Dopico *Polynomial and rational matrices with prescribed data*	(SIBLT1) chair: Christian Mehl
	Javad Mashreghi *Linear polynomial approximation schemes*	(CHLT) chair: Marek Ptak

0.0.5. *Abstracts.*

Barbara Kaltenbacher, University of Klagenfurt

Inverse problems for fractional PDEs

Abstract. This talk is based on recent joint work with William Rundell, Texas A&M University, USA, cf. [1,2,3]. In particular, we plan to focus on two prototypical problems in which the order of the fractional time derivative in the governing PDE has a crucial influence on the degree of ill-posedness of the inverse problem under consideration. In the first one, this effect will be exploited for designing a regularization method with the time fractional differentiation order as a regularization parameter; in the second one, the order of the fractional derivative is not at our disposal but imposed by the physical model. Modelling with fractional derivatives, in particular in ultrasonics, will be therefore be topic of this talk as well. If time permits, some selected further inverse problems for fractional PDEs will be highlighted.

References

[1] Barbara Kaltenbacher and William Rundell. Regularization of a backwards parabolic equation by fractional operators. *Inverse Problems and Imaging*, 13(2):401–430, 2019.
[2] Barbara Kaltenbacher and William Rundell. Some inverse problems for wave equations with fractional derivative attenuation. *Inverse Problems*, 37(4):045002, 2021.
[3] Barbara Kaltenbacher and William Rundell. Inverse Problems for Fractional Partial Differential Equations. Graduate Texts in Mathematics. AMS, 2023

Stéphane Gaubert, Inria and CMAP, École polytechnique

Tropical geometry, operators and games

Abstract. Tropical convex sets arise as log-limits of parametric families of convex sets, or as images by a non-archimedean valuation of convex sets over order non-archimedean fields. This sets up a correspondence between several classes of zero-sum repeated games that have an unsettled complexity, and several classes of convex semialgebraic programs. This has unexpected consequences, leading to a proof that interior point methods in linear programming are not strongly polynomial. An operator approach, studying zero-sum games by means of nonexpansive and order preserving mappings (dynamic programming operators), plays an essential role here. We will provide an introductive survey of these developments which combine methods from tropical geometry, non-linear Perron-Frobenius theory and game theory.

References

[1] Xavier Allamigeon, Pascal Benchimol, Stéphane Gaubert, and Michael Joswig, What Tropical Geometry Tells Us about the Complexity of Linear Programming, *SIAM Rev.*, **63**(1), 123–164, 2021
[2] Xavier Allamigeon, Stéphane Gaubert and Nicolas Vandame, No self-concordant barrier interior point method is strongly polynomial, *Proceedings of STOC 2022: 54th Annual ACM Symposium on Theory of Computing.*
[3] Marianne Akian, Stéphane Gaubert and Antoine Hochart, A game theory approach to the existence and uniqueness of nonlinear Perron-Frobenius eigenvectors, *Discrete & Continuous Dynnamical Systems - A*, **40**:207, 2020

Sabine Bögli, Durham University

On the discrete eigenvalues of Schrödinger operators with complex potentials

Abstract. In this talk I shall present constructions of Schrödinger operators with complex-valued potentials whose spectra exhibit interesting properties. One example shows that for sufficiently large p, the discrete eigenvalues need not be bounded in modulus by the L^p norm of the potential. This is a counterexample to the Laptev–Safronov conjecture (Comm. Math. Phys. 2009). Another construction proves optimality (in some sense) of generalisations of Lieb–Thirring inequalities to the non-selfadjoint case - thus giving us information about the accumulation rate of the discrete eigenvalues to the essential spectrum. This talk is based on joint works with Jean-Claude Cuenin (Loughborough) and František Štampach (Prague).

Froilán Dopico, Universidad Carlos III de Madrid, Spain

Polynomial and rational matrices with prescribed data

Abstract. We study necessary and sufficient conditions for the existence of polynomial and rational matrices with different prescribed data. First, we consider the problem for polynomial matrices when the size, degree, rank, invariant factors, infinite elementary divisors, and the minimal indices of their left and right null spaces are prescribed and prove that a polynomial matrix with these data exists if and only if these data satisfy a surprisingly simple unique condition related to a fundamental constraint known as the "index sum theorem". In the second place, we extend this result to rational matrices when the size, rank, invariant rational functions, invariant orders at infinity, and minimal indices of their left and right null spaces are prescribed. The data prescribed so far are called in the literature the *complete structural data* or the *complete eigenstructure* of the polynomial or rational matrix. Finally, in addition to the complete eigenstructure, we also prescribe the minimal indices of the row and column spaces and show that the simple condition found in the previous problems must be completed with a majorization relation among the involved indices. The results presented in this talk are based on the references [1], [3] and the work still in preparation [2].

References

[1] L. M. Anguas, F. M. Dopico, R. Hollister, D. S. Mackey, Van Dooren's index sum theorem and rational matrices with prescribed structural data, *SIAM J. Matrix Anal. Appl.* **40**, (2019), 720–738.
[2] I. Baragaña, F. M. Dopico, S. Marcaida, A. Roca, Polynomial and rational matrices with the invariant rational functions and the four sequences of minimal indices prescribed, in preparation.
[3] F. De Terán, F. M. Dopico, P. Van Dooren, Matrix polynomials with completely prescribed eigenstructure, *SIAM J. Matrix Anal. Appl.* **36**, (2015), 302–328.

This work has been partially supported by the Agencia Estatal de Investigación of Spain MCIN/AEI/10.13039/501100011033/ through grants PID2019-106362GB-I00 and RED2022-134176-T.

This talk is based on joint work with L. M. Anguas, I. Baragaña, F. De Terán, R. Hollister, D. S. Mackey, S. Marcaida, A. Roca, and P. Van Dooren.

Javad Mashreghi, Laval University

Linear polynomial approximation schemes

Abstract. In this presentation, we explore polynomial approximation schemes within function spaces. While Taylor polynomials are fundamental in polynomial approximation theory, there are instances where they may not be the most suitable candidates. Without entering into technical details, we will discuss some summation methods, with a particular emphasis on the well-known Cesaro means. Our focus remains primarily on Hardy and Dirichlet spaces, although other function spaces also make appearances in the discussion. Moreover, within the broader context of super-harmonically weighted Dirichlet spaces, we establish that Fejer polynomials and de la Vallee Poussin polynomials serve as appropriate approximation schemes.

This work has evolved over an extended period and is the result of collaborative efforts with O. El-Fallah, E. Fricain, K. Kellay, H. Klaja, M. Nasri, P. Parisé, M. Shirazi, V. Verreault, T. Ransford, and M. Withanachchi in various combinations.