Objective Priors from Scoring rules for N-mixture models

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N-MIXTURE MODELS

N-mixture models[1] are a class of hierarchical models that are very commonly used to estimate the absolute abundance of a species based on survey sampling.

Count data C_{ij} are obtained during $j = 1, \ldots, T$ sampling occasions at $i = 1, \ldots, M$ sites. The Binomial N-mixture model can be defined as follows:

Abundance process: $N_i \sim g(N_i; \lambda, \gamma)$

Detection process : $C_{ij}|N_i \sim \text{Binomial}(N_i, p)$

where N_i denotes local abundance, λ represents expected abundance, γ represents an optional parameter for dispersion in the abundance process and p represents detection probability.

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[1] Royle, J. A. (2004). N-mixture models for estimating population size from spatially replicated counts.Biometrics, 60(1):108-115.

USE OF N-MIXTURE MODELS

N-mixture models have been used to:

- evaluate the effectiveness of conservation actions (Romano et al., 2017)[2],
- better understand absolute abundance and population dynamics (Studds et al., 2017) [3],

[2] Romano, A., Costa, A., Basile, M., Raimondi, R., Posillico, M., Roger, D. S., Crisci, A., Piraccini, R., Raia, P., Matteucci, G., et al. (2017). Conservation of salamanders in managed forests: Methods and costs of monitoring abundance and habitat selection. Forest ecology and management, 400:12-18

[3] Studds, C. E., Kendall, B. E., Murray, N. J., Wilson, H. B., Rogers, D. I., Clemens, R. S., Gosbell, K., Hassell, C. J., Jessop, R., Melville, D. S., et al. (2017). Rapid population decline in migratory shorebirds relying on yellow sea tidal mudats as stopover sites. Nature communications, 8:14895.

Use of N-mixture models

N-mixture models have been used to:

- predict population responses to differing conservation scenarios (Ladin et al., 2016)[4] and
- forecast shifts in species distribution (Hunter et al., 2017)[5].

[4] Ladin, Z. S., D'Amico, V., Baetens, J. M., Roth, R. R., and Shriver, W. G. (2016). Predicting metapopulation responses to conservation in human-dominated landscapes. Frontiers in Ecology and Evolution, 4:122.

[5] Hunter, E., Nibbelink, N., and Cooper, R. (2017). Divergent forecasts for two salt marsh specialists in response to sea level rise. Animal Conservation, 20(1):20-28.

ISSUES WITH N-MIXTURE MODELS

- Parameter identifiability:
    - Dennis et al. (2015)[6] showed that when detection probability and the number of sampling occasions are small, infinite estimates of absolute abundance can arise.
    - Barker et al. (2018)[7] showed that the loss of individual information resulting from count surveys is critical and causes parameter identifiability issues in Poisson Binomial(P-B) N-mixture models.

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[6] Dennis, E. B., Morgan, B. J. T., and Ridout, M. S. (2015). Computational aspects of N-mixture models. Biometrics, 71(1):237-246.

[7] Barker, R. J., Schofield, M. R., Link, W. A., and Sauer, J. R. (2018). On the reliability of N-mixture models for count data. Biometrics, 74(1):369-377.

ISSUES WITH N-MIXTURE MODELS

- Sensitivity to model assumptions:
  - Link et al. (2018)[8] showed that unmodeled variation in population size over time as well as unmodeled variation in detection probability over time lead to biased estimation of average abundance.
- Model selection:
  - Kéry et al. (2005) [9] showed that Negative-Binomial Binomial(NB-B) N-mixture models may lead to unrealistic high abundance estimates, even though the NB model may be strongly preferred by AIC over Poisson or zero inflated Poisson mixtures.

[8] Link, W. A., Schofield, M. R., Barker, R. J., and Sauer, J. R. (2018). On the robustness of N-mixture models. Ecology, 99(7):1547-1551.

[9] Kéry M., Royle, J. A., and Schmid, H. (2005). Modelling avian abundance from replicated counts using Binomial N-mixture models. Ecological applications, 15(4):1450-1461.

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- We consider fitting N-mixture models within a Bayesian framework.
- An important question in Bayesian inference is: *how does one select a prior distribution* $p(\theta)$?
  - Subjective prior
  - Objective prior
- Objective priors are often used in ecology due to the lack of information about model parameters.

Motivation

- Toribio et al. (2012)[10] used *improper objective priors* to study the robustness of a Bayesian approach to fitting the N-mixture model for pseudo-replicated count data.
- Use of improper priors can result in improper posterior distributions.
- General results that allow one to assess if a given improper prior yields a proper distribution are yet to be found.
- Use of improper priors are also problematic in model selection via Bayes factor.

[10] Toribio, S., Gray, B., and Liang, S. (2012). An evaluation of the Bayesian approach to fitting the N-mixture model for use with pseudo-replicated count data. Journal of Statistical Computation and Simulation, 82(8):1135-1143.

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- We apply a new class of objective priors to N-mixture models: "Objective priors from Scoring rules". These priors may overcome the weakness of improper objective prior as they can be chosen to be proper.
- We test the performance of proper objective priors from scoring rules on P-B N-mixture models by preforming an extensive simulation study that considers both small and large values of λ and p.
- Using proper objective priors from scoring rules, we preforming model selection via Bayes factor to assess whether one can discern between P-B N-mixture models and NB-B N-mixture models.

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Objective priors from Scoring rules

- Leisen et al. (2018)[11] introduced a novel approach that uses proper scoring rules, $S(\theta, p)$, to create a class of objective prior distributions $p(\theta)$. These objective priors are constructed such that

$$S(\theta, p) = \text{constant} \quad \forall\, \theta \in \Theta$$

where Θ denotes the parameter space.

- This construction provide two desirable properties:

  1. The objective prior distributions are not model dependent but based on the sole knowledge of Θ.

  2. The priors can be proper.

[11] Leisen, F., Villa, C., Walker, S. G., et al. (2018). On a class of objective priors from scoring rules. Bayesian Analysis.

Simulation results

Improper objective priors

λ	p	$\hat{\lambda}$	Cov_λ	$RMSE_\lambda$	B_λ	$\hat{p}$	Cov_p	$RMSE_p$	B_p
2	0.25	2.347	95	2.10	0.174	0.223	92	0.449	-0.108
2	0.5	1.974	93	0.229	-0.013	0.498	92	0.149	-0.004
5	0.25	5.341	94	3.347	0.068	0.236	94	0.410	0.400
5	0.5	4.927	96	0.223	-0.015	0.498	95	0.139	-0.003

Objective priors from scoring rule

λ	p	$\hat{\lambda}$	Cov_λ	$RMSE_\lambda$	B_λ	$\hat{p}$	Cov_p	$RMSE_p$	B_p
2	0.25	2.209	98	1.026	0.104	0.264	90	0.390	0.059
2	0.5	2.045	90	0.224	0.022	0.501	94	0.158	0.002
5	0.25	4.94	94	2.106	-0.012	0.258	94	0.341	0.032
5	0.5	4.975	94	0.179	-0.005	0.505	96	0.127	0.010

Simulation results

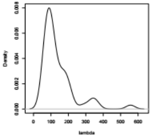
Improper objective priors

λ	p	$\hat{\lambda}$	Cov_λ	$RMSE_\lambda$	B_λ	$\hat{p}$	Cov_p	$RMSE_p$	B_p
100	0.25	107.282	92	1.571	0.073	0.235	92	0.435	-0.058
100	0.5	101.950	94	0.179	0.0195	0.494	94	0.146	-0.012
500	0.25	662.708	90	1.546	0.325	0.192	90	0.446	-0.232
500	0.5	522.873	97	0.1692	0.046	0.474	97	0.139	-0.051
1000	0.25	1149.98	93	1.172	0.150	0.218	93	0.403	-0.127
1000	0.5	1028.65	90	0.199	0.029	0.486	90	0.151	-0.026

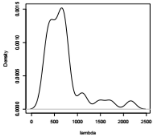
Objective priors from scoring rules

λ	p	$\hat{\lambda}$	Cov_λ	$RMSE_\lambda$	B_λ	$\hat{p}$	Cov_p	$RMSE_p$	B_p
100	0.25	103.597	94	1.020	0.035	0.241	94	0.397	-0.034
100	0.5	100.849	94	0.168	0.008	0.497	94	0.140	-0.005
500	0.25	626.643	93	0.937	0.253	0.199	93	0.405	-0.204
500	0.5	514.364	100	0.151	0.028	0.488	100	0.129	-0.023
1000	0.25	1069.87	93	0.707	0.070	0.235	93	0.371	-0.058
1000	0.5	1013	97	0.170	0.014	0.494	97	0.137	-0.010

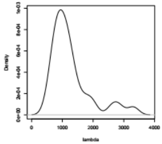
DENSITY PLOTS FOR POSTERIOR MEDIAN OF λ USING OBJECTIVE PRIORS FROM SCORING RULES.



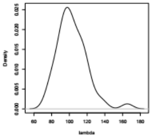
(a) $\lambda = 100, p = 0.25$



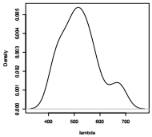
(b) $\lambda = 500, p = 0.25$



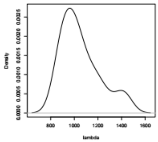
(c) $\lambda = 1000, p = 0.25$



(a) $\lambda = 100, p = 0.5$



(b) $\lambda = 500, p = 0.5$



(c) $\lambda = 1000, p = 0.5$

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Model selection via Bayes Factor

Bayes Factor (BF)

Let M_1: P-B N-mixture model with $\boldsymbol{\theta} = (\lambda, p)$ and M_2: NB-B N-mixture model with $\boldsymbol{\phi} = (p, r, s)$, the Bayes factor (BF_{12}) can be defined as:

$$\text{BF}_{12} = \frac{p(y|M_1)}{p(y|M_2)} = \frac{\int p(y|\theta, M_1)p_1(\theta)d\theta}{\int p(y|\phi, M_2)p_2(\phi)d\phi}$$

We use the naive Monte Carlo to estimate $p(y|M_1)$ and $p(y|M_2)$ respectively.

Results

Table: Simulation results for 100 runs where the true model is the P-B N-mixture model(M_1).

λ	p	$\mathrm{BF}_{12} > 1$	$\mathrm{Min}_{\mathrm{BF}_{12}}$	$\mathrm{Max}_{\mathrm{BF}_{12}}$
5	0.25	97	0.34	13.08
5	0.5	34	$5.24e^{-05}$	11.17
5	0.9	7	$2.91e^{-17}$	10.550

Results

Table: Simulation results for 100 runs where the true model is the NB-B N-mixture model(M_2).

p	r	s	$\text{BF}_{12} < 1$	$\text{Min}_{\text{BF}_{12}}$	$\text{Max}_{\text{BF}_{12}}$
0.25	2	0.5	3	0.446	9.934
0.5	2	0.5	43	$1.72e^{-11}$	10.927
0.9	2	0.5	83	0	11.440

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1. Objective priors from scroing rules produce results similar to improper objective priors for N-mixture models. Being proper objective priors, they allow for use in N-mixture model without the need to assess whether the posterior is proper and enables model selection via Bayes factor.

2. Based on Bayes factor, it seems the ability to discern between the P-B N-mixture model and the NB-B N-mixture model depends on the detection probability.

Thank you!
Any questions/comments?